CS445

Nathan Sprague

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Expectation, Variance

- Expectation (continuous) (also referred to as the "mean" or "first moment")

$$\mu = \mathbb{E}[x] = \int x f(x) dx$$

- Expectation (discrete)

$$\mathbb{E}[X] = \sum_{1}^{n} P(x_i) x_i$$

- Variance (also referred to as the "second moment")

$$\sigma^2 = \mathbb{E}[(x - \mathbb{E}[x])^2]$$

Quiz

$$\mathbb{E}[X] = \sum_{1}^{n} P(x_i)x_i$$

$$\sigma^2 = \mathbb{E}[(x - \mathbb{E}[x])^2]$$

Imagine we are rolling a four-sided die. The following probability distribution describes the probability for each number that we could roll:

$P(X = 1) = .7$
$P(X = 2) = .1$
$P(X = 3) = .1$
$P(X = 4) = .1$

What is the expected value of this distribution? What is the variance?

Sample Mean and Variance

Expectation and variance are properties of distributions. We can also calculate the **sample mean** and the **sample variance** for a given data set:
$\{x_1, x_2, \ldots, x_n\}$.

- Sample mean

$$m = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- Sample variance

$$s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - m)^2$$

Normal Distribution

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

(*Normal* because of the central limit theorem.)
All distributions