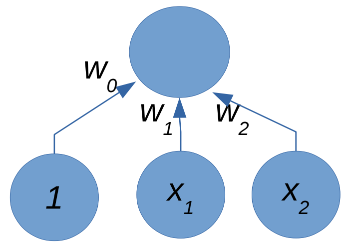
CS444 Gradient Descent Exercises

1. Consider the following, very simple, "neural network":



where the activation of the output unit is just the dot product between the input and the weight vector: $h(\mathbf{x}) = \mathbf{w}^T\mathbf{x}$

Assume that the current weights are $w_0 = 0, w_1 = 1, w_2 = .5$ and we have the following set of examples E:

$$\mathbf{X} = \begin{bmatrix} 1 & 2 \\ -2 & 5 \\ 0 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 1 \\ 6 \\ 1 \end{bmatrix}$$

Where each row in $\mathbf{X}$ represents a data point and each entry in $\mathbf{y}$ represents the corresponding target value.

- What output will this network produce for $\mathbf{x} = \begin{bmatrix} 2 & 3 \end{bmatrix}^T$?

- Calculate

$$Error(\mathbf{w}) = \frac{1}{2}\sum_{e \in E}(y_e - \mathbf{w}^T\mathbf{x}_e)^2$$

  for this data set.

- Can you find a set of weights that would result in less error?

2. Calculate the partial derivatives of $f(x, y) = \sqrt{x^2 + y^2}$. Show your work.

3. Recall the least-squares error function for linear regression:

$$Error(\boldsymbol{w}) = \frac{1}{2}(y - \boldsymbol{w}^T\boldsymbol{x})^2$$

(This is the error associated with a single training sample with input $\boldsymbol{x}$ and target value y.)

This objective function encodes a belief that bigger errors are *much* worse than smaller errors: in particular, that the penalty for making a mistake should grow with the square of the magnitude of the mistake. That seems reasonable[1], but it isn't the only possible error function. One problem with using a squared error function is that outliers can have a big impact on the result.

An alternative that is more robust to outliers is the absolute error (or L1 error):

$$\begin{aligned} Error(\boldsymbol{w}) = & |y - \boldsymbol{w}^T\boldsymbol{x}| \\ & |y - (w_0x_0 + ... + w_ix_i + ... + w_nx_n)| \end{aligned}$$

Your goal in this exercise is to develop a gradient-descent learning rule for this new objective function. (It will be helpful to know that $\frac{d}{dx}|u| = \frac{u}{|u|} \times \frac{d}{dx}u$)

Your final rule should have the form:

$$w_i \leftarrow w_i - \eta???$$

[1] In fact, there are good statistical reasons for using this error function, particularly if the noise in the data is normally distributed.

4. Perform one round of gradient descent updates using the data set in question 1) and $\eta = .1$. Recall that the learning rule for our sum-squared error term is:

$$\mathbf{w} \leftarrow \mathbf{w} + \eta \sum_{e \in E} (y_e - \mathbf{w}^T \mathbf{x}_e) \mathbf{x}_e$$

Recalculate the the error with the new weights to confirm it went down.