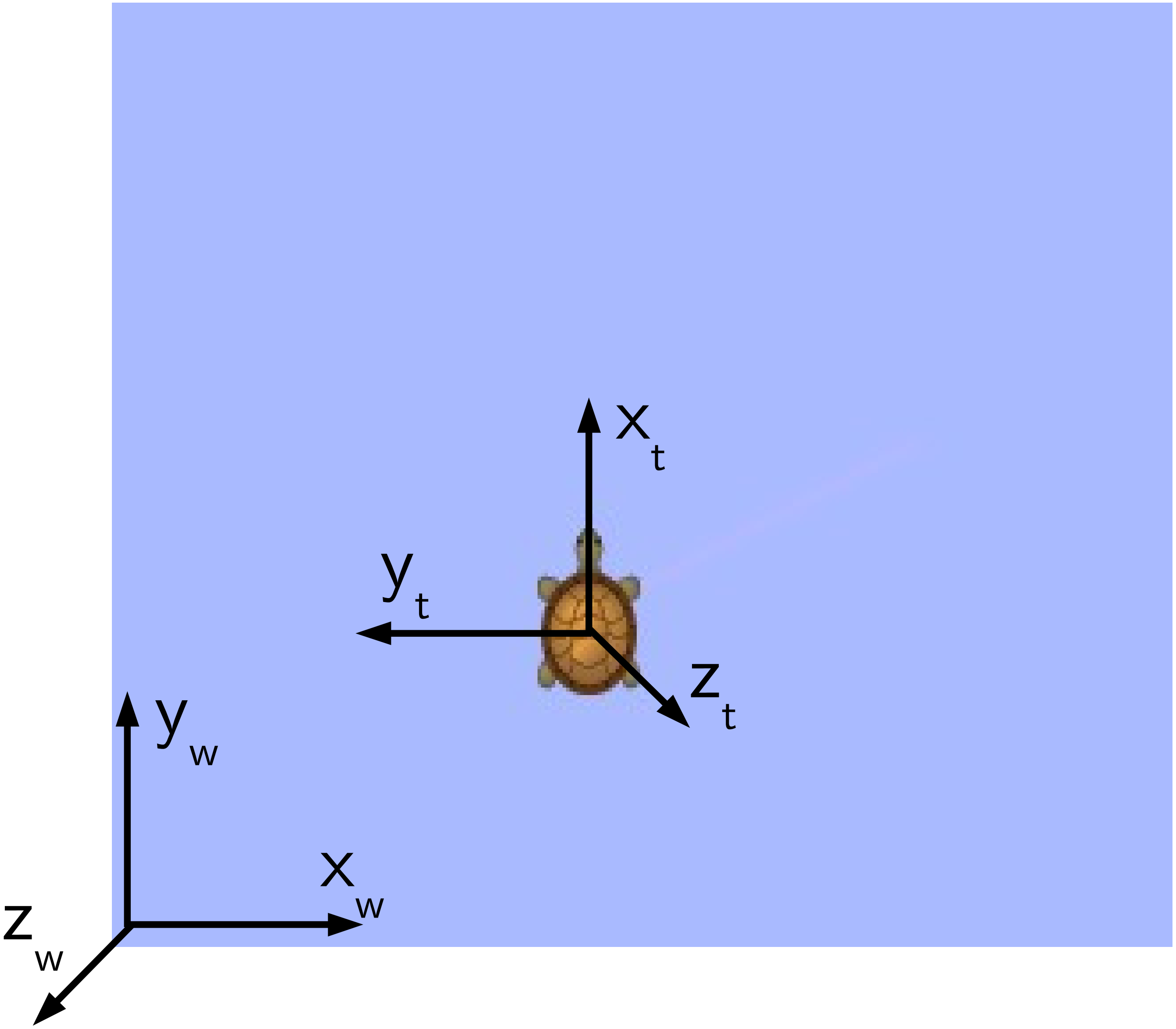
CS354

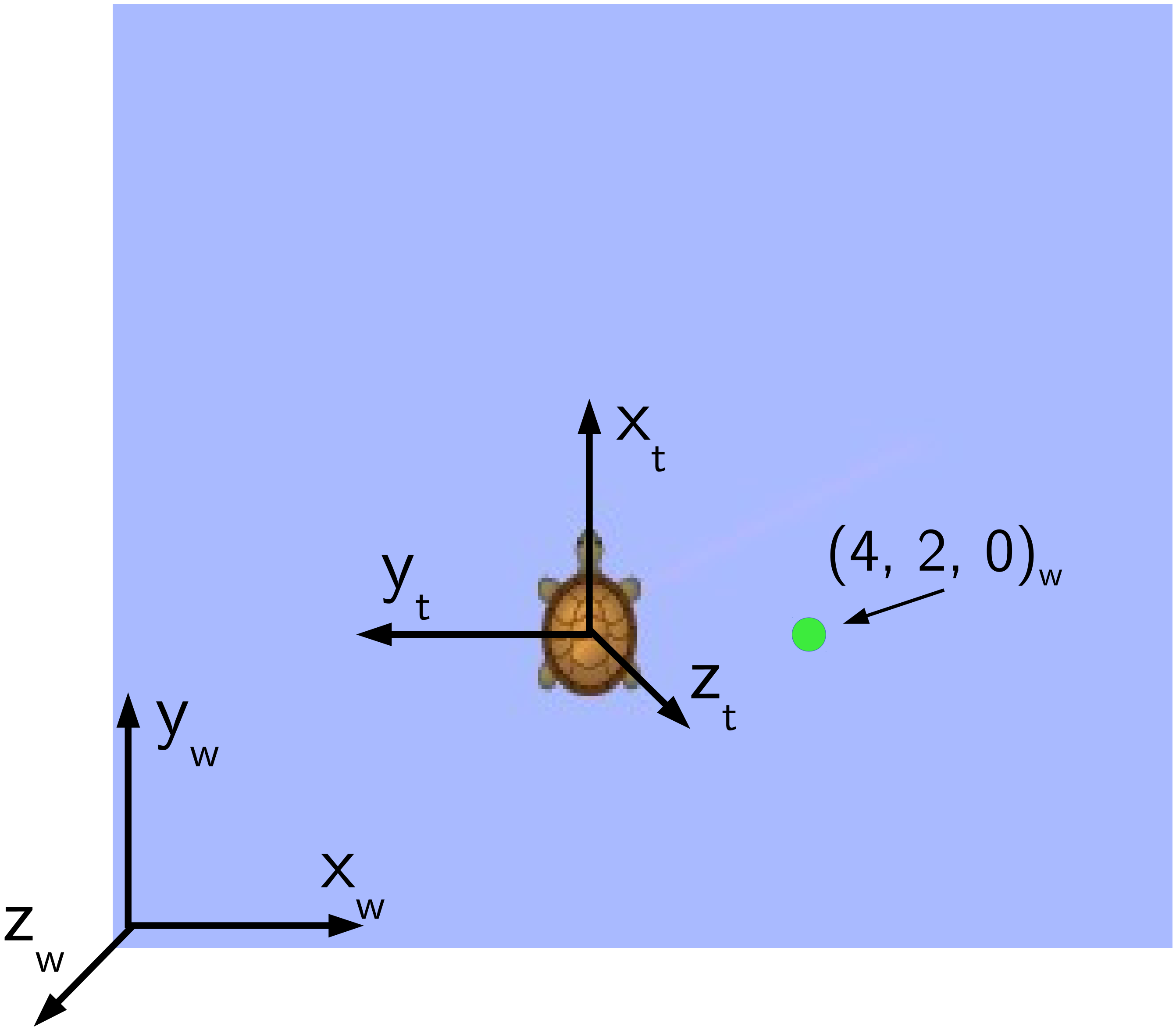
JAMES MADISON UNIVERSITY®

Turtle Navigation



Two coordinate frames: turtle (t) and world (w)

