CS354

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State Estimation

- The goal is to estimate the state of the robot from a history of observations:

$$Bel(X_t) = P(X_k \mid Z_1, Z_2, ..., Z_k)$$

- We make some (true-ish) simplifying assumptions:
  - Markov Assumption:

$$P(X_k \mid X_1, X_2, ..., X_{k-1}) = P(X_k \mid X_{k-1})$$

  - Assumption that the current observation only depends on the current state:

$$P(Z_t \mid X_1, Z_1, X_2, ..., Z_{t-1}, X_t) = P(Z_t \mid X_t)$$

Probabilistic State Representations: Grid-Based

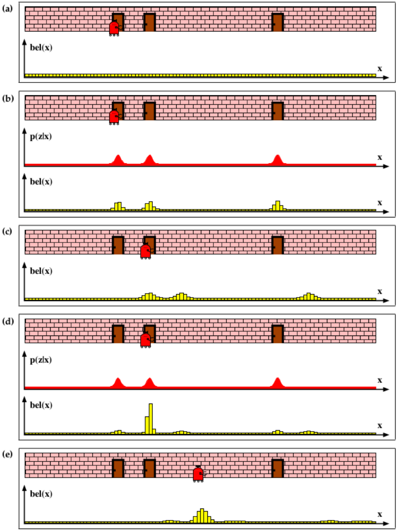

Figure 8.1 Grid localization using a fine-grained metric decomposition. Each picture depicts the position of the robot in the hallway along with its belief $bel(x_t)$, represented by a histogram over a grid.

Probabilistic Robotics. Thrun, Burgard, Fox, 2005

The Answer! Recursive State Estimation

- Two Steps:
  - Prediction based on system dynamics:

$$Bel^{-}(X_t) = \sum_{x_{t-1} \in X} P(X_t \mid x_{t-1}) Bel(x_{t-1})$$

  - Correction based on sensor reading:

$$Bel(X_t) = \eta P(Z_t \mid X_t) Bel^{-}(X_t)$$

Repeat forever.
Again η is a normalizing constant chosen to make the distribution sum to 1.

Prediction Example

- The robot is now moving Right! (or trying to)
- Motion model: Robot is 80% likely to move the direction he intends to move. 20% likely to fail and not move.
- Assume we know that the robot starts in position a, $Bel(X_0) =$

a	b	c	d
1	0	0	0

- Or:
  $Bel(X_0 = a) = 1$
  $Bel(X_0 = b) = 0$
  ...

Prediction Example

- Run one step of prediction:

$$
\begin{aligned}
Bel^{-}(X_1 = a) = \sum_{x_0 \in X} P(x_1 = a \mid x_0) Bel(x_0) \\
= & P(X_1 = a \mid X_0 = a) Bel(X_0 = a) + \\
& P(X_1 = a \mid X_0 = b) Bel(X_0 = b) + \\
& P(X_1 = a \mid X_0 = c) Bel(X_0 = c) + \\
& P(X_1 = a \mid X_0 = d) Bel(X_0 = d)
\end{aligned}
$$

$$= .2 \times 1 + 0 \times 0 + 0 \times 0 + .8 \times 0$$

$$= .2$$

Prediction Example

- Similarly
  $Bel^{-}(X_1 = b) = .8 \times 1 + .2 \times 0 + 0 \times 0 + 0 \times 0 = .8$
  $Bel^{-}(X_1 = c) = 0$
  $Bel^{-}(X_1 = d) = 0$
- Unsurprisingly, $Bel^{-}(X_1) =$

a	b	c	d
.2	.8	0	0

Estimation

Now that we have a prediction, we can update it based on the latest sensor reading:

$$Bel(X_t) = \eta P(Z_t \mid X_t) Bel^{-}(X_t)$$

This is *exactly* what we did when we talked about using Bayes rule to update a prior state estimate based on a sensor reading.

The process is then repeated indefinitely.