Heuristic Search

The Story So Far...

- The central problem of this course:

$$\arg\max_X \; Smartness(X)$$

  - Possibly with some constraints on *X*.

- (Alternatively: $\arg\min_X \; Stupidness(X)$)

Properties of *Smartness(X)*

- Possible categories for $\arg\max_X \; Smartness(X)$, in decreasing order of desirability:
  - Efficient closed-form solution.
    - Linear Regression etc.
  - Differentiable and convex.
    - Quadratic programming (SVMs) etc.
  - Differentiable and non-convex.
    - Multi-layer Perceptrons etc.
  - Non differentiable
    - ??

Hill Climbing

- Assume that our search problem allows us to take one of a fixed number of moves.
- Each move transforms the current state to a successor state.
- States can be evaluated by an objective function.
- Hill Climbing Algorithm:
  - Start in an arbitrary state.
  - Choose the move that results in the best successor state.
  - Repeat until converged.

Properties of Hill Climbing

- Easy to program.
- Often finds good solutions quickly.
- Very susceptible to local maxima.
- Can be inefficient if many moves are available.
- Lots of variations:
  - Stochastic hill climbing – randomly choose an uphill move.
  - Random restart hill climbing – redo search until satisfied with result.
  - Simulated Annealing...

Simulated Annealing

- Avoiding local maxima requires us to take some steps downhill.
- Simulated Annealing attempts to find the right trade off between uphill and downhill moves:

Start in a random state

** Select a random move

* If the move results in improvement, keep it

* If the move does *not* result in improvement, keep it with probability P^{SA}.

* Return to **.

Computing P^{SA}

$$P^{SA} = e^{\frac{-(E_{cur} - E_{move})}{T_{cur}}}$$

- P^{SA} decreases as $(E_{cur} - E_{move})$ increases. I.e. The worse the move, the less likely we are to keep it.
- P^{SA} also decreases as T_{cur} (temperature) decreases.
  - High temperature: random search.
  - Low temperature: randomized hill climbing.
- If we decrease temperature *infinitely* slowly, we can guarantee that the global optimum will be found :)

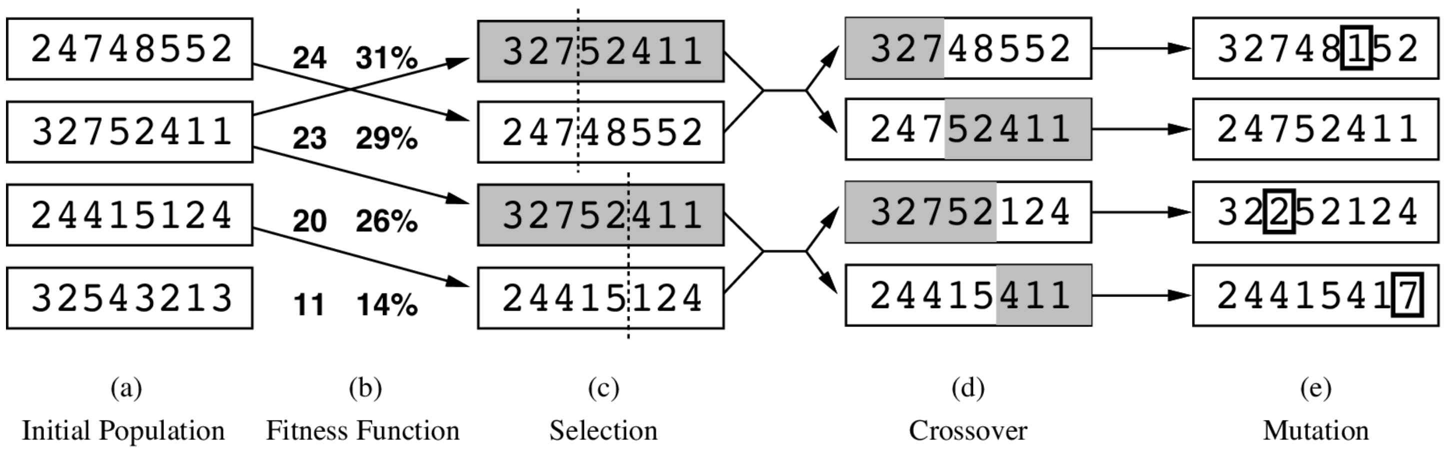
Genetic Algorithm Design

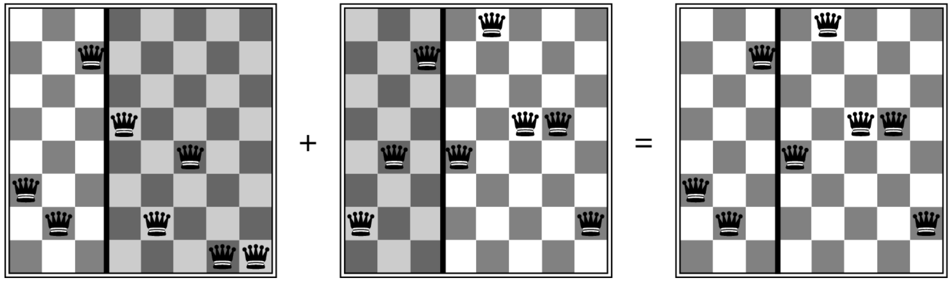
- (Basic algorithm due to John Holland, 1975)
- First, contrive a mapping from bit strings to your problem space.
  - Genotype -> Phenotype.
- Next, contrive an objective function for evaluating solutions:
  - fitness function.
  - F(phenotype) = fitness.

Genetic Algorithm

- Generate *K* bit strings randomly: a population of individuals.
- **Evaluate the fitness of each individual.
- Assign a probability to each individual proportional to it's fitness.
- Generate a new population:
  - Select 2 individuals (parents) according to probability assigned above.
  - Crossover: Pick a random bit position, and swap all bits after that position.
  - Mutation: flip individual bits with a small independent probability.
  - Repeat until we have K new individuals.
- Return to ** unless satisfied.

GA Illustration





Analysis of GA

- Schema Theory:
  - A schema is something like ***10**.
  - The *'s can be anything, the 10 is fixed.
  - It can be shown that schemas with short fixed regions tend to increase exponentially, if average instances of those schema have above average fitness.
  - We can make an argument from decision theory that this is the right thing to do.
  - (Why not long schemas? They tend to be broken up by crossover.)
- This has implications for code design.
- Why mutation? Seems to help. Avoid premature convergence.

Does it Work?

- How well it works has a lot to do with the structure of the fitness surface.
- Getting it to work well requires carefully engineering the genotype->phenotype mapping.
- MANY variations:
  - Different methods for selecting individuals, rank ordered instead of proportional, keep the fittest, etc. etc...
  - Co-evolving parasites – evolving test cases.
  - Independently evolving “island” populations.