Name: ____________________________

CS354 HW#4, Spring 2014

1. **Multivariate Normal Distributions and the Kalman Filter**

Recall that the Kalman filter requires both a linear system model and a linear measurement model. The system model (without control) can be expressed as $\mathbf{x}_{t+1} = \Phi\mathbf{x}_t + \mathbf{v}_t$, where $\mathbf{x}_t$ represents the system state, Φ expresses the state dynamics, and $\mathbf{v}_t$ is a noise term. The measurement model can be expressed as $\mathbf{z}_t = \Lambda\mathbf{x}_t + \mathbf{w}_t$, where $\mathbf{z}_t$ is a measurement value, Λ expresses how sensor values are related to the system state, and $\mathbf{w}_t$ is sensor noise.

For this question assume that we want to use a Kalman filter to track an object moving in three dimension with a fixed velocity.

The following difference equations describe the system dynamics:

$$
\begin{aligned}
x_{t+1} &= x_t + \dot{x}_t \Delta t \\
y_{t+1} &= y_t + \dot{y}_t \Delta t \\
z_{t+1} &= z_t + \dot{z}_t \Delta t \\
\dot{x}_{t+1} &= \dot{x}_t \\
\dot{y}_{t+1} &= \dot{y}_t \\
\dot{z}_{t+1} &= \dot{z}_t
\end{aligned}
$$

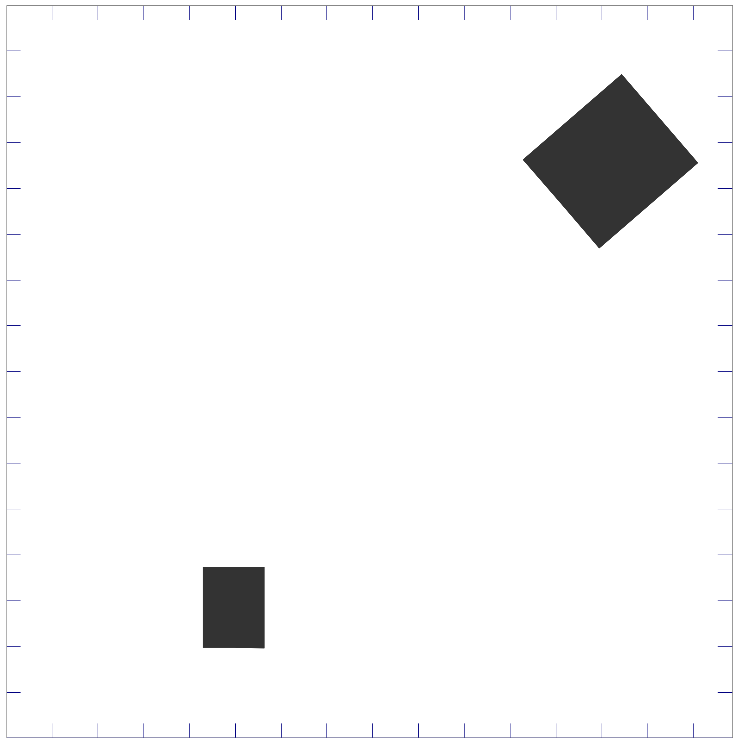
Where x_t is the object position in the x dimension, $\dot{x}_t$ is the velocity in the x dimension and Δt is the size of the time step.

- Assuming that the state of the system is encoded as: $\mathbf{x}_t = \begin{bmatrix} x_t & y_t & z_t & \dot{x}_t & \dot{y}_t & \dot{z}_t \end{bmatrix}^T$, what Φ matrix corresponds to the difference equations above? (4pts)

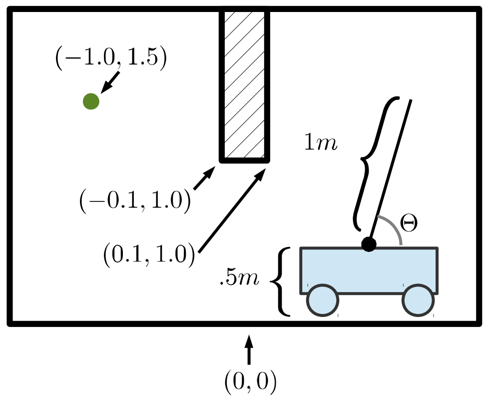
- What should the Λ matrix be to represent the fact that we have a sensor that provides an estimate of the object position, but no information about velocity or acceleration? (4pts)

2. Complete the in-class planning exercise from 3/27. (12pts)

3. Draw the quadtree decomposition of the following room. The hash marks indicate the resolution limit. (5pts)



4. The figure below illustrates a robotic cart. The cart can move to the left or right, and can change the angle of the attached pole. The angle of the pole is indicated by Θ, where $\Theta = 0$ when the pole is rotated all the way to the right and $\Theta = \pi$ when the pole is rotated all the way to the left. The green dot is a goal location for the pole end-point and the hashed box is an obstacle.



- Draw the configuration space for this robot, with x on the horizontal axis and Θ on the vertical axis. Draw $\mathcal{C}_{obs}$ as a shaded region. (5pts)

- Draw a valid trajectory in your configuration space from the robot's current configuration to the goal configuration. (2pts)

- Which would be more appropriate for planning in this configuration space A^* or RRT? Justify your answer. (3pts)

5. The following recursive state estimation formula can be used to estimate the state of a system X given a sensor model $P(Z_k|X_k)$ and a model of state dynamics $P(X_k|x_{k-1})$.

$$Bel(X_k) = \mu P(Z_k|X_k) \sum_{x_{k-1} \in X} P(X_k|x_{k-1}) Bel(x_{k-1})$$

A particle filter is one approach to estimating $Bel(X_k)$ when it cannot be calculated exactly.

- Why might it be inefficient to calculate $Bel(X_k)$ exactly? (2pts)
- Which term in the equation above corresponds to the collection of particles in a particle filter? (2pts)
- One of the steps of the particle filtering algorithm involves calculating weights for the particles. Which term in the equation above corresponds to this step? (2pts)
- What is the purpose of the resampling stage of the particle filter algorithm? (2pts)