

# Lowest-Cost-First Search (Dijkstra's Algorithm)

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(Some material from Artificial Intelligence: Foundations of Computational Agents 2<sup>nd</sup> ed., Poole and Mackworth, 2017.)

# Path Nodes

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<b>PathNode</b>
+state: Node +parent: PathNode +path_cost: Number
+PathNode(State: Node, parent: PathNode, step_cost: Number) <i>path_cost = step_cost + parent.path_cost</i>

- For our purposes:
  - A “Node” corresponds to a state in the problem. There may be infinitely many Nodes.
  - A “PathNode” is a data type that represents a partial solution and the associated cost.
- (This is not standard terminology. Our textbook does not discuss PathNodes; in Russel & Norvig, they are just called “Nodes”.)

# Depth First Search (With PathNodes!)

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```
procedure DepthFirstSearch(G, S, goal)
```

**Inputs**

*G*: graph with nodes *N* and arcs *A*

*s*: start node

*goal*: Boolean function of states

*cost*: The cost function for arcs

**Output**

path from a member of *S* to a node for which *goal* is true

or  $\perp$  if there are no solution paths

**Local**

*Frontier*: a stack of PathNodes

*Explored*: set of nodes that have been expanded

```
Frontier  $\leftarrow$  Empty Stack
```

```
Frontier.push(PathNode(s, None, 0))
```

```
Explored  $\leftarrow$  {}
```

```
while (Frontier is not empty)
```

```
  Pop pNode from Frontier
```

```
  Explored  $\leftarrow$  Explored  $\cup$  {pNode.state}
```

```
  if ( goal(pNode.state)) then
```

```
    return The path represented by pNode
```

```
  For all {(nk, n) : (nk, n)  $\in$  A  $\wedge$  n  $\notin$  Frontier  $\wedge$  n  $\notin$  Explored}
```

```
    Frontier.push(PathNode(n, pNode, cost(nk, n)))
```

```
return  $\perp$ 
```

# Lowest-Cost-First Search (Dijkstra's Algorithm)

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**procedure** *LowestCostSearch*(*G, S, goal*)

**Inputs**

*G*: graph with nodes *N* and arcs *A*

*s*: start node

*goal*: Boolean function of states

*cost*: The cost function for arcs

**Output**

path from a member of *S* to a node for which *goal* is true  
or  $\perp$  if there are no solution paths

**Local**

*Frontier*: a Priority Queue of PathNodes ordered by cost

*Explored*: set of nodes that have been expanded

*Frontier*  $\leftarrow$  Empty Stack

*Frontier.enqueue*(PathNode(*s*, None, 0))

*Explored*  $\leftarrow$  {}

**while** (*Frontier* is not empty)

Pop *pNode* from *Frontier*

*Explored*  $\leftarrow$  *Explored*  $\cup$  {*pNode.state*}

**if** ( *goal*(*pNode.state*)) **then**

**return** The path represented by *pNode*

For all { $\langle n_k, n \rangle : \langle n_k, n \rangle \in A \wedge n \notin \text{Frontier} \wedge n \notin \text{Explored}$ }

*Frontier.enqueue*(PathNode(*n*, *pNode*, *cost*( $\langle n_k, n \rangle$ )))

**return**  $\perp$

# Lowest-Cost-First Search (Dijkstra's Algorithm)

```
procedure LowestCostSearch(G, S, goal)
```

## Inputs

*G*: graph with nodes *N* and arcs *A*  
*s*: start node  
*goal*: Boolean function of states  
*cost*: The cost function for arcs

## Output

path from a member of *S* to a node for which *goal* is true  
or  $\perp$  if there are no solution paths

## Local

*Frontier*: a Priority Queue of PathNodes ordered by cost  
*Explored*: set of nodes that have been expanded

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Frontier  $\leftarrow$  Empty Stack
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Frontier.enqueue(PathNode(s, None, 0))
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Explored  $\leftarrow$  {}
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  Pop pNode from Frontier
```

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  Explored  $\leftarrow$  Explored  $\cup$  {pNode.state}
```

```
  if ( goal(pNode.state) ) then
```

```
    return The path represented by pNode
```

```
  For all {(nk, n) : (nk, n)  $\in$  A  $\wedge$  n  $\notin$  Frontier  $\wedge$  n  $\notin$  Explored}
```

```
    Frontier.enqueue(PathNode(n, pNode, cost((nk, n)))
```

```
return  $\perp$ 
```

Missing detail:  
if *s* is already in  
the frontier,  
then it's  
PathNode  
should be  
replaced if the  
new node  
would have a  
lower path  
cost.

