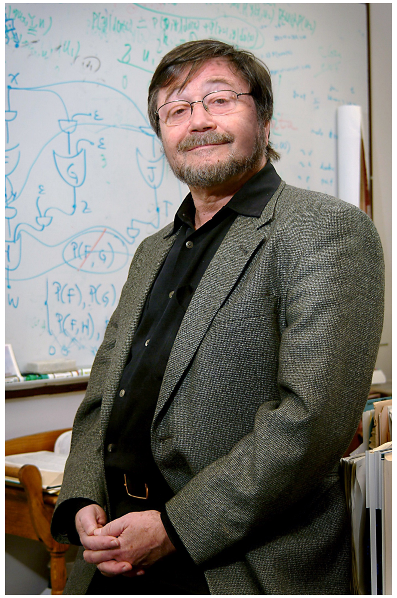
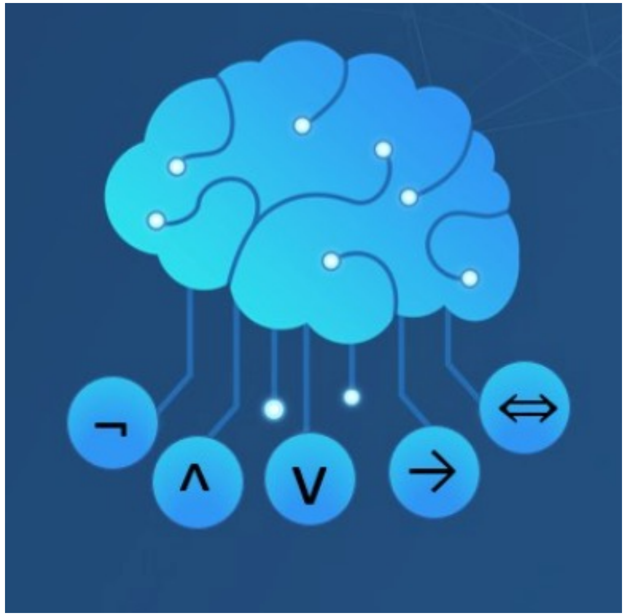


Artificial Intelligence



First Order Logic (part 1)

CS 444 – Spring 2021

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JAMES MADISON UNIVERSITY

Much of this lecture is taken from
Dan Klein and Pieter Abbeel AI class at UC Berkeley

Example Knowledge Base

The law says that it is a crime for an American to sell weapons to hostile nations. The country, Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is an American.

Prove that Col. West is a criminal.

Represent this problem in FOL and show a prove that Col. West is a criminal.

Example Knowledge Base

The law says that it is a crime for an American to sell weapons to hostile nations. The country, Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is an American.

... it is a crime for an American to sell weapons to hostile nations:

$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Longrightarrow Criminal(x)$$

Nono .. Has some missiles

$$\exists x\ Owns(Nono, x) \wedge Missile\ (x):$$

$$Owns(Nono, M_1) \text{ and } Missle(M_1)$$

.. All of its missiles were sold to it by Col. West:

$$\forall x\ Owns(Nono, x) \wedge Missile\ (x) \Longrightarrow Sells(West, x, Nono)$$

Missiles are weapons:

$$\forall x\ Missile\ (x) \Longrightarrow Weapon(x)$$

An enemy of America counts as hostile: $\forall\ x\ Enemy(x, America) \Longrightarrow Hostile(x)$

Example Knowledge Base

The law says that it is a crime for an American to sell weapons to hostile nations. The country, Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is an American.

… it is a crime for an American to sell weapons to hostile nations:

$$American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \implies Criminal(x)$$

Nono .. Has some missiles

$$\exists x\ Owns(Nono, x) \land Missile(x):$$

.. All of its missiles were sold to it by Col. West:

$$\forall x\ Owns(Nono, x) \land Missile(x) \implies Sells(West, x, Nono)$$

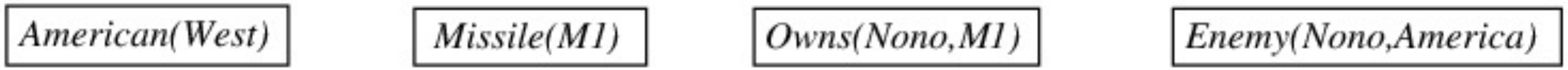
Missiles are weapons: $\forall x\ Missile(x) \implies Weapon(x)$

