Red Black Trees $+$ Tree Review

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Red-Black Trees

- Another self-balancing binary search tree.
- Five Rules:
  - All nodes are labeled either red or black
  - The root must be black
  - All (empty) leaves are black
  - If a node is red, all children are black
  - Every path from the root to a leaf contains the same number of black nodes
- The **black height** of a tree is the number of black nodes on any path from the root to a leaf.
- The **black depth** of a node is the number of black nodes from the root to that node.

Socrative

- Is this a valid red-black tree?
  - A) No – violates 1 rule
  - B) No – violates 2 rules
  - C) No – violates 3 rules
  - D) No – violates 4 rules
  - E) No – violates 5 rules
  - F) Yes

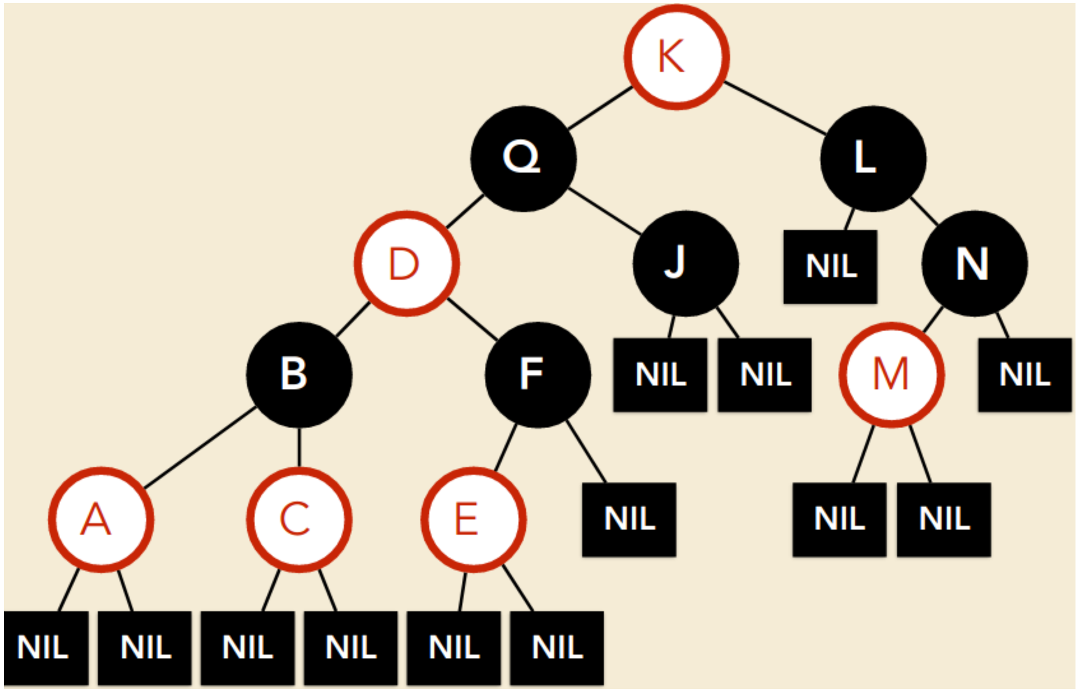


Image credit: Michael Kirkpatrick

Socrative 2

- The left subtree of the root of a particular red-black tree has a black height of 12. Which of the following could be the *height* of the right subtree.

  A) 0

  B) 10

  C) 20

  D) 30

  E) None of the above are possible

  F) Any of the above are possible

Insertion/Removal

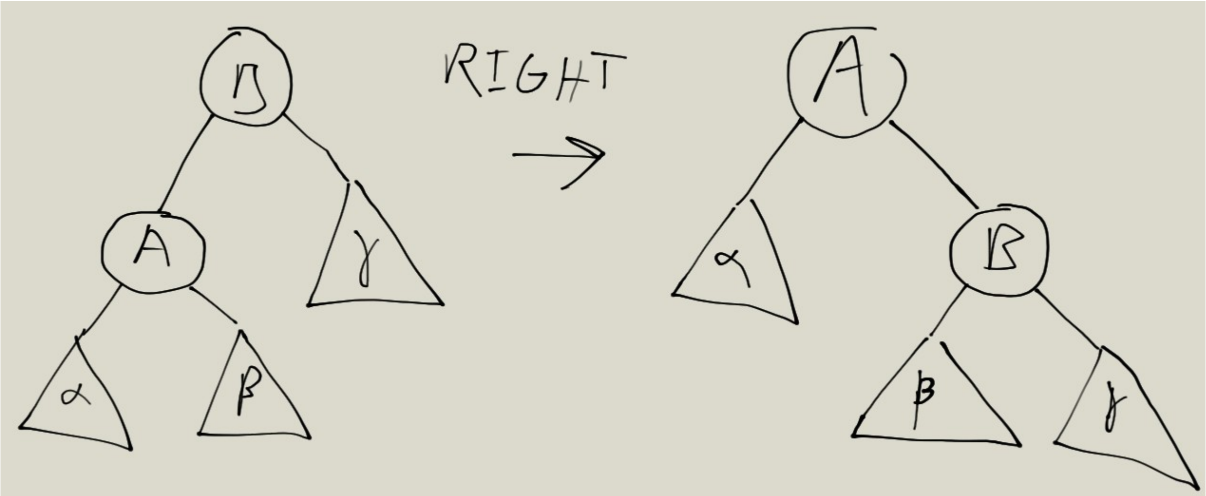
- Newly inserted nodes are colored red
- Perform rotations and recolorings to restore the red-black tree properties
- (We won't worry about the details of insertion and removal)

Red-Black vs. AVL

- Both ensure O(log n) insertion, removal and lookup.
  - Max depth of a red-black tree: $2 \log_2(n+1)$
  - Max depth of an AVL Tree: $\approx 1.44 \log_2(n+2) - 3.28$

- AVL Trees are shorter by a constant factor, but require more rotations.
- Java's TreeMap and TreeSet use red-black trees.

Rotation Reminder



- This does not change the in-order traversal order

  α A β B γ

Exercise (if time)

Insert B, A, F, E into the AVL tree on the right