CS444 Probability Review

Useful identities:

Definition of conditional probability	$P(a \mid b) = \dfrac{P(a \wedge b)}{P(b)}$
Product rule	$P(a \wedge b) = P(a \mid b)P(b)$
Bayes' rule	$P(h \mid e) = \dfrac{P(e \mid h)P(h)}{P(e)}$
Law of total probability	$P(e) = \sum_{h \in H} P(e \mid h)P(h)$
Independence	$P(X, Y) = P(X)P(Y)$ iff X and Y are independent.

1. Assume that A, B, and C, are three mutually independent random variables, and that $P(A = \mathit{true}) = .4$, $P(B = \mathit{true}) = .3$, $P(C = \mathit{true}) = .9$. Find the probabilities that:

   (a) All three are true.

   (b) Exactly two of the three are true.

   (c) None of the three is true.

   (d) Fill in the full joint probability distribution for these three variables. (Make sure the rows sum to 1!)

A	B	C	Probability
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

2. Compute the quantities below by referring to the following joint probability distribution:

A	B	C	Probability
T	T	T	.1
T	T	F	.05
T	F	T	.01
T	F	F	.02
F	T	T	.3
F	T	F	.2
F	F	T	.2
F	F	F	.12

   (a) $P(a \wedge b \wedge \neg c)$

   (b) $P(\neg b)$

   (c) $P(\neg b \vee c)$

   (d) $P(c|\neg b)$

3. Assume that we are building a probabilistic model involving 32 Boolean random variables. We need to be able to represent the full joint probability distribution across all variables. Roughly how much memory will be required to represent the distribution assuming that:

   (a) All 32 variables are mutually independent.

   (b) Each variable is dependent on every other variable.

4. You work at the airport as a passenger screener. You know the following things:

   (a) One passenger in one hundred tries to sneak a bomb through screening.

   (b) The conditional probability that the alarm will go off, given that the passenger has a bomb is .5.

   (c) The conditional probability that the alarm will go off given that the passenger does not have a bomb is .1.

   The alarm goes off. What is the probability that the passenger has a bomb?