# CS480

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# Probability Notation

- Probability Functions/Distributions:
  - $P(A)$ is a function that maps from all possible values of $A$ to the probability of the corresponding event.
  - Examples:
    - $P(A = true) = .9$
      $P(A = false) = .1$
    - $P(B = red) = .8$
      $P(B = blue) = .1$
      $P(B = green) = .1$
- $P(A)$ is also referred to as a prior probability.

# Conditional Probability

- $P(A|B)$ Expresses the probability of assignments to $A$ given assignments to $B$.
  - $P(SQUISHED = true) = .0001$
  - $P(SQUISHED = true|UNDER\_FALLING\_PIANO = true) = .9$

$$P(A|B) = \frac{P(A \wedge B)}{P(B)}$$

# Bayes Rule

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Very handy for updating our beliefs on the basis of evidence.

# Bayes Rule Example

- Robot is in a simple four room maze, rooms are labeled a-d.
- Initially, he is equally likely to be in any room, $P(X = a) = .25$, $P(X = b) = .25$, ...

| a | b | c | d |
|---|---|---|---|
| .25 | .25 | .25 | .25 |

# Bayes Rule Example

- Robot has a sensor designed to tell him what room he is in.
- Sensor is not perfect: only 80% likely to report he is in the correct room. 20% of the time the sensor is off by one. (Errors at the edge wrap around.)
- Distribution of sensor readings when robot is in $a$:

| a | b | c | d |
|---|---|---|---|
| .8 | .1 | 0 | .1 |

- In probability notation, where $X$ is the position and $Z$ is sensor reading.
  - $P(Z = a|X = a) = .8$
    $P(Z = b|X = a) = .1$
    $P(Z = c|X = a) = 0$
    $P(Z = d|X = a) = .1$

# Bayes Rule Example

- Given that we have a sensor model, Baye's rule enables us to update our prior beliefs based on sensor input:

$$P(X|Z) = \frac{P(Z|X)P(X)}{P(Z)}$$

# Bayes Rule Example

- Let's calculate $P(X = a|Z = b)$

$$P(X = a|Z = b) = \frac{P(Z = b|X = a)P(X = a)}{P(Z = b)}$$

- $P(Z = b|X = a) = .1$ (From our sensor model)
- $P(X = a) = .25$ (Our prior)
- $P(Z = b)$ (??)

# Bayes Rule Example

To calculate $P(Z = b)$, we can use the following identity:

$$P(Z) = \sum_{i}^{N} P(X = x_i)P(Z|X = x_i)$$

We can also treat $P(Z)$ as an unknown constant,

$$P(X|Z) = \eta P(Z|X)P(X)$$

and set it to whatever value makes $P(X|Z)$ sum to 1. The two approaches are equivalent.

# Bayes Rule Example

Back to work...

$$P(X = a|Z = b) = \frac{P(Z = b|X = a)P(X = a)}{P(Z = b)}$$

$$= \eta \times .1 \times .25 = .025\eta$$

Similarly:

$$P(X = b|Z = b) = \eta \times .8 \times .25 = .2\eta$$

$$P(X = c|Z = b) = \eta \times .1 \times .25 = .025\eta$$

$$P(X = d|Z = b) = \eta \times 0 \times .25 = 0$$

# Bayes Rule Example

Therefore, after our sensor reading, the updated distribution over possible robot locations is:

| a | b | c | d |
|---|---|---|---|
| $.025\eta$ | $.2\eta$ | $.025\eta$ | 0 |

We know the robot is *somewhere*, so we know that:

$$.025\eta + .2\eta + .025\eta = 1$$

$$\eta = \frac{1}{.025 + .2 + .025} = 1/.25 = 4$$

# Bayes Rule Example

Finally, we have an updated belief about the robot location:

| a | b | c | d |
|---|---|---|---|
| .1 | .8 | .1 | 0 |

We may use this as our new prior, and incorporate additional sensor readings.