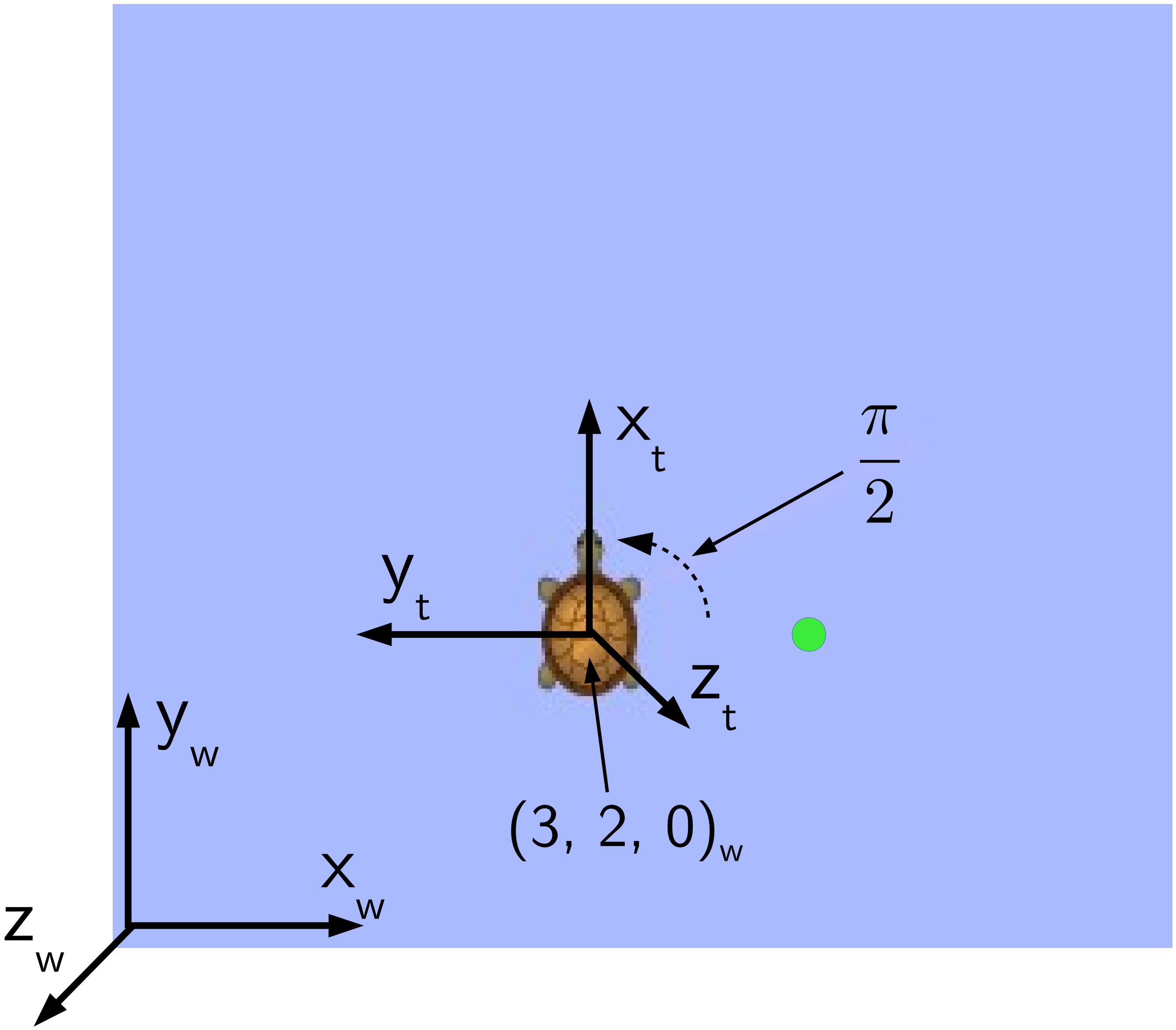
Turtle Navigation



Turtle wants to move to the goal at position

$(4, 2, 0)_w$

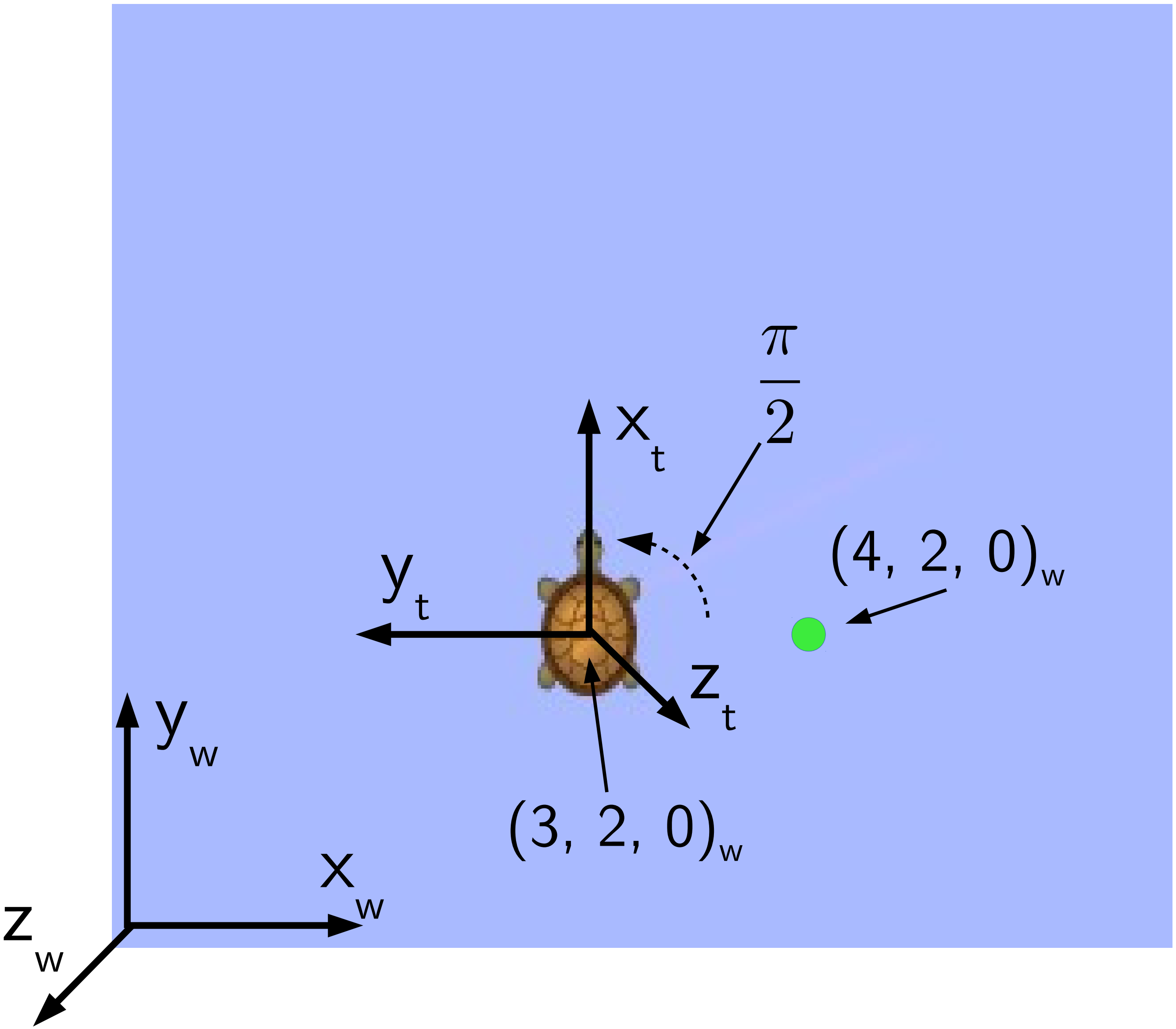
Turtle Navigation



Turtle's position is $(3, 2, 0)_w$ his orientation is

$$\Theta = \frac{\pi}{2}$$

Turtle Navigation



Life would be easier if the goal position was in in the turtle coordinate frame:

- Positive y $\rightarrow$ move right
- Positive x $\rightarrow$ move forward
- Etc.

Quiz: What are the coordinates of the goal in the turtle coordinate frame?

Turtle Navigation

Transforming from one coordinate frame to another can be accomplished through a matrix multiplication:

Goal point in

Homogeneous coordinates.

$$\mathbf{g}_w = \begin{bmatrix} 4 & 2 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} 4 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Goal point in Turtle coordinate frame

$$\mathbf{g}_t = \mathbf{T}_w^t \mathbf{g}_w$$

Appropriate 4x4 transformation matrix

Finding Transformation Matrix: Moving Axes Approach

- Determine a sequence of rotations and translations that would "move" the target axis to the origin axis.
- Each translation or rotation is performed relative to the previous steps.
- Each operation has a corresponding matrix:

