CS354

Nathan Sprague

February 21, 2019

Probability and Robotics

What will probability allow us to do?

1. Update our existing beliefs on the basis of new sensor data
2. Combine multiple (conflicting) sources of information
3. Combine uncertain predictive models with noisy sensor data to obtain better state estimats than either source alone could provide

Today we will focus 1.

Probability Notation

- Probability Functions/Distributions:
  - $P(A)$ is a function that maps from all possible values of A to the probability of the corresponding event.
  - Examples:
    - $P(A = true) = .9$
      $P(A = false) = .1$
    - $P(B = red) = .8$
      $P(B = blue) = .1$
      $P(B = green) = .1$

Sample Spaces and Joint Probability Distributions

- Sample space is the set of all possible outcomes.
- The full joint probability distribution assigns a probability to each element of the sample space:
  - S - Squished, U - Under falling Piano

S	U	$P(S, U)$
T	T	.008
T	F	.002
F	T	.001
F	F	.989

Conditional Probability

- $P(A \mid B)$ Expresses the probability of assignments to A given assignments to B.
  - $P(SQUISHED = true) = .01$
  - $P(SQUISHED = true \mid UNDER\_PIANO = true) \approx .89$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Bayes Rule

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

Very handy for updating our beliefs on the basis of evidence.

Bayes Rule Example

- Robot is in a simple four room maze, rooms are labeled a-d.
- Initially, we think he is most likely to be in the left half, $P(X = a) = .4$, $P(X = b) = .4$, ...

a	b	c	d
.4	.4	.1	.1

Bayes Rule Example

- Robot has a sensor designed to tell him what room he is in.
- Sensor is not perfect: only 80% likely to report he is in the correct room. 20% of the time the sensor is off by one. (Errors at the edge wrap around.)
- Distribution of sensor readings when robot is in *a*:

a	b	c	d
.8	.1	0	.1

- In probability notation, where X is the position and Z is sensor reading.
  - $P(Z = a \mid X = a) = .8$
    $P(Z = b \mid X = a) = .1$
    $P(Z = c \mid X = a) = 0$
    $P(Z = d \mid X = a) = .1$

Bayes Rule Example

- Given that we have a sensor model, Baye's rule enables us to update our prior beliefs based on sensor input:

$$P(X \mid Z) = \frac{P(Z \mid X)P(X)}{P(Z)}$$

Bayes Rule Example

- Let's calculate $P(X = a \mid Z = b)$

$$P(X = a \mid Z = b) = \frac{P(Z = b \mid X = a)P(X = a)}{P(Z = b)}$$

- $P(Z = b \mid X = a) = .1$ (From our sensor model)
- $P(X = a) = .4$ (Our prior)
- $P(Z = b)$ (??)

Bayes Rule Example

To calculate $P(Z = b)$, we can use the total probability theorem:

$$P(Z) = \sum_{i}^{N} P(X = x_i) P(Z \mid X = x_i)$$

We can also treat $P(Z)$ as an unknown constant,

$$P(X \mid Z) = \eta P(Z \mid X) P(X)$$

and set it to whatever value makes $P(X \mid Z)$ sum to 1. The two approaches are equivalent.

Bayes Rule Example

Back to work...

$$P(X = a \mid Z = b) = \frac{P(Z = b \mid X = a)P(X = a)}{P(Z = b)}$$

$$= \eta \times .1 \times .4 = .04\eta$$

Similarly:

$$P(X = b \mid Z = b) = \eta \times .8 \times .4 = .32\eta$$

$$P(X = c \mid Z = b) = \eta \times .1 \times .1 = .01\eta$$

$$P(X = d \mid Z = b) = \eta \times 0 \times .1 = 0$$

Bayes Rule Example

Therefore, after our sensor reading, the updated distribution over possible robot locations is:

a	b	c	d
$.04\eta$	$.32\eta$	$.01\eta$	0

We know the robot is *somewhere*, so we know that:

$$.04\eta + .32\eta + .01\eta = 1$$

$$\eta = \frac{1}{.04 + .32 + .01} = 1/.37 \approx 2.70$$

Bayes Rule Example

Finally, we have an updated belief about the robot location:

a	b	c	d
.108	.865	.027	0

We may use this as our new prior, and incorporate additional sensor readings.