Name: ____________________________

HW #4

1. **Grid-Based Localization and Tracking**

   We've discussed the application of grid-based localization to the problem of tracking a robot moving in a circular four-room maze. For this question we will track the same robot. The robot may choose to move left or right, and we know that the actions succeed 50% of the time. When an action does not succeed, the robot remains in the same location. Our sensor model tells us that there is an 80% chance that his room sensor will output the true location, and a 20% that it will indicate one of the rooms to the immediate left or right of the true location.

   Initially, the robot is 75% likely to be in room "a" and 25% likely to be in room "b":

   $Bel(X_0) =$

a	b	c	d
.75	.25	0	0

   The robot's first action is "right" and the first sensor output is "b".

   - What will the belief distribution be after one step of prediction (before the sensor update)? Show your work.

   $Bel^{-}(X_1) =$

a	b	c	d

   - What will the belief distribution be after the sensor update? Show your work.

   $Bel(X_0) =$

a	b	c	d

2. **Multivariate Normal Distributions and the Kalman Filter**

Recall that the Kalman filter requires both a linear system model and a linear measurement model. The system model (without control) can be expressed as

$\mathbf{x}_t = A\mathbf{x}_{t-1} + \mathbf{w}_{t-1}$,

where $\mathbf{x}_t$ represents the system state, A expresses the state dynamics, and $\mathbf{w}_t$ is a noise term. The measurement model can be expressed as

$\mathbf{z}_t = H\mathbf{x}_t + \mathbf{v}_t$,

where $\mathbf{z}_t$ is a measurement value, H expresses how sensor values are related to the system state, and $\mathbf{v}_t$ is sensor noise.

For this question assume that we want to use a Kalman filter to track an object moving in one dimension with a fixed acceleration.

The following difference equations describe the system dynamics:

$$\begin{aligned} x_t &= x_{t-1} + \dot{x}_{t-1}\Delta t \\ \dot{x}_t &= \dot{x}_{t-1} + \ddot{x}_{t-1}\Delta t \\ \ddot{x}_t &= \ddot{x}_{t-1} \end{aligned}$$

Where x_t is the object position, $\dot{x}_t$ is the velocity and $\ddot{x}_t$ is acceleration and Δt is the size of the time step.

- Assuming that the state of the system is encoded as: $\mathbf{x}_t = \begin{bmatrix} x_t \\ \dot{x}_t \\ \ddot{x}_t \end{bmatrix}$, what A matrix corresponds to the difference equations above?

- What should the H matrix be to represent the fact that we have a one-dimensional sensor that provides an estimate of the object position, but no information about velocity or acceleration?

- Recall that the multivariate normal distribution is parametrized by a mean vector μ and a covariance matrix Σ. Cross out any of the following parameter values that do *not* correspond to a valid normal distribution.

$\mu_A = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\mu_B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \Sigma_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\mu_C = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_C = \begin{bmatrix} 2 & -.5 \\ .5 & 1 \end{bmatrix}$
$\mu_D = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_D = \begin{bmatrix} 1 & 0 \\ 0 & .2 \end{bmatrix}$	$\mu_E = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_E = \begin{bmatrix} 1 & -.3 \\ -.3 & 1 \end{bmatrix}$	$\mu_F = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma_F = \begin{bmatrix} 0 & .8 \\ .8 & 0 \end{bmatrix}$

- Each of the following figures illustrates one of the probability density functions parametrized above. Label each figure with the matching parametrization (A-F).