CS354

Nathan Sprague

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Probabilistic State Representations: Continuous

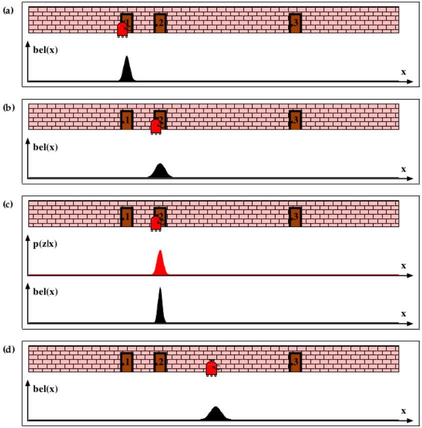

Figure 7.6 Application of the Kalman filter algorithm to mobile robot localization. All densities are represented by unimodal Gaussians.

Probabilistic Robotics. Thrun, Burgard, Fox, 2005

Probability Density Functions

Represent probability distributions over random variables:

- Properties:
  - $f(x) \geq 0$
  - $\int_{-\infty}^{\infty} f(x)dx = 1$
- Interpretation:
  - $P(a \leq x \leq b) = \int_a^b f(x)dx$

Expectation, Variance

- Expectation (continuous)

$$\mu = \mathbb{E}[x] = \int xf(x)dx$$

- Expectation (discrete)

$$\mathbb{E}[X] = \sum_{1}^{n} P(x_i)x_i$$

- Variance

$$\sigma^2 = \mathbb{E}[(x - \mathbb{E}[x])^2]$$

Normal Distribution

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

(*Normal* because of the central limit theorem.)

Vector-Valued State

- We'll need to generalize all of this to the case where the state of the system can't be represented as a single number.
- Use a vector $\mathbf{x}$ to represent the state.

Covariance

$$cov(x, y) = \mathbb{E}[(x - \mu_x)(y - \mu_y)]$$

- Properties:
  - $cov(x, y) = cov(y, x)$
  - If x and y are independent, $cov(x, y) = 0$
  - If $cov(x, y) > 0$, y tends to increase when x increases.
  - If $cov(x, y) < 0$, y tends to decrease when x increases.

Covariance Matrix

- Covariance matrix:

$$cov(\mathbf{x}) = \Sigma_{\mathbf{x}} = \mathbb{E}[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T]$$

- Where $\mathbf{x}$ is a random vector and $\hat{\mathbf{x}}$ is the vector mean.
- The entry at row i, column j in the matrix is cov($\mathbf{x}_i$,$\mathbf{x}_j$)
- Multivariate normal distribution is parameterized by the mean vector and covariance matrix.

Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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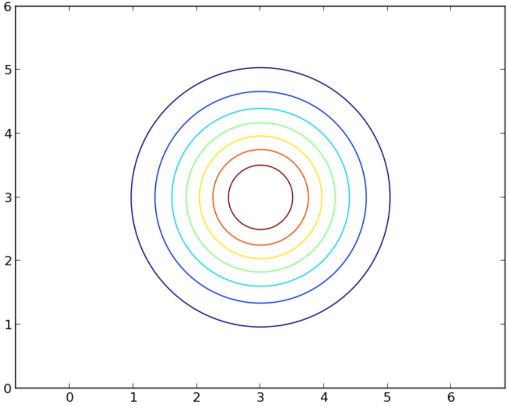

Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma = \begin{bmatrix} .2 & 0 \\ 0 & 1 \end{bmatrix}$$

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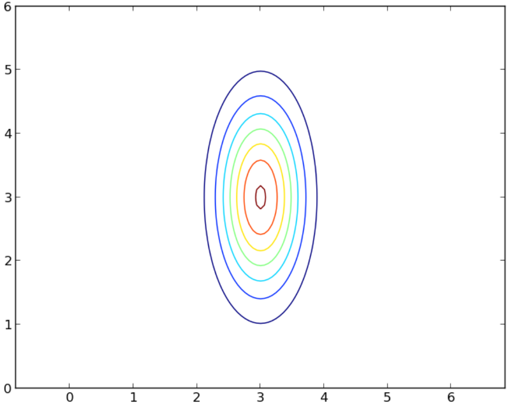
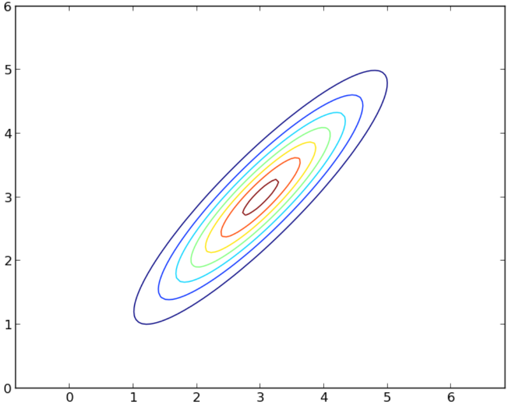


Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma = \begin{bmatrix} 1 & .9 \\ .9 & 1 \end{bmatrix}$$

Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma = \begin{bmatrix} 1 & .9 \\ .9 & 1 \end{bmatrix}$$

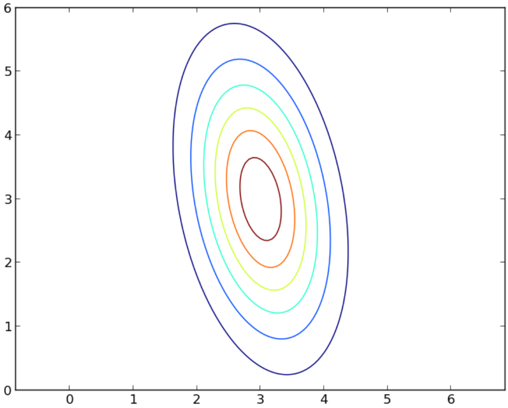


Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \ \Sigma = \begin{bmatrix} .5 & -.3 \\ -.3 & 2 \end{bmatrix}$$

Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \ \Sigma = \begin{bmatrix} .5 & -.3 \\ -.3 & 2 \end{bmatrix}$$



Can We Do Recursive State Estimation?

- Two Steps:
  - Prediction based on system dynamics:

$$Bel^{-}(x_t) = \int p(x_t \mid x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

  - Correction based on sensor reading:

$$Bel(x_t) = \eta p(z_t \mid x_t) Bel^{-}(x_t)$$

YES. The Kalman filter. Next time.