An enemy of America counts as hostile: $\forall x\ Enemy(x, America) \implies Hostile(x)$

West, who is an American.. → American(West)

The country Nono, an enemy of America → Enemy(Nono, America)

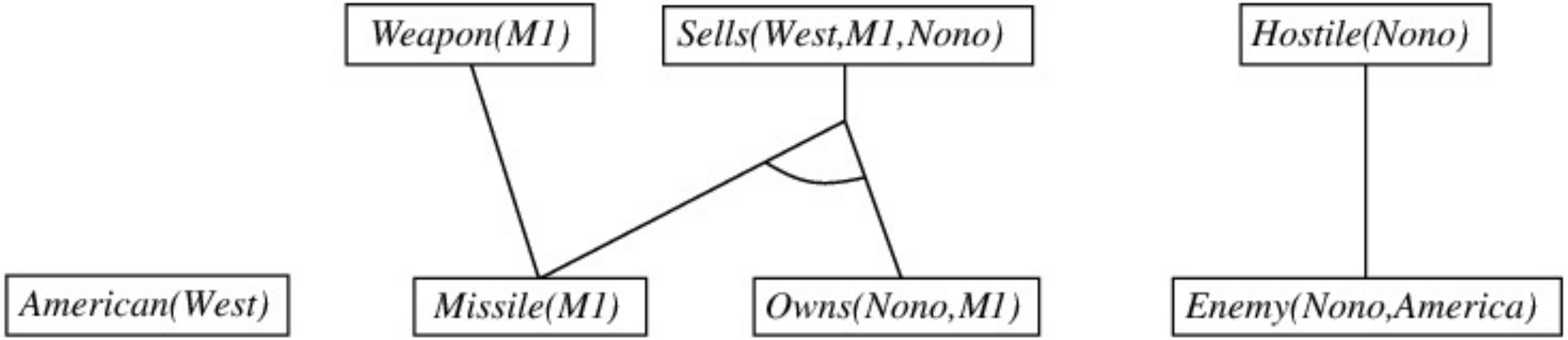
Forward Chaining Example



Forward Chaining Example

$\forall x\ Owns(Nono, x) \land Missile(x) \Longrightarrow Sells(West, x, Nono)$

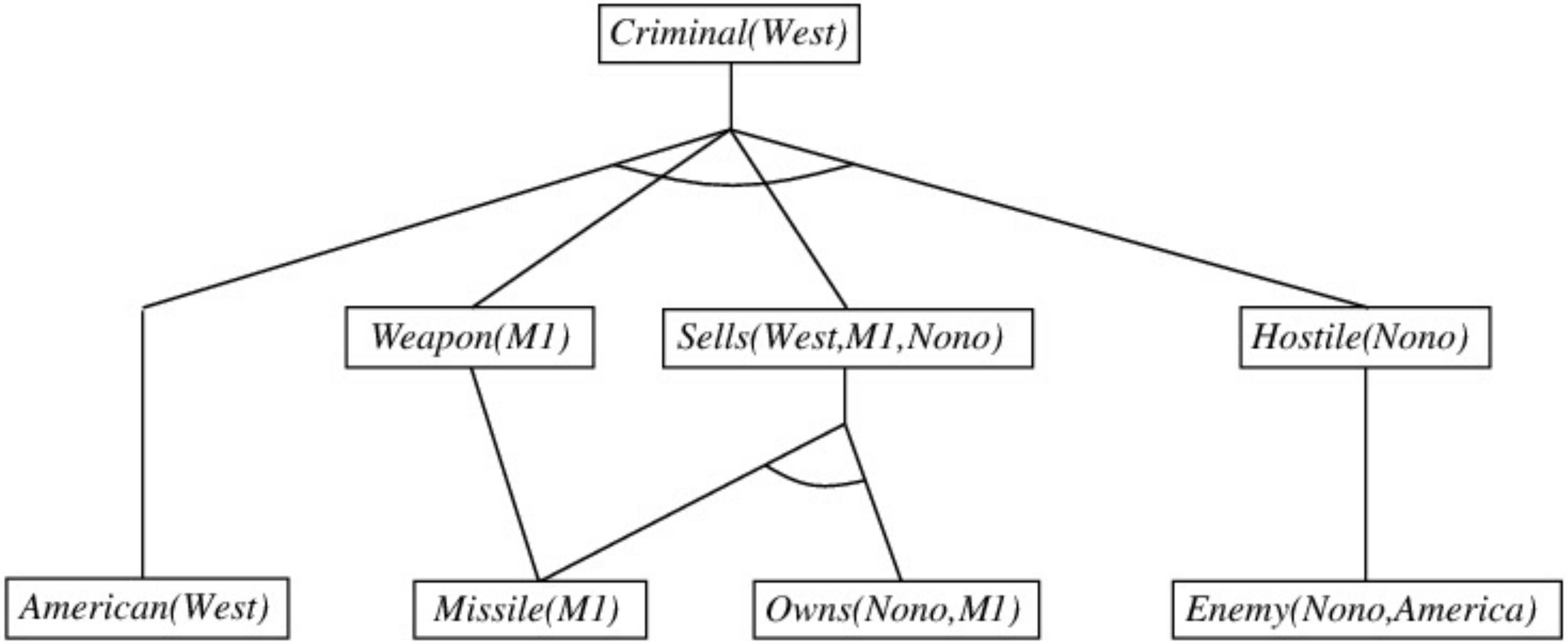
$\forall x\ Missile(x) \Longrightarrow Weapon(x)$



