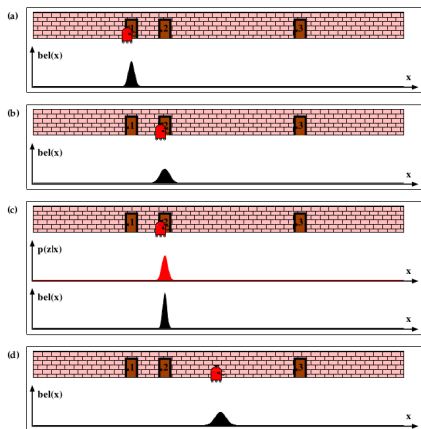


# CS354

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# Probabilistic State Representations: Continuous



Probabilistic Robotics. Thrun, Burgard, Fox, 2005

Figure 7.6 Application of the Kalman filter algorithm to mobile robot localization. All densities are represented by unimodal Gaussians.

# Probability Density Functions

Represent probability distributions over random variables:

- Properties:

- $f(x) \geq 0$

- $\int_{-\infty}^{\infty} f(x)dx = 1$

- Interpretation:

- $P(a \leq x \leq b) = \int_a^b f(x)dx$

# Expectation, Variance

- Expectation (continuous)

$$\mu = \mathbb{E}[x] = \int xf(x)dx$$

- Expectation (discrete)

$$\mathbb{E}[X] = \sum_1^n P(x_i)x_i$$

- Variance

$$\sigma^2 = \mathbb{E}[(x - \mathbb{E}[x])^2]$$

# Normal Distribution

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

(*Normal* because of the central limit theorem.)

# Vector-Valued State

- We'll need to generalize all of this to the case where the state of the system can't be represented as a single number.
- Use a vector  $\mathbf{x}$  to represent the state.

$$\text{cov}(x, y) = \mathbb{E}[(x - \mu_x)(y - \mu_y)]$$

## ■ Properties:

- $\text{cov}(x, y) = \text{cov}(y, x)$
- If  $x$  and  $y$  are independent,  $\text{cov}(x, y) = 0$
- If  $\text{cov}(x, y) > 0$ ,  $y$  tends to increase when  $x$  increases.
- If  $\text{cov}(x, y) < 0$ ,  $y$  tends to decrease when  $x$  increases.

# Covariance Matrix

- Covariance matrix:

$$\text{cov}(\mathbf{x}) = \Sigma_{\mathbf{x}} = \mathbb{E}[(\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^T]$$

- Where  $\mathbf{x}$  is a random vector and  $\hat{\mathbf{x}}$  is the vector mean.
- The entry at row  $i$ , column  $j$  in the matrix is  $\text{cov}(\mathbf{x}_i, \mathbf{x}_j)$
- Multivariate normal distribution is parameterized by the mean vector and covariance matrix.

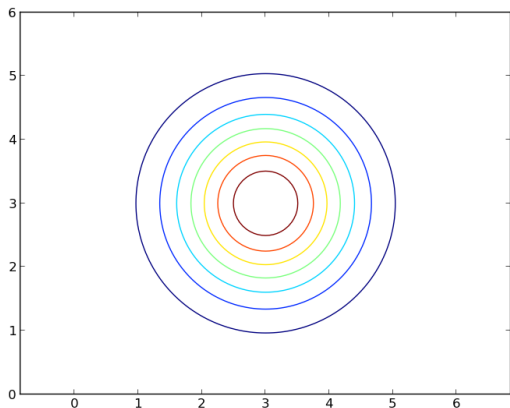


# Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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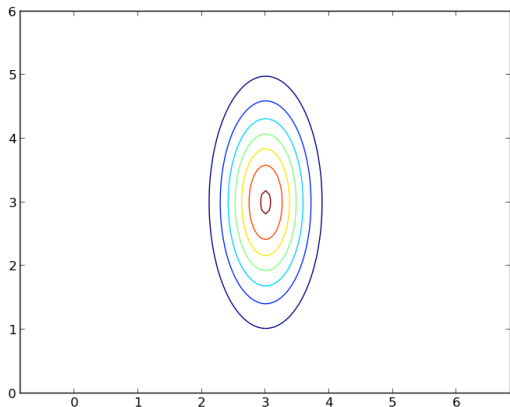


# Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma = \begin{bmatrix} .2 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma = \begin{bmatrix} .2 & 0 \\ 0 & 1 \end{bmatrix}$$

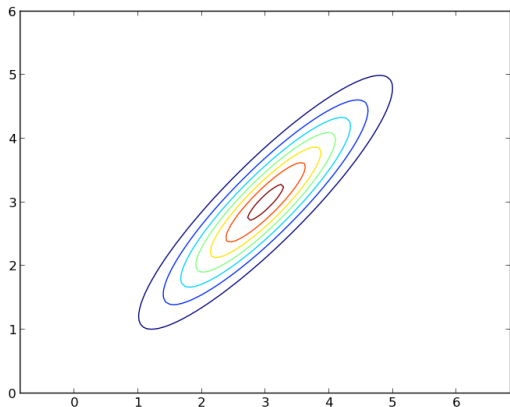


# Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma = \begin{bmatrix} 1 & .9 \\ .9 & 1 \end{bmatrix}$$

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$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma = \begin{bmatrix} 1 & .9 \\ .9 & 1 \end{bmatrix}$$

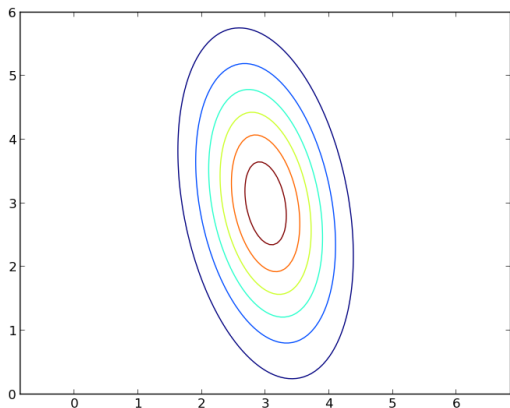


# Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \Sigma = \begin{bmatrix} .5 & -.3 \\ -.3 & 2 \end{bmatrix}$$

# Multivariate PDF Example

$$\mathbf{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} .5 & -.3 \\ -.3 & 2 \end{bmatrix}$$





# Can We Do Recursive State Estimation?

- Two Steps:
  - **Prediction** based on system dynamics:

$$Bel^-(x_t) = \int p(x_t | x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

- **Correction** based on sensor reading:

$$Bel(x_t) = \eta p(z_t | x_t) Bel^-(x_t)$$

YES. The Kalman filter. Next time.