CS444

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Matrices and Vectors

- A matrix is an $m \times n$ array of values, usually denoted with an upper-case letter.
- Example 2×3 matrix:

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix}$$

- A column vector is a $m \times 1$ matrix, usually denoted with a lower-case letter:
- Example four-dimensional vector:

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Transpose

- Taking the transpose of a matrix exchanges the rows with the columns:

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \quad A^T = \begin{bmatrix} a_{00} & a_{10} \\ a_{01} & a_{11} \\ a_{02} & a_{12} \end{bmatrix}$$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad x^T = \begin{bmatrix} x_0 & x_1 & x_2 & x_3 \end{bmatrix}$$

Addition and Subtraction

- Addition and subtraction require compatible shapes:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Multiplying a Matrix by a Scalar

$$e\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ea & eb \\ ec & ed \end{bmatrix}$$

Matrix Multipication

- Inner dimensions must agree 2×3 with 3×2 is OK:

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} p & q \\ r & s \\ t & u \end{bmatrix} = \begin{bmatrix} ap + br + ct & aq + bs + cu \\ dp + er + ft & dq + es + fu \end{bmatrix}$$

- 3×2 with 3×2 is not:

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} p & q \\ r & s \\ t & u \end{bmatrix} = UNDEFINED$$

Identity Matrix

- Square matrix with ones on the diagonal:

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- It's called the identity matrix because AI = A = IA
- Example

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \times 2 + 0 \times 3 + 0 \times 5 \\ 0 \times 2 + 1 \times 3 + 0 \times 5 \\ 0 \times 2 + 0 \times 3 + 1 \times 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Matrix Inverse (1/2)

- Depending on A, there may or may not exist a matrix A^{-1} such that $AA^{-1} = I$
- Useful for solving systems of linear equations: $Ax = b$
- In "normal" algebra we would divide both sides by A (which is the same as multiplying by $\frac{1}{A}$, which is the same as multiplying by A^{-1}):

$$\frac{Ax}{A} = \frac{b}{A}$$

$$x = \frac{b}{A}$$

Matrix Inverse (2/2)

- We can do exactly the same thing in linear algebra:

$$A^{-1}Ax = A^{-1}b$$

$$Ix = A^{-1}b$$

$$x = A^{-1}b$$

- Unfortunately *finding* the matrix inverse is a chore