CS240 HW #2

YOUR NAME HERE

Answers to the following exercises should be submitted through Canvas as a .pdf file. Don't forget to include your name and an honor code statement.

Hints for solving recurrences (these may or may not apply to the problems below):

- Two sums often show up in solving these recurrences. One is the geometric sum:

  $$\sum_{j=0}^{n} r^j = \frac{1 - r^{n+1}}{1 - r}.$$

  From which it follows that:

  $$\sum_{j=0}^{n-1} r^j = \frac{1 - r^{n}}{1 - r}.$$

  A second is the arithmetic sum:

  $$\sum_{j=0}^{n} j = \frac{n(n+1)}{2}$$

  From which it follows that:

  $$\sum_{j=0}^{n-1} = \frac{(n-1)n}{2}.$$

- Remember that $a^{\log_2 b} = b^{\log_2 a}$. This helps, for instance, with simplifying statements like $4^{\log_2 n} = n^{\log_2 4} = n^2$.

- Wolfram alpha (www.wolframalpha.com) is able to solve recurrences like these automatically. Feel free to use this tool to check your answers. Your score will be based on showing the steps required to solve the recurrences using backward substitution.

1. Write the value of the following recurrences for $n = 0$ to 4. (3pts)

   (a) $T(0) = 1$
   $T(n) = 2T(n-1)$

   (b) $T(0) = 1$
   $T(1) = 2$
   $T(n) = 2n + 2T(n-2) + T(n-1)$

   (c) $T(0) = 2$
   $T(1) = 2$
   $T(n) = 3n^2 + T(n-2)$

2. Write the value of the following recurrences for $n = 1$, 2, 4 and 8. (3pts)

   (a) $T(1) = 5$
   $T(n) = 4T(n/2)$

   (b) $T(1) = 5$
   $T(n) = n + T(n/2)$

   (c) $T(1) = 0$
   $T(n) = 3n^2 + n + T(n/2)$

3. Find a closed form solution for each of the following recurrences (for b you may assume n is a power of 4). (4 pts)

   (a) $T(0) = 5$
   $T(n) = 1 + 5T(n-1)$

   (b) $T(1) = 2$
   $T(n) = n + 4T(n/4)$

4. Write recurrence relations that describe the number of times that `System.out.println` is called by each of the following recursive functions. (4pts)

```
public static void fun1(int n) {
    if (n == 0) {
        System.out.println("fun1 line");
    } else {
        for (int i = 0; i < 4; i++) {
          System.out.println("fun1 line");
        }
        fun1(n - 1);
    }
}

public static void fun2(int n) {
    if (n == 0) {
        System.out.println("fun2 line");
    } else {
        fun2(n - 1);
        for (int i = 0; i < 5; i++) {
            System.out.println("fun2 line");
        }
        fun2(n - 1);
    }
}
```

Another hint:

- Since we are counting println statements, you should be able to double check your solution by executing the code and counting the number of lines printed.

5. Use the method of backward substitution (or the tree method) to solve the recurrences from the previous exercise. Show your work. (4pts)

Helpful LaTeX Hints

Thanks to Dr. Kirkpatrick...

- Use $\frac{1}{2}$ instead of 1/2.
- Use $\log n$ instead of $logn$ or $log\ n$.
- Compare n^{123} with n^123.
- Compare $\sqrt{n+3}$ with $\sqrt{n}+3$.
- Use $\Theta(n)$ or $\Omega(n)$ as needed. Note that parentheses don't have a special meaning in math mode when used like this.
- Compare $\lim_{n\to\infty}\frac{n^2}{n^3}$ with $\displaystyle\lim_{n\to\infty}\frac{n^2}{n^3}$.

- Math mode has a lot of built-in equality symbols, such as $<, >, \leq, \geq, \neq, \in, \subset, \subseteq, \equiv$. Thes can be negated by adding \not before them, such as $\not\in, \not\subset, \not\equiv$.
- You can make things **bold** or **bold**. You can also switch to `monospace fonts`. Use \{emph} to make things *italic*. You can also switch to SMALL CAPS FONT or sans-serif font.

Paragraphs

If you want to create a new paragraph, it's really easy to do. Just put a blank line between two paragraphs.

The new paragraph will automatically be indented and you can continue writing. Sometimes, you want more than that, and you want to force an extra blank line. There are a few ways. One is to add \\ at the end, like here:

The new paragraph will start with a blank line in between. You can adjust that spacing with a measurement in brackets, such as using what's at the end of this paragraph to add a 1-inch space.

Now we have even more space in between. Another helpful spacing tool is the tilde character (˜). You can add spaces between words or c h a r a c t e r s. You can also use this in math mode if you want to make a difference between $3n^2$ and $3~n^2$.