CS228 - Strong Induction

Nathan Sprague

February 12, 2014

Material in these slides is from "Discrete Mathematics and Its Applications 7e", Kenneth Rosen, 2012.

Strong Induction

To prove that $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function, we complete two steps:

- Basis Step: We verify that $P(1)$ is true.
- Inductive Step: We show that the conditional statement $P(1) \wedge P(2) \wedge \ldots \wedge P(k) \rightarrow P(k+1)$ is true for all positive integers k.

The inductive hypothesis, "$P(1) \wedge P(2) \wedge \ldots \wedge P(k)$", may also be written "$P(j)$ for $1 \leq j \leq k$".

Strong Induction (Generalized)

To prove that $P(n)$ is true for all integers $n \geq b$ for a fixed integer b, where $P(n)$ is a propositional function, we complete two steps:

- Basis Step: We verify that $P(b), P(b+1), ..., P(b+c)$ are true.
- Inductive Step: We show that the conditional statement $P(b) \wedge P(b+1) \wedge ... \wedge p(k) \rightarrow P(k+1)$ is true for all integers k with $k >= b + c$.

The inductive hypothesis, "$P(b) \wedge P(b+1) \wedge ... \wedge P(k)$", may also be written "$P(j)$ for $b + c \leq j \leq k$".

Example

Proof that if n is an integer greater than 1, then n can be written as the producet of primes:

Let $P(n)$ be the proposition that n can be written as the product of primes.

Example

Proof that if n is an integer greater than 1, then n can be written as the producet of primes:

Let $P(n)$ be the proposition that n can be written as the product of primes.

Basis Step: $P(2)$ is true, because 2 is prime.

Example

Proof that if n is an integer greater than 1, then n can be written as the producet of primes:

Let $P(n)$ be the proposition that n can be written as the product of primes.

Basis Step: $P(2)$ is true, because 2 is prime.

Inductive Step: The inductive hypothesis is the assumption that $P(j)$ is true for all integers j with $2 \leq j \leq k$. In other words, all integers j such that $2 \leq j \leq k$ may be written as a product of primes.

It must be shown that $k + 1$ is a product of primes.

Two cases: $k + 1$ is prime or it is not. If prime, it can clearly be written as the product of primes. If not then it can be written as the product of two positive integers a and b, both of which are between 2 and k.