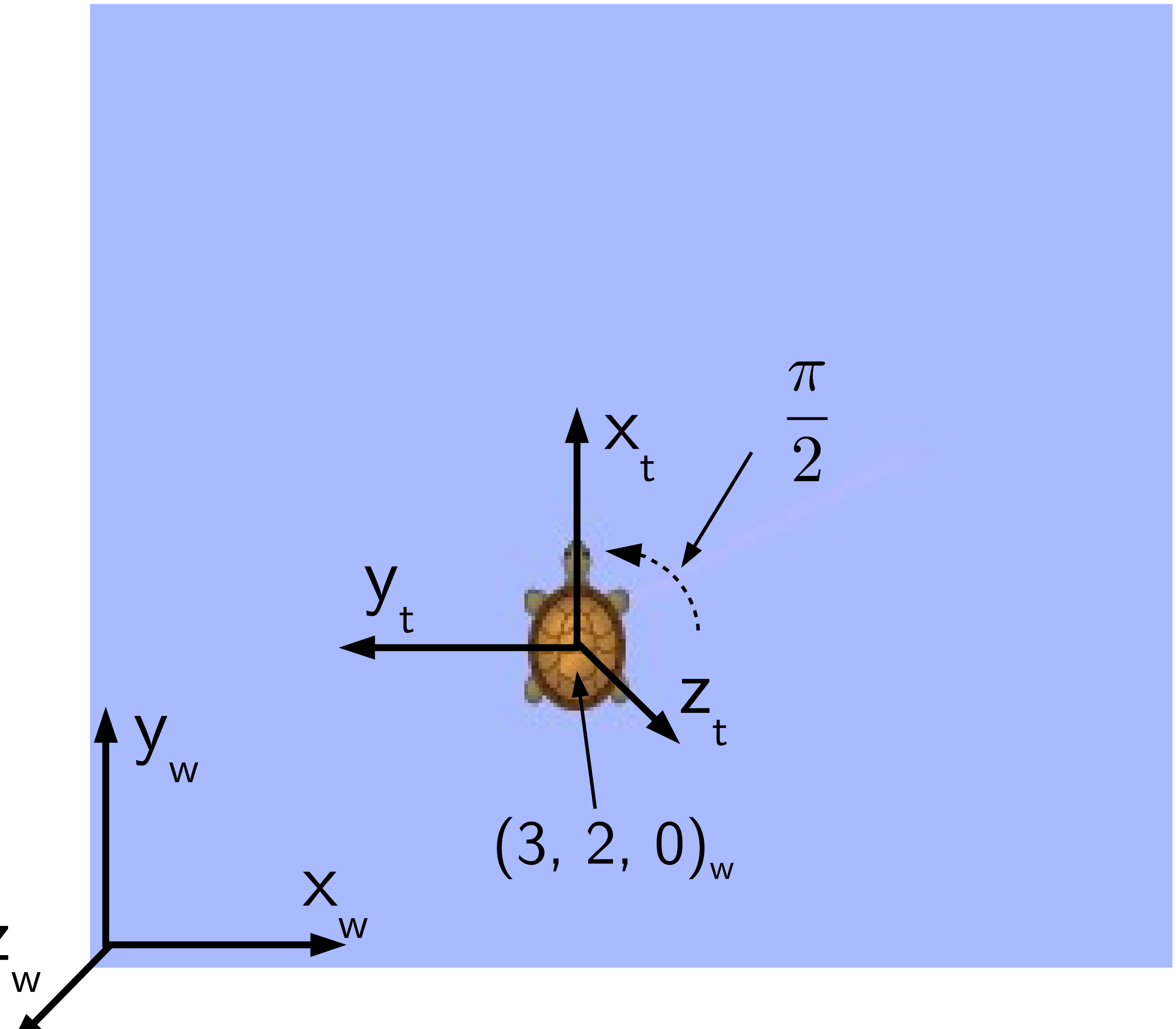
$$Trans(a,b,c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Roty(\Theta) = \begin{bmatrix} \cos\Theta & 0 & \sin\Theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\Theta & 0 & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rotx(\Theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\Theta & -\sin\Theta & 0 \\ 0 & \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

