

AdaBoost Exercises

1. AdaBoost

The AdaBoost algorithm can be summarized as follows:

During training, repeat the following for T iterations:

- Create a training set by sampling with replacement according to the sample weights \mathbf{w} (initially all weights are equal.)
- Train a base C_i classifier on the sampled data.
- Apply the classifier to all training data and calculate the weighted error:

$$\epsilon_i = \sum_{j=1}^N w_j \delta(C_i(x_j) \neq y_j)$$

- Calculate the importance of the new classifier:

$$\alpha_i = \frac{1}{2} \ln \frac{1 - \epsilon_i}{\epsilon_i}$$

- Update the sample weights according to:

$$w_j^{(i+1)} = \frac{w_j^{(i)}}{Z_i} \times \begin{cases} e^{-\alpha_i} & \text{if } C_i(x_j) = y_j \\ e^{\alpha_i} & \text{if } C_i(x_j) \neq y_j \end{cases}$$

where Z_i is selected to make the weights sum to 1.

At prediction time:

$$C^*(x) = \arg \max_y \sum_1^T \alpha_i \delta(C_i(x) = y)$$

Consider the following training set and initial weights:

x:	.1	.3	.6	.9
y:	-1	1	1	-1
\mathbf{w}_0 :	.25	.25	.25	.25

- (a) Assume that C_0 has been created, with the following result:

x:	.1	.3	.6	.9
correct?	N	Y	Y	Y

What is ϵ_0 ? What is α_0 ? Show your work.

(b) Fill in the table below with the updated weights. Show your work.

x:	.1	.3	.6	.9
\mathbf{w}_1 :				

(c) Now assume that C_1 performs as follows:

x:	.1	.3	.6	.9
correct?	Y	Y	Y	N

Calculate ϵ_1 , α_1 and the updated weights:

x:	.1	.3	.6	.9
\mathbf{w}_2 :				

(d) Given that C_0 classifies a particular point as -1 and C_1 classifies it as 1, what class will be selected by the ensemble? Justify your answer.