CS480

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State Estimation

- The goal is to estimate the state of the robot from a history of actions and observations:

$$P(X_k|O_k, A_{k-1}, O_{k-1}, A_{k-2}, ..., O_0)$$

- We make some (true-ish) simplifying assumptions:
  - Markov Assumption:

$$P(X_k|X_{k-1}, A_{k-1}, X_{k-2}, A_{k-2}, ..., X_0) = P(X_k|X_{k-1}, A_{k-2})$$

  - Assumption that the current observation only depends on the current state:

$$P(O_k|X_k, O_{k-1}, X_{k-2}, ..., O_0) = P(O_k|X_k)$$

Simplifying the Notation...

- Looks nicer if we let actions be implicit (They aren't really random variables, because we choose them.)
- The goal is to estimate the state of the robot from a history of observations:

$$Bel(X_k) = P(X_k|Z_k, Z_{k-1}, ..., Z_0) = P(X_k|\mathbf{Z}_{0:k})$$

  - Markov Assumption:

$$P(X_k|X_{k-1}, X_{k-2}, ..., X_0) = P(X_k|X_{k-1})$$

  - Assumption that the current observation only depends on the current state:

$$P(Z_k|X_k, Z_{k-1}, X_{k-2}, ..., Z_0) = P(Z_k|X_k)$$

The Answer!

$$Bel(X_k) = \mu P(Z_k|X_k) \sum_{x_{k-1} \in X} P(X_k|x_{k-1}) Bel(x_{k-1})$$

Again μ is a normalizing constant chosen to make the distribution sum to 1.

(With continuous variables, this would be written:

$$Bel(X_k) = \mu p(z_k|x_k) \int p(x_k|x_{k-1}) Bel(x_{k-1}) dx_{k-1}$$

)

Digesting the Answer

$$Bel(X_k) = \mu P(Z_k|X_k) \sum_{x_{k-1} \in X} P(X_k|x_{k-1}) Bel(x_{k-1})$$

- We can compute this in two stages:
  - *Prediction* – What is the state distribution BEFORE the the latest sensor reading:

$$P(X_k|\mathbf{Z}_{0:k-1}) = \sum_{x_{k-1} \in X} P(X_k|x_{k-1}) Bel(x_{k-1})$$

  - *Estimation* – How does the latest sensor reading modify our belief:

$$Bel(X_k) = \mu P(Z_k|X_k) P(X_k|\mathbf{Z}_{0:k-1})$$

Prediction Example

- The robot is now moving Right! (or trying to)
- 80% chance to move right, 20% chance stays in place.
- Assume we know that the robot starts in position a, $P(X_0) =$

a	b	c	d
1	0	0	0

- Or:
  $Bel(X_0 = a) = 1$
  $Bel(X_0 = b) = 0$
  ...

Prediction Example

- Run one step of prediction:

$$P(X_1 = a|Z_0) = \sum_{x_0 \in X} P(x_1 = a|x_0)Bel(x_0)$$

$$= P(X_1 = a|X_0 = a)Bel(X_0 = a) +$$

$$P(X_1 = a|X_0 = b)Bel(X_0 = b) +$$

$$P(X_1 = a|X_0 = c)Bel(X_0 = c) +$$

$$P(X_1 = a|X_0 = d)Bel(X_0 = d)$$

$$= .2 \times 1 + 0 \times 0 + 0 \times 0 + .8 \times 0$$

$$= .2$$

$$P(X_1 = b|Z_0) = .8 \times 1 + .2 \times 0 + 0 \times 0 + 0 \times 0 = .8$$

Prediction Example

- Similarly
  $P(X_1 = b|Z_0) = .8 \times 1 + .2 \times 0 + 0 \times 0 + 0 \times 0 = .8$
  $P(X_1 = c|Z_0) = 0$
  $P(X_1 = d|Z_0) = 0$
- Unsurprisingly, $P(X_1|Z_0) =$

a	b	c	d
.2	.8	0	0

Estimation

Now that we have a prediction, we can update it based on the latest sensor reading:

$$Bel(X_k) = \mu P(Z_k|X_k)P(X_k|\mathbf{Z}_{0:k-1})$$

This is *exactly* what we did when we talked about using Bayes rule to update a prior state estimate based on a sensor reading.

The process is then repeated indefinitely.