CS228 - Bayes' Theorem

Nathan Sprague

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Material in these slides is from "Discrete Mathematics and Its Applications 7e", Kenneth Rosen, 2012.

Bayes' Theorem

Theorem

$$p(F|E) = \frac{p(E|F)p(F)}{p(E)}$$

Where F *and* E *are events such that* $p(F) \neq 0$ *and* $p(E) \neq 0$.

Bayes' Theorem Derivation

By definition of conditional probability:

$$p(F|E) = \frac{p(F \cap E)}{p(E)} \quad \text{and} \quad p(E|F) = \frac{p(F \cap E)}{p(F)}$$

Therefore:

$$p(F \cap E) = p(F|E)p(E) = p(E|F)p(F)$$

Substituting:

$$p(F|E) = \frac{p(E|F)p(F)}{p(E)}$$

Law of Total Probality

Partition sample space into disjoint events
$S = A_1 \cup \ldots \cup A_n$
Then $p(B) = \sum_i p(B \cap A_i)$
Then $p(B) = \sum_i p(B|A_i)p(A_i)$

Binary case: $p(B) = p(B|A)p(A) + p(B|\overline{A})p(\overline{A})$

Revised Bayes' Theorem

Theorem

$$p(F|E) = \frac{p(E|F)p(F)}{p(E|F)p(F) + p(E|\overline{F})p(\overline{F})}$$

Where F and E are events such that $p(F) \neq 0$ and $p(E) \neq 0$.

Spam Detection Example

Gather some statistics about email:

$p(Spam) = .9, \qquad p(\overline{Spam}) = .1$
$p(Viagra|Spam) = .2, \qquad p(Viagra|\overline{Spam}) = .001$

Spam Detection Example

Gather some statistics about email:

$p(Spam) = .9, \qquad p(\overline{Spam}) = .1$
$p(Viagra|Spam) = .2, \qquad p(Viagra|\overline{Spam}) = .001$

Use this data to classify an incoming message that contains the word Viagra:

$$\begin{aligned} P(Spam|Viagra) &= \frac{P(Viagra|Spam)P(Spam)}{P(Viagra)} \\ &= \frac{P(Viagra|Spam)P(Spam)}{p(Viagra|Spam)p(Spam) + p(Viagra|\overline{Spam})p(\overline{Spam})} \\ &= \frac{.2 \times .9}{.2 \cdot .9 + .001 \times .1} \approx 0.999 \end{aligned}$$

(In practice, we would combine evidence from a large number of words.)