CS354

Nathan Sprague

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Probabilistic State Representations: Continuous

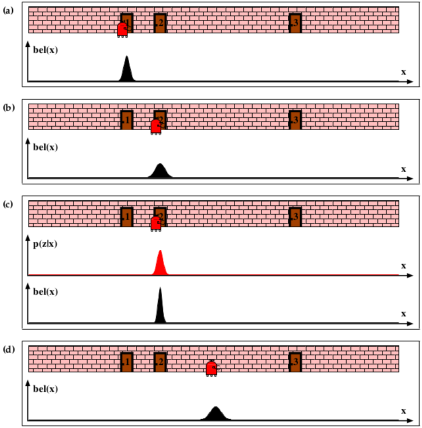

Figure 7.6 Application of the Kalman filter algorithm to mobile robot localization. All densities are represented by unimodal Gaussians.

Probabilistic Robotics. Thrun, Burgard, Fox, 2005

Combining Evidence

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  - x_2 with variance σ_2^2
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- We want to find weights that minimize variance (uncertainty) in the estimate:
  - $\sigma^2 = E[(\hat{x} - E[\hat{x}])^2]$

Combining Evidence – Solution

(Derivation not shown...)

$$\hat{x} = \frac{\sigma_2^2 x_1 + \sigma_1^2 x_2}{\sigma_2^2 + \sigma_1^2}$$

$$\sigma^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_2^2 + \sigma_1^2}$$

Vector-Valued State

- Kalman filter generalizes this to multivariate data.
- Typically the two sources of evidence are coming from:
  - Sensor
  - System Model

Linear System Models

- State can include information other than position. E.g. velocity.
- Linear model of an object moving with a fixed velocity in 2d:
  - $x_{t+1} = x_t + \dot{x}_t dt$
  - $y_{t+1} = y_t + \dot{y}_t dt$
  - $\dot{x}_{t+1} = \dot{x}_t$
  - $\dot{y}_{t+1} = \dot{y}_t$
- dt is time.
- $\dot{x}_t$ is velocity along the x axis.

Linear System Model in Matrix Form

This is equivalent to the last slide:

$$\mathbf{x}_t = \begin{bmatrix} x_t \\ y_t \\ \dot{x}_t \\ \dot{y}_t \end{bmatrix}$$

$$\mathbf{x}_{t+1} = \begin{bmatrix} 1 & 0 & dt & 0 \\ 0 & 1 & 0 & dt \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x}_t$$

Kalman Filter

- Assumes:
  - Linear state dynamics
  - Linear sensor model
  - Normally distributed noise in the state dynamics
  - Normally distributed noise in the sensor model
- State Transition Model:
  - $\mathbf{x}_t = A\mathbf{x}_{t-1} + B\mathbf{u}_{t-1} + \mathbf{w}_{t-1}$
  - $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ (Normal distribution with mean 0 and covariance $\mathbf{Q}$)
- Sensor Model:
  - $\mathbf{z}_t = H\mathbf{x}_t + \mathbf{v}_t$
  - $\mathbf{v} \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$

Kalman Filter in One Slide

- Predict:
  Project the state forward:

$$\hat{\mathbf{x}}_t^- = A\hat{\mathbf{x}}_{t-1} + B\mathbf{u}_{t-1}$$

  Project the covariance of the state estimate forward:

$$\mathbf{P}_t^- = A\mathbf{P}_{t-1}A^T + \mathbf{Q}$$

- Correct:
  Compute the Kalman gain:

$$\mathbf{K}_t = \mathbf{P}_t^- H^T (H\mathbf{P}_t^- H^T + \mathbf{R})^{-1}$$

  Update the estimate with the measurement:

$$\hat{\mathbf{x}}_t = \hat{\mathbf{x}}_t^- + \mathbf{K}_t(\mathbf{z}_t - H\hat{\mathbf{x}}_t^-)$$

  Update the estimate covariance:

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t H)\mathbf{P}_t^-$$