Name: ______________________________

State Estimation Exercises

1. Recursive State Estimation

   In the game Plinko, a disk is placed at the top of a tilted board. It slides down the board, bouncing off any pins in its path. Players gamble on where the disk will end up when it reaches the bottom.

   Consider a simplified version of the game in which the disk moves downward on an $n \times n$ grid. On each time step, the disk moves to the grid space directly below it unless that space is occupied by a pin (represented by large black dots in the figures below). If the disk's path is obstructed by a pin, then it randomly ends up on either the left or right side of the pin with a .5 probability of each outcome.

   Assuming that the disk is equally likely to enter the board at any of the n positions in the top row, your task is to calculate the probability distribution over possible disk positions for each time step in the game. Times $T = 0$ and $T = 1$ are already completed; you need to fill in values for the remaining time steps (on the next page).

   As a reminder, the equation for recursive state updates in a Markov Model can be expressed as follows:

$$\boldsymbol{P}(X_{t+1}) = \sum_{x_t} \boldsymbol{P}(X_{t+1}|x_t)P(x_t)$$

   Note that this equation doesn't include observations.

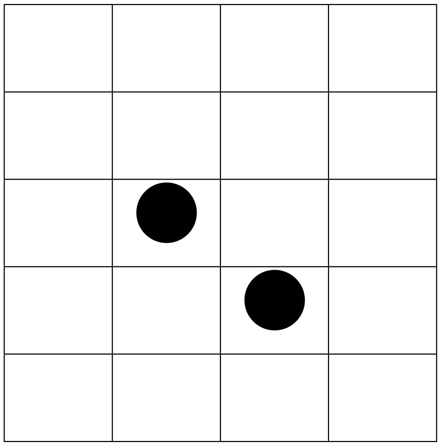
   (The position labeled J is the jackpot position, you can ignore it.)

.25	.25	.25	.25
0	0	0	0
0	●	0	0
0	0	●	0
0	0 (J)	0	0

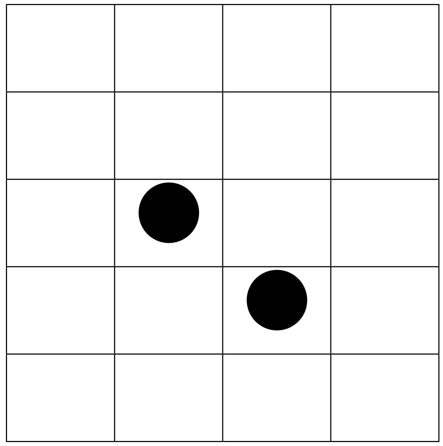
$T = 0$

0	0	0	0
.25	.25	.25	.25
0	●	0	0
0	0	●	0
0	0	0	0

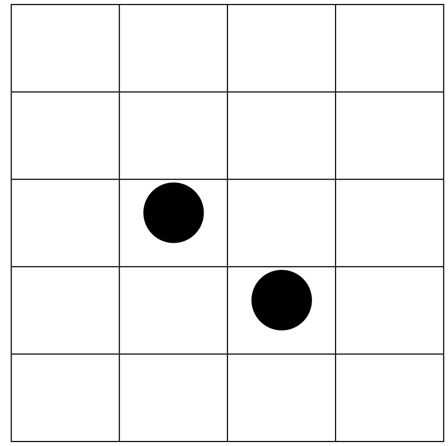
$T = 1$



$T = 2$



$T = 3$



$T = 4$

2. In Agility Plinko, the goal is to catch the disk as it drops out of the bottom of the board. Unfortunately, there is a semi-opaque screen over the board that makes it hard to see the disk. The observation model is as follows: there is a .7 probability that the disk will be observed in its true location. The remaining .3 probability is evenly destributed between the other open positions in the same row. Recompute the probability distribution from the previous question given the following series of observations (starting at $T = 1$): (1,2) (2,3), (3,3), (4,2). (These observations refer to grid locations, where the (0,0) is the upper left grid position.

   As a reminder, the state update equation (with observations) can be written as follows:

$$P(X_{t+1} \mid e_{1:t}) = \sum_{x_t} P(X_{t+1} \mid x_t) P(x_t \mid e_{1:t})$$

   And the observation update can be written like this:

$$P(X_{t+1} \mid e_{1:t+1}) = \alpha P(e_{t+1} \mid X_{t+1}) P(X_{t+1} \mid e_{1:t})$$

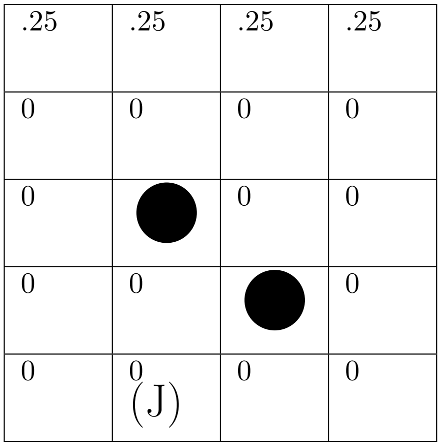
   These two equations can be rewritten with belief distributions as:

   (Belief distribution before observation)

