CS240

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Merge Sort Analysis

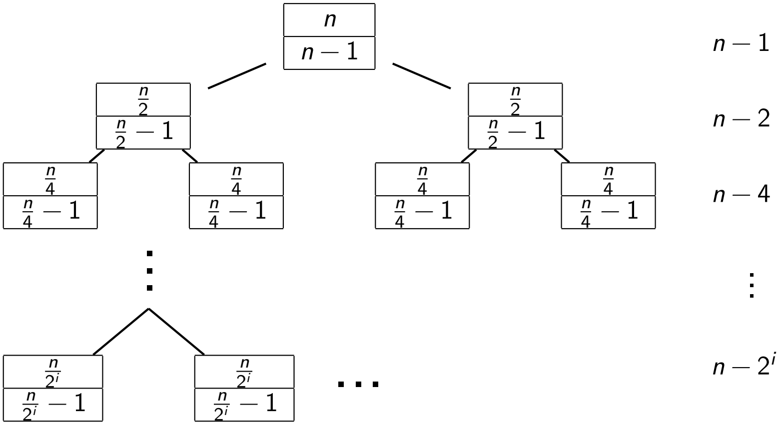
- Basic operation: comparisons.
- Worst case recurrence:

$$W(1) = 0$$
$$W(n) = n - 1 + 2W(\tfrac{n}{2})$$

Merge Sort Analysis

- We could solve this with backward subsitution.
- Instead we will draw the recursion tree.
- On the next slide:
  - Each box represents one instance of our recurrance.
  - Top of the box represents the input value.
  - Bottom of the box represents the amount contributed.
- Sum contributions accross each row of the tree...

Recursion Tree



Merge Sort Analysis

Now we can write down the overall sum:

$$\sum_{j=0}^{i} n - 2^j$$

i is determined by the initial condition...
$n/2^i = 1$
$i = \log_2 n$

Substituting:

$$\sum_{j=0}^{\log_2 n} n - 2^j$$

Merge Sort Analysis

Useful fact: $\sum_{j=0}^{n} 2^j = 2^{n+1} - 1$

$$\sum_{j=0}^{\log_2 n} n - 2^j$$

$$= \sum_{j=0}^{\log_2 n} n - \sum_{j=0}^{\log_2 n} 2^j$$

$$= n(\log_2 n + 1) - \sum_{j=0}^{\log_2 n} 2^j$$

(Apply useful fact.)

$$= n(\log_2 n + 1) - (2^{\log_2 n + 1} - 1)$$

$$= n \log_2 n + n - (2n - 1)$$

$$= n \log_2 n - n + 1$$