Name: ____________________________

HW #3

1. **Bayes' Rule and Probability Notation**[1]

   A common technique for the localization of robots in industrial settings is to augment the local environment with specific landmarks (often visual) that can be sensed by the robot. Suppose we have such a landmark scheme. We are interested in two Boolean random variables: D - the robot has detected a landmark and L - there is actually a landmark present. Express each of the following in probability notation using these variables:

   (a) When a landmark is present, the system correctly identifies it with probability .9.

   (b) Landmarks are identified falsely (i.e. when no landmark is present) with probability .01.

   (c) The probability of a landmark being present is .05.

   Answer the following questions by writing the desired quantity in probability notation, then determining the solution.

   (d) What is the missed detection probability (probability of missing a landmark even though one is present)?

   (e) What is the probability that a target is present given that one is detected?

   (f) Complete the following table with the full joint distribution over detections and landmark presence:

D	L	$P(D, L)$
T	T	
T	F	
F	T	
F	F	

[1] Question is based on question 5 from Appendix A of Computational Principles of Mobile Robotics.

2. **Bayes' Rule and Localization** Consider the robot localization scenario described in these slides. What is the final probability distribution over room locations given the following series of sensor readings?

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   b (done for you in the slides)
   a
   b
   ```

   Assume that the initial distribution is

.4	.4	.1	.1

   that the robot is not moving, and that sensor readings are independent given the robot's position. (I.e. one incorrect sensor reading does not increase the probability of seeing the same incorrect sensor reading in the future.)

   You should treat the distribution calculated after each sensor reading as the prior distribution for the next sensor reading.

3. **Grid-Based Localization and Tracking**

We've discussed the application of grid-based localization to the problem of tracking a robot moving in a circular four-room maze. For this question we will track the same robot. The robot may choose to move left or right, and we know that the actions succeed 50% of the time. When an action does not succeed, the robot remains in the same location. Our sensor model tells us that there is an 80% chance that his room sensor will output the true location, and a 20% that it will indicate one of the rooms to the immediate left or right of the true location.

Initially, the robot is 75% likely to be in room "a" and 25% likely to be in room "b":

$Bel(X_0) =$

a	b	c	d
.75	.25	0	0

The robot's first action is "right" and the first sensor output is "b".

(a) What will the belief distribution be after one step of prediction (before the sensor update)? Show your work.

$Bel^{-}(X_1) =$

a	b	c	d

(b) What will the belief distribution be after the sensor update? Show your work.

$Bel(X_0) =$

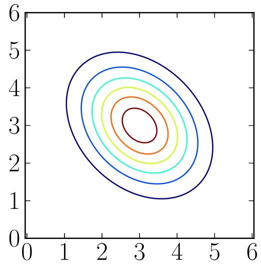
a	b	c	d

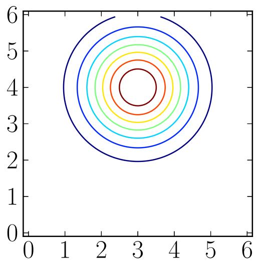
4. **Multivariate Normal Distribution**

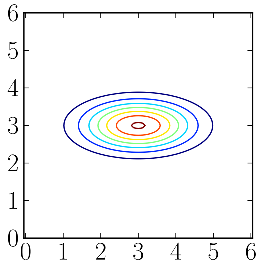
(a) Recall that the multivariate normal distribution is parameterized by a mean vector μ and a covariance matrix Σ. Cross out any of the following parameter values that do *not* correspond to a valid normal distribution.

$\mu_A = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\mu_B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \Sigma_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\mu_C = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_C = \begin{bmatrix} 2 & -.5 \\ .5 & 1 \end{bmatrix}$
$\mu_D = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_D = \begin{bmatrix} 1 & 0 \\ 0 & .2 \end{bmatrix}$	$\mu_E = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_E = \begin{bmatrix} 1 & -.3 \\ -.3 & 1 \end{bmatrix}$	$\mu_F = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_F = \begin{bmatrix} 0 & .8 \\ .8 & 0 \end{bmatrix}$

(b) Each of the following figures illustrates one of the probability density functions parameterized above. Label each figure with the matching parameterization (A-F).







5. **The Kalman Filter**

Recall that the Kalman filter requires both a linear system model and a linear measurement model. The system model (without control) can be expressed as

$\mathbf{x}_t = A\mathbf{x}_{t-1} + \mathbf{w}_{t-1}$,

where $\mathbf{x}_t$ represents the system state, A expresses the state dynamics, and $\mathbf{w}_t$ is a noise term. The measurement model can be expressed as

$\mathbf{z}_t = H\mathbf{x}_t + \mathbf{v}_t$,

where $\mathbf{z}_t$ is a measurement value, H expresses how sensor values are related to the system state, and $\mathbf{v}_t$ is sensor noise.

For this question assume that we want to use a Kalman filter to track an object moving in one dimension with a fixed acceleration.

The following difference equations describe the system dynamics:

$$\begin{aligned} x_t &= x_{t-1} + \dot{x}_{t-1}\Delta t \\ \dot{x}_t &= \dot{x}_{t-1} + \ddot{x}_{t-1}\Delta t \\ \ddot{x}_t &= \ddot{x}_{t-1} \end{aligned}$$

Where x_t is the object position, $\dot{x}_t$ is the velocity and $\ddot{x}_t$ is acceleration and Δt is the size of the time step.

(a) Assuming that the state of the system is encoded as: $\mathbf{x}_t = \begin{bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{bmatrix}$, what A matrix corresponds to the difference equations above?

(b) What should the H matrix be to represent the fact that we have a one-dimensional sensor that provides an estimate of the object position, but no information about velocity or acceleration?

6. **Particle Filters**

   Complete the unfinished methods in particle_demo.py. You should be able to use the completed script to re-calculate approximate answers for the grid-based localization question from Question 3.

   You may also test the correctness of your methods by running the tests in tests.py.