CS480

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February 19, 2013

Expectation, Variance

- Expectation (continuous)

$$\mu = \mathbb{E}[x] = \int x f(x) dx$$

- Expectation (discrete)

$$\mathbb{E}[X] = \sum_{1}^{n} P(x_i) x_i$$

- Variance

$$\sigma^2 = \mathbb{E}[(x - \mathbb{E}[x])^2]$$

Covariance

$$cov(x, y) = \mathbb{E}[(x - \mu_x)(y - \mu_y)]$$

- Properties:
  - $cov(x, y) = cov(y, x)$
  - If x and y are independent, $cov(x, y) = 0$
  - If $cov(x, y) > 0$, y tends to increase when x increases.
  - If $cov(x, y) < 0$, y tends to decrease when x increases.

Covariance Matrix

- Covariance matrix:

$$cov(\mathbf{x}) = \Sigma_{\mathbf{x}} = \mathbb{E}[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T]$$

- Where $\mathbf{x}$ is a random vector and $\hat{\mathbf{x}}$ is the vector mean.
- The entry at row i, column j in the matrix is cov($\mathbf{x}_i$,$\mathbf{x}_j$)

Kalman Filter

- Assumes:
  - Linear state dynamics
  - Linear sensor model
  - Normally distributed noise in the state dynamics
  - Normally distributed noise in the sensor model
- State Transition Model:
  - $\mathbf{x}(k+1) = \Phi\mathbf{x}(k) + \Gamma\mathbf{u}(k) + \mathbf{v}(k)$
- Sensor Model:
  - $\mathbf{z}(k) = \Lambda\mathbf{x}(k) + \mathbf{w}(k)$

Kalman Filter in One Slide

- Predict:
  Project the state forward:
  $$\hat{\mathbf{x}}(k+1|k) = \Phi\hat{\mathbf{x}}(k) + \Gamma\mathbf{u}(k)$$
  Project the covariance of the state estimate forward:
  $$\mathbf{P}(k+1|k) = \Phi\mathbf{P}(k)\Phi^T + \mathbf{C}_v$$
- Correct:
  Compute the Kalman gain:
  $$\mathbf{K}(k+1) = \mathbf{P}(k+1|k)\Lambda^T(\Lambda\mathbf{P}(k+1|k)\Lambda^T + \mathbf{C}_w)^{-1}$$
  Update the estimate with the measurement:
  $$\hat{\mathbf{x}}(k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{K}(k+1)(\mathbf{z}(k+1) - \Lambda\hat{\mathbf{x}}(k+1|k))$$
  Update the estimate covariance:
  $$\mathbf{P}(k+1) = (\mathbf{I} - \mathbf{K}(k+1)\Lambda)\mathbf{P}(k+1|k)$$