# AdaBoost Exercises

1. **AdaBoost**

   The AdaBoost algorithm can be summarized as follows[1]:

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   During training, repeat the following for $T$ iterations:

   - Create a training set by sampling with replacement according to the sample weights $\mathbf{w}$ (initially all weights are equal.)

   - Train a base $C_i$ classifier on the sampled data.

   - Apply the classifier to all training data and calculate the weighted error:

   $$\epsilon_i = \sum_{j=1}^{N} w_j \delta \left( C_i(x_j) \neq y_j \right)$$

   - Calculate the importance of the new classifier:

   $$\alpha_i = \frac{1}{2} \ln \frac{1 - \epsilon_i}{\epsilon_i}$$

   - Update the sample weights according to:

   $$w_j^{(i+1)} = \frac{w_j^{(i)}}{Z_i} \times \begin{cases} e^{-\alpha_i} & \text{if } C_i(x_j) = y_j \\ e^{\alpha_i} & \text{if } C_i(x_j) \neq y_j \end{cases}$$

   where $Z_i$ is selected to make the weights sum to 1.

   At prediction time:

   $$C^*(x) = \arg\max_y \sum_{1}^{T} \alpha_i \delta(C_i(x) = y)$$

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   Consider the following training set and initial weights:

   | x: | .1 | .3 | .6 | .9 |
   |---|---|---|---|---|
   | y: | -1 | 1 | 1 | -1 |
   | $\mathbf{w}_0$: | .25 | .25 | .25 | .25 |

   (a) Assume that $C_0$ has been created, with the following result:

   | x: | .1 | .3 | .6 | .9 |
   |---|---|---|---|---|
   | correct? | N | Y | Y | Y |

   What is $\epsilon_0$? What is $\alpha_0$? Show your work.

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[1]This is essentially the same as Algorithm 4.6 in our textbook, but fixes some minor errors.

(b) Fill in the table below with the updated weights. Show your work.

| x: | .1 | .3 | .6 | .9 |
|---|---|---|---|---|
| $\mathbf{w}_1$: | | | | |

(c) Now assume that $C_1$ performs as follows:

| x: | .1 | .3 | .6 | .9 |
|---|---|---|---|---|
| correct? | Y | Y | Y | N |

Calculate $\epsilon_1$, $\alpha_1$ and the updated weights:

| x: | .1 | .3 | .6 | .9 |
|---|---|---|---|---|
| $\mathbf{w}_2$: | | | | |

(d) Given that $C_0$ classifies a particular point as -1 and $C_1$ classifies it as 1, what class will be selected by the ensemble? Justify your answer.