Linear Algebra and Python

What Are We Missing...

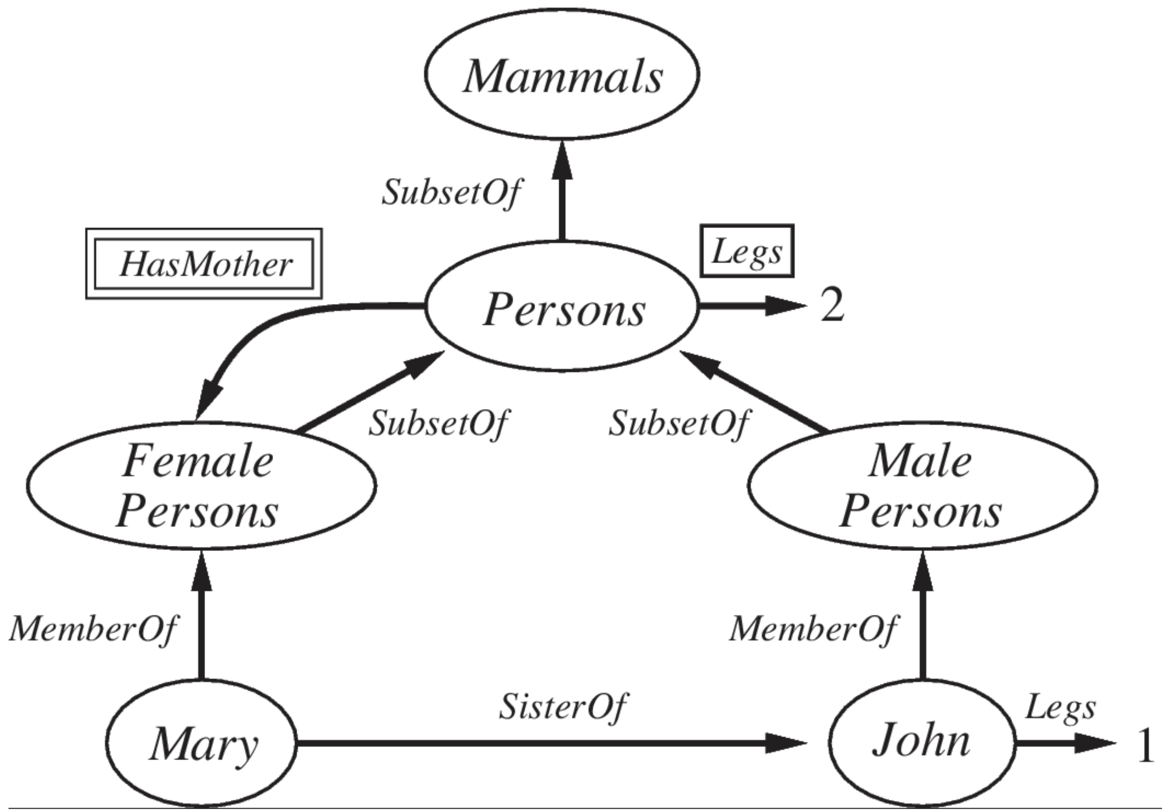
- Chapter 10, Classical Planning...

$Init(On(A, Table) \wedge On(B, Table) \wedge On(C, A)$
$\quad \wedge Block(A) \wedge Block(B) \wedge Block(C) \wedge Clear(B) \wedge Clear(C))$
$Goal(On(A, B) \wedge On(B, C))$
$Action(Move(b, x, y),$
$\quad$ PRECOND: $On(b, x) \wedge Clear(b) \wedge Clear(y) \wedge Block(b) \wedge Block(y) \wedge$
$\qquad (b \neq x) \wedge (b \neq y) \wedge (x \neq y),$
$\quad$ EFFECT: $On(b, y) \wedge Clear(x) \wedge \neg On(b, x) \wedge \neg Clear(y))$
$Action(MoveToTable(b, x),$
$\quad$ PRECOND: $On(b, x) \wedge Clear(b) \wedge Block(b) \wedge (b \neq x),$
$\quad$ EFFECT: $On(b, Table) \wedge Clear(x) \wedge \neg On(b, x))$

Figure 10.3 A planning problem in the blocks world: building a three-block tower. One solution is the sequence $[MoveToTable(C, A), Move(B, Table, C), Move(A, Table, B)]$.

What Are We Missing...

- Chapter 12, Knowledge Representation



Linear Algebra Basics

- Linear algebra allows concise manipulation of multi-dimensional data.
- Matrix – M x N array of values, indicated with uppercase bold characters.
  - Example 3 x 2 matrix:

$$\boldsymbol{X} = \begin{bmatrix} x_{1,1} & x_{1,2} \\ x_{2,1} & x_{2,2} \\ x_{3,1} & x_{3,2} \end{bmatrix}$$

- Vector – M x 1 matrix, indicated with lowercase bold characters.

$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

Arithmetic

- Matrix arithmetic requires compatibility in dimensions

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$$

Matrix Multiplication

- Scalar times a matrix:

$$e\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ea & eb \\ ec & ed \end{bmatrix}$$

- Matrix times a matrix (2 x 3 and 3 x 2 = OK)

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix} = \begin{bmatrix} ag+bi+ck & ah+bj+cl \\ dg+ei+fk & dh+ej+fl \end{bmatrix}$$

- 3 x 2 and 3 x 2 = NOT OK

$$\begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix}\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} = \texttt{Undefined}$$

Miscellany

- Transpose – swap rows and columns:

$$\boldsymbol{X} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \qquad \boldsymbol{X}^{\boldsymbol{T}} = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

- Dot Product:

$$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}^{\boldsymbol{T}} \boldsymbol{b} = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \sum_{i=1}^{3} a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3$$

The Dot Product and Geometry

- Two vectors are orthogonal (perpendicular) if their dot product is zero.
- The length of a vector, denoted $\|\boldsymbol{x}\|$, is $\sqrt{\boldsymbol{x} \cdot \boldsymbol{x}}$
- The angle θ , between two vectors x and y is given by

$$\cos\theta = \frac{\boldsymbol{x} \cdot \boldsymbol{y}}{\|\boldsymbol{x}\| \, \|\boldsymbol{y}\|}$$

- The projection of $\boldsymbol{x}$ onto $\boldsymbol{y}$ is given by:

$$\frac{(\boldsymbol{x} \cdot \boldsymbol{y})\, \boldsymbol{y}}{\|\boldsymbol{y}\|^2}$$

Matrix Inverse

- The identity matrix is a square matrix with 1's on the diagonal:

$$\boldsymbol{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- So called because: $\boldsymbol{A}\,\boldsymbol{I} = \boldsymbol{A}$
- Depending on ***A***, there may or may not exist a matrix $\boldsymbol{A}^{-1}$ such that $\boldsymbol{A}\,\boldsymbol{A}^{-1} = \boldsymbol{I}$, this is the inverse.