## Bias/Variance Quiz

## 1. Interpreting the Bias Variance Formula

The two figures below illustrate the result of fitting many data sets drawn from a single distribution. The data distribution and the sizes of the data sets are the same for both figures, but the parameters of the model differs.



Recall that the bias variance decomposition can be expressed as follows:

$$
\begin{align*}
E\left[(y-\hat{f}(x))^{2}\right]= & (E[\hat{f}(x)]-f(x))^{2}+ \\
& E\left[(\hat{f}(x)-E[\hat{f}(x)])^{2}\right]+  \tag{1}\\
& \sigma^{2}
\end{align*}
$$

(a) Label $E[\hat{f}(x)], \hat{f}(x)$, and $f(x)$ in each of the figures above. If the item you are labeling appears more than once, just pick one instance to label.
(b) Notice that the bias, $\operatorname{Bias}(x)=E[\hat{f}(x)]-f(x)$, is a function of $x$ and may be either positive or negative. What is the approximate value of $\operatorname{Bias}(x)$ for $x=.3$ in the figure on the left?
(c) Which model appears to have a higher average squared bias? Which model appears to have a higher variance? Justify your answer.
2. Now let's re-run our experiment under a different scenario using the model represented in the figure on the right above (a degree-10 polynomial):

(a) What do you think changed between the two experiments? Justify your answer.
3. Imagine we are now able to collect data using less noisy sensor. Where will this have the biggest impact on expected squared error? The Bias term, the Variance term or the irreducible error term? Justify your answer.

