CS228 - Dijkstra's Algorithm

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Dijkstra's Algorithm

procedure DIJKSTRA(G : weighted, connected simple graph with all weights positive, a: start vertex, z: goal vertex.)

    for $i := 1$ to n
        $L(v_i) := \infty$
    $L(a) := 0$
    $S := \emptyset$
    while $z \notin S$
        $u := $ a vertex not in S with $L(u)$ minimal
        $S := S \cup \{u\}$
        for all vertices v not in S
            if $L(u) + w(u, v) < L(v)$ **then**
                $L(v) := L(u) + w(u, v)$
    return $L(z)$

Running time: $O(n^2)$

Dijkstra's Algorithm Correctness

Outline of inductive proof:

Inductive hypothesis is the following assertion:

At the kth iteration

- the label of every vertex v in S is the length of a shortest path from a to this vertex, and
- the label of every vertex not in S is the length of a shortest path from a to this vertex that contains only (besides the vertex itself) vertices in S.

Example 1

Step	S	$L(a)$	$L(b)$	$L(c)$	$L(d)$	$L(e)$	$L(z)$
0	$\emptyset$	0	INF	INF	INF	INF	INF

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1	a	0	4	2	INF	INF	INF

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0	$\emptyset$	0	INF	INF	INF	INF	INF
1	a	0	4	2	INF	INF	INF
2	a, b	0	4	2	INF	5	INF

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Step	S	$L(a)$	$L(b)$	$L(c)$	$L(d)$	$L(e)$	$L(z)$
0	∅	0	INF	INF	INF	INF	INF
1	a	0	4	2	INF	INF	INF
2	a, b	0	4	2	INF	5	INF
3	a, b, c	0	4	2	3	5	INF