CS444 Markov Model Exercises

1. Consider the following Markov Model:

$$P(X_0 = sick) = .5$$
$$P(X_0 = healthy) = .5$$

X_{t-1}	X_t	$P(X_t \mid X_{t-1})$
healthy	*healthy*	.9
healthy	*sick*	.1
sick	*healthy*	.3
sick	*sick*	.7

Use the "Mini-Forward" algorithm described in the video,

$$P(x_t) = \sum_{x_{t-1}} P(x_t \mid x_{t-1}) P(x_{t-1})$$

to complete the following exercises:

- Calculate the probability distribution for X_1.
- Calculate the probability distribution for X_2.

2. Use the AIspace Belief Network Tool to create a belief network corresponding to the Markov Model above. (In principle this Markov Model is infinitely large. You only need to include the first three time steps.) Use the model to verify that your calculations above were correct.

3. Modify the Bayes net from the previous question to be a Hidden Markov Model with an observation variable named Fever.

The conditional probability distribution for your fever variable should be as follows:

X_t	$Fever_t$	$P(Fever_t \mid X_t)$
healthy	*fever*	.2
healthy	$\neg fever$	.8
sick	*fever*	.4
sick	$\neg fever$	.6

Experiment with setting observations for one or both of the observation variables. Does the impact on the distribution of the hidden variables seem reasonable?

4. Given enough time steps, most Markov models will converge to a *stationary distribution*. This is a fixed distribution that doesn't depend on the initial state of the system. Finding the stationary distribution can be useful. For example, if the example from question 1 is an accurate model of disease, the stationary distribution will tell us what percentage of a person's life is spent sick.

   At the stationary distribution probabilities don't change with additional time steps:

$$P(x_\infty) = P(x_{\infty+1}) = \sum_{x_\infty} P(x_{\infty+1} \mid x_\infty)P(x_\infty)$$

   This observation can be used to solve for $P(x_\infty)$ as a series of n equations in n unknowns. For example:

$$\begin{aligned} P(x_\infty = \mathit{healthy}) =& P(x_{\infty+1} = \mathit{healthy} \mid x_\infty = \mathit{healthy})P(x_\infty = \mathit{healthy}) \\ &+ P(x_{\infty+1} = \mathit{healthy} \mid x_\infty = \mathit{sick})P(x_\infty = \mathit{sick}) \\ P(x_\infty = \mathit{sick}) =& P(x_{\infty+1} = \mathit{sick} \mid x_\infty = \mathit{healthy})P(x_\infty = \mathit{healthy}) \\ &+ P(x_{\infty+1} = \mathit{sick} \mid x_\infty = \mathit{sick})P(x_\infty = \mathit{sick}) \end{aligned}$$

   Find the stationary distribution for the Markov Model in question 1.