CS444 Predicate Logic Proofs

1. Consider the following knowledge base:[1]

   (a) $has\_access(X, library) \leftarrow student(X)$.

   (b) $has\_access(X, library) \leftarrow faculty(X)$.

   (c) $has\_access(X, library) \leftarrow has\_access(Y, library) \wedge parent(Y, X)$.

   (d) $faculty(diane)$.

   (e) $faculty(ming)$.

   (f) $student(william)$.

   (g) $parent(diane, karen)$.

   (h) $parent(diane, robin)$.

   (i) $parent(karen, todd)$.

   Provide an SLD derivation of the query `ask` $has\_access(todd, library)$. Quoting from our textbook:

   > The main difference between this and the propositional top-down proof procedure is that, for clauses with variables, the proof procedure must take copies of clauses from the knowledge base. The copying renames the variables in the clause with new names. This is both to remove name clashes between variables and because a single proof may use different instances of a clause.

   I've started the derivation for you, by creating the answer clause, and making the first two substitutions. For the sake of consistency, your derivation should always attempt to match the left-most atom in the body of the current answer clause. See figure 13.5 in our textbook for an example of appropriate formatting.

   $yes() \leftarrow has\_access(todd, library)$

   Resolve with (c) above using the substitution $\{X/todd\}$.

   $yes() \leftarrow has\_access(Y_1, library) \wedge parent(Y_1, todd)$

   Resolve with (c) above using the substitution $\{X/Y1\}$.

   $yes() \leftarrow has\_access(Y_2, library) \wedge parent(Y_2, Y_1) \wedge parent(Y_1, todd)$

[1] This is a slightly simplified version of question 12.7 from *Artificial Intelligence: Foundations of Computational Agents*, David Poole and Alan Mackworth, Cambridge University Press, 2010.

2. Express the following knowledge base in first order logic using the predicates
   $T(x, y)$ - x is taller than y.
   $S(x, y)$ - x can see over the head of y.
   $C(x)$ - x can ride the coaster.
   $H(x)$ - x has a ticket.

   - Anyone who has a ticket and is taller than Alice can ride the roller coaster.
   - Anyone who can see over another person's head is taller than that person.
   - Bob can see over Alice's head.
   - Bob has a ticket or no one has a ticket.
   - Alice has a ticket.

3. Convert your first order knowledge base from the previous question to clausal form. (Conjunctive normal form with all quantifiers removed).

4. Construct a resolution proof that Bob can ride the roller coaster.