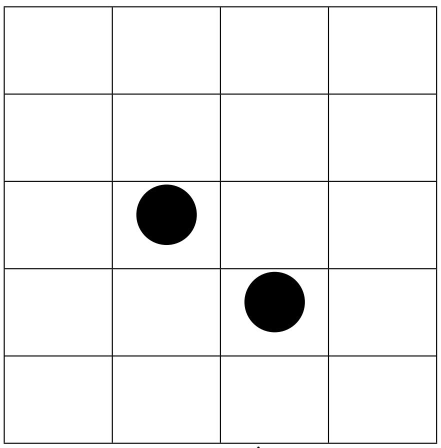
$$B'(X_{t+1}) = \sum_{x_t} P(X_{t+1}|x_t) B(x_t)$$

   (Belief distribution after observation)

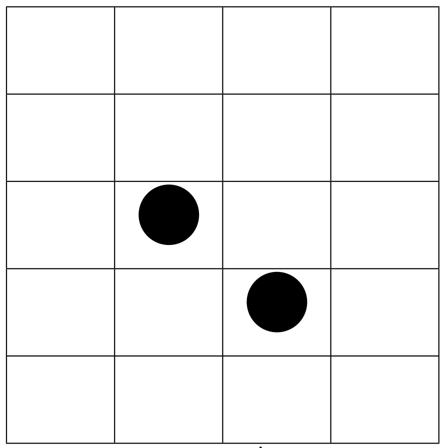
$$B(X_{t+1}) = \alpha P(e_{t+1}|X_{t+1}) B'(X_{t+1})$$



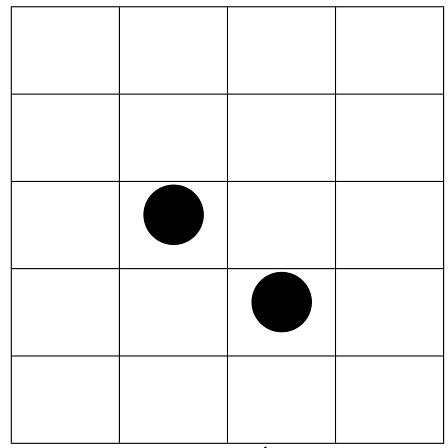
$T = 0$



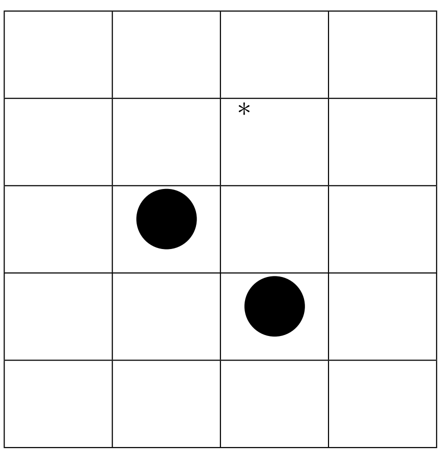
$T = 1'$



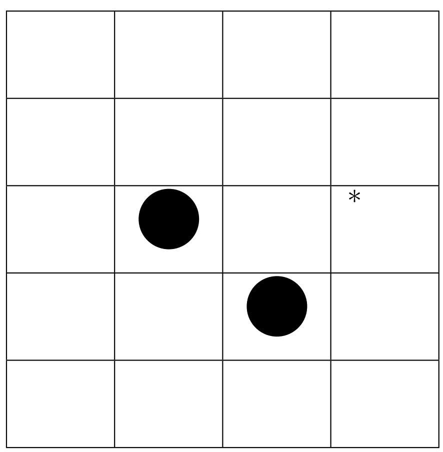
$T = 2'$



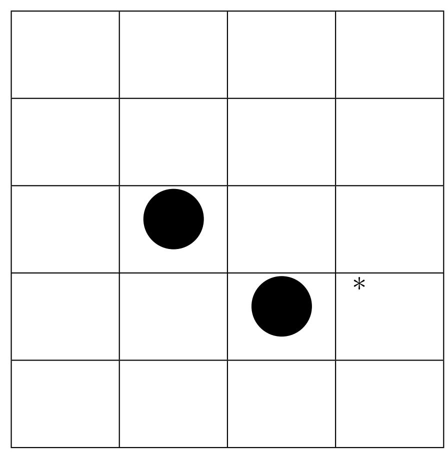
$T = 3'$



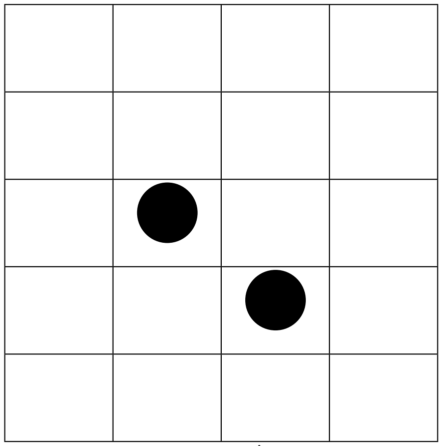
$T = 1$



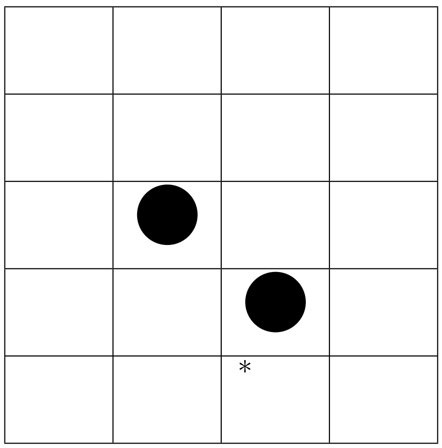
$T = 2$



$T = 3$



$T = 4'$



$T = 4$