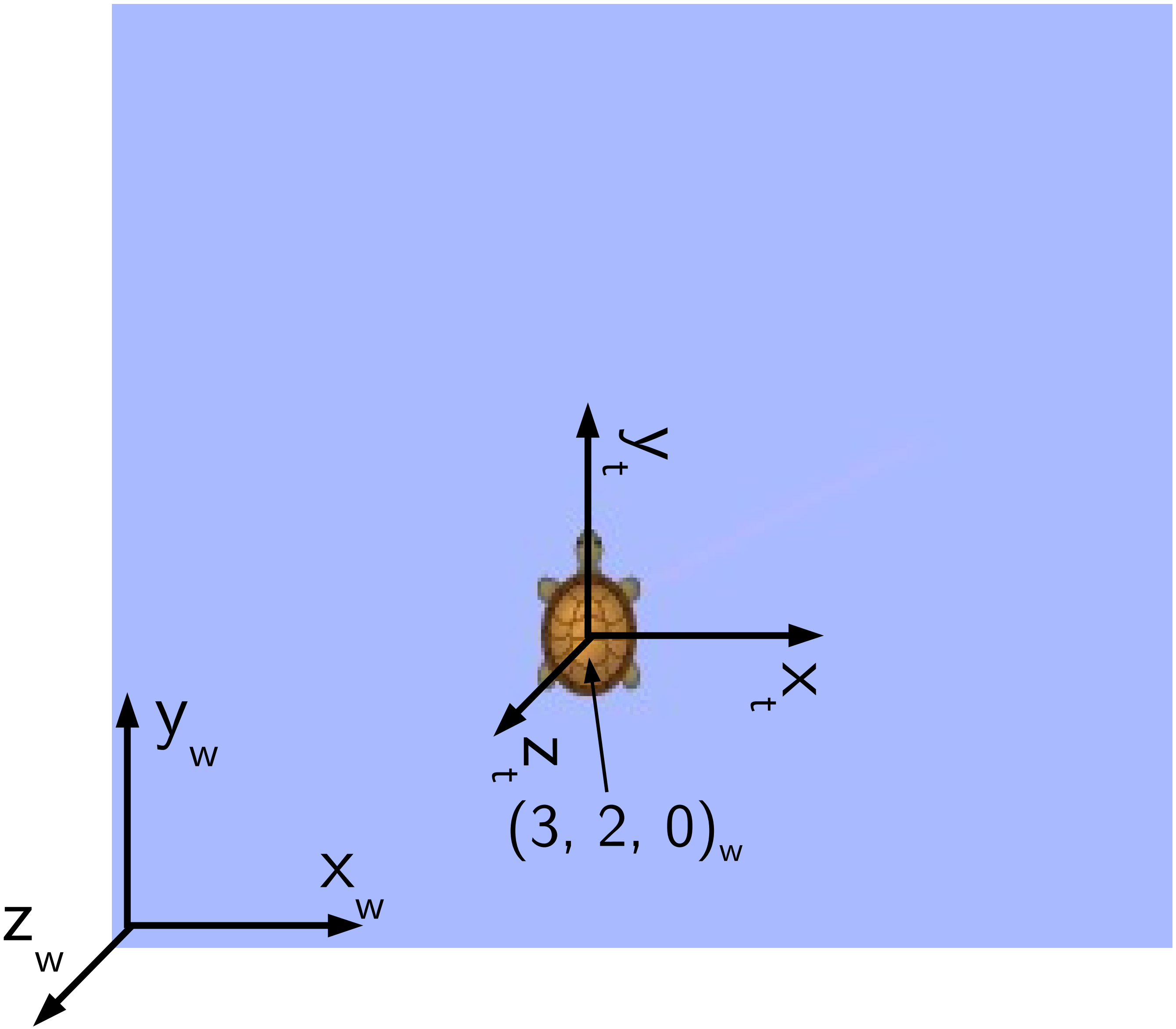
$$Rotz(\Theta) = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 & 0 \\ \sin\Theta & \cos\Theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Moving Axes



Step 1: Rotate around the z axis by $-\frac{\pi}{2}$ radians.

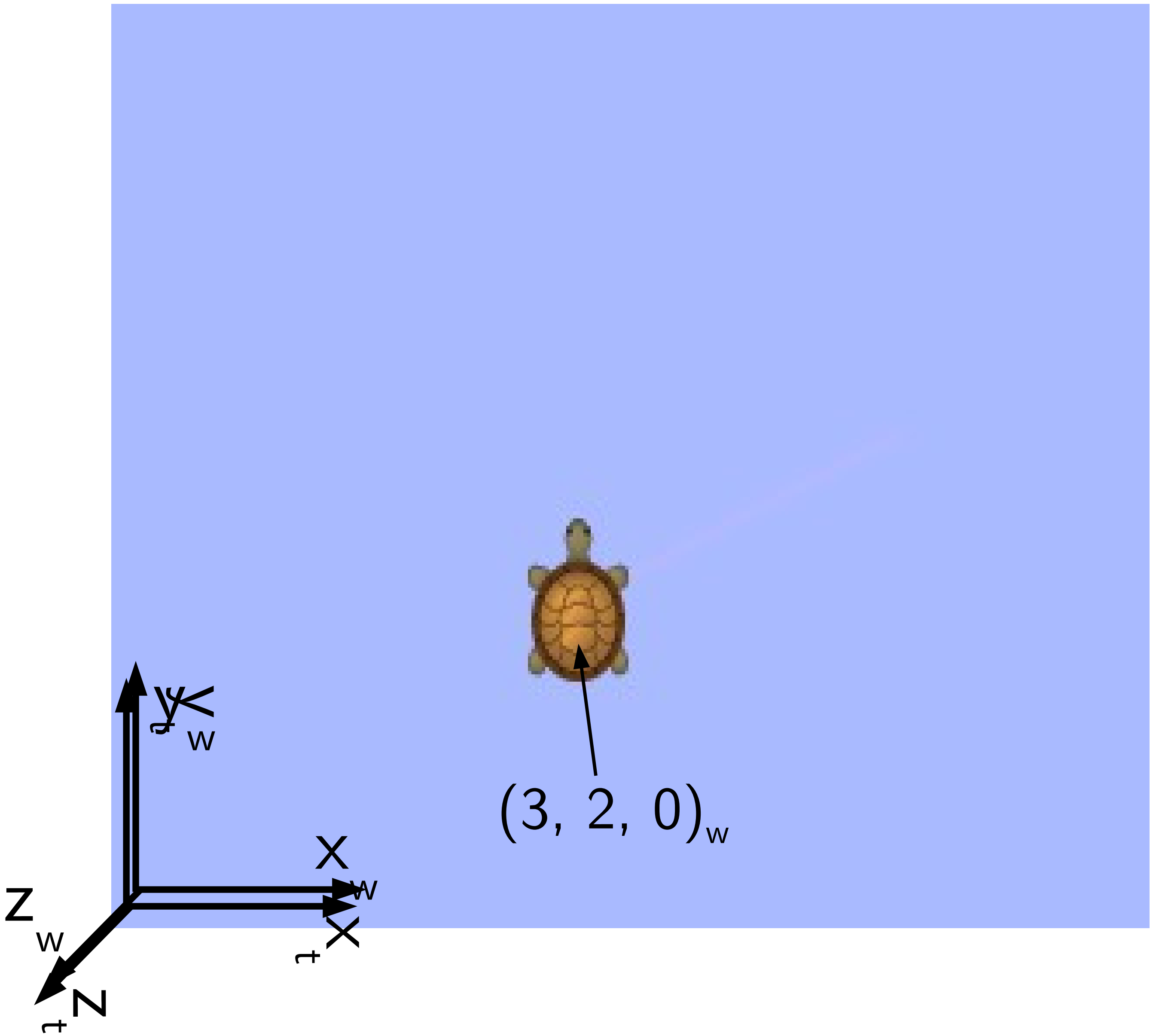
Moving Axes



Step 1: Rotate around the z axis by $-\frac{\pi}{2}$ radians.

Step 2: Translate -3 meters along the (new) x-axis, and -2 meters along the (new) y axis

Moving Axes



Step 1: Rotate around the z axis by - $\frac{\pi}{2}$ radians.

Step 2: Translate -3 meters along the (new) x-axis, and -2 meters along the (new) y axis

SUCCESS!

Moving Axes

- Now we can calculate $\mathbf{T}_w^t$

$$\mathbf{T}_w^t = Rotz\left(-\frac{\pi}{2}\right) \times Trans(-3,-2,0)$$

$$\mathbf{T}_w^t = \begin{bmatrix} \cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) & 0 & 0 \\ \sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_w^t = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -2 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Moving Axes

- This is what we originally wanted to calculate:

$$\mathbf{g}_t = \mathbf{T}_w^t \mathbf{g}_w$$

$$\mathbf{g}_t = \begin{bmatrix} 0 & 1 & 0 & -2 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

SUCCESS!