An enemy of America counts as hostile: $\forall\ x\ Enemy(x, America) \Longrightarrow Hostile(x)$

Forward Chaining Example

$$American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Longrightarrow Criminal(x)$$



Forward Chaining Algorithm

Operates the same as with propositional logic. Combines sentences

until it reaches a fixed point. When KB is written with definite clauses, and does not contain functional symbols, it is **complete**. It is bound by $O(pn^k)$ where *k* is the maximum arity of all predicate functions, *n* is the number of constant symbols, and *p* is the number of predicates.

Is it complete for knowledge bases with functional symbols?

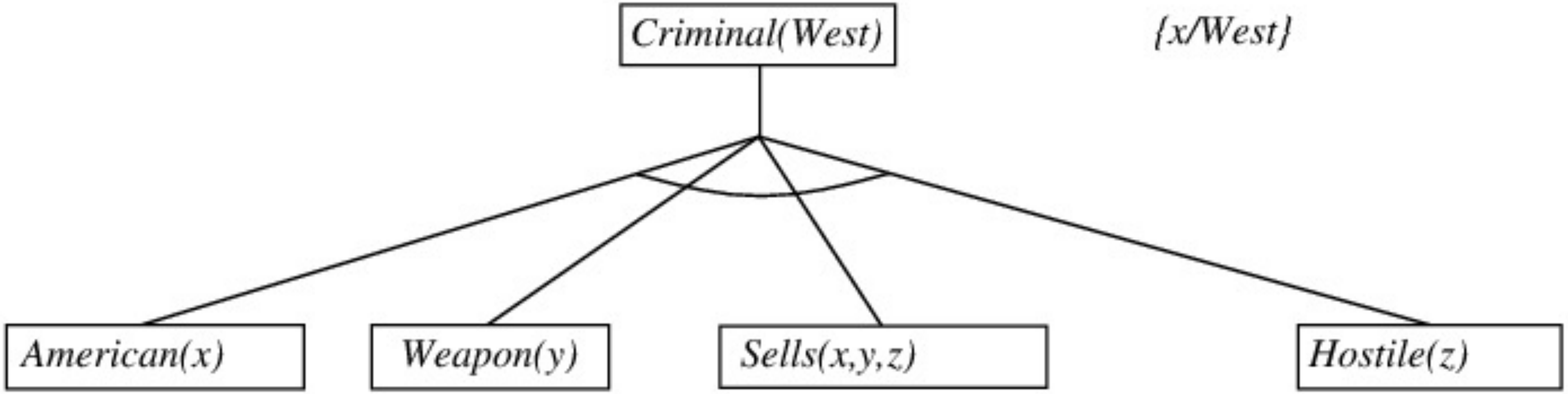
The short answer is yes, but logically complete means that if the statement is true, we can proof that in a finite amount of time.

Because functions can be applied recursively, (recall S(S(S(n))) for example. So, it is not possible to decide whether the knowledge base entails some fact (when it doesn't, we would loop forever). When it does, you can imagine a routine like IDS (iterative deepening search), where the level of recursion is controlled. We call this **semi-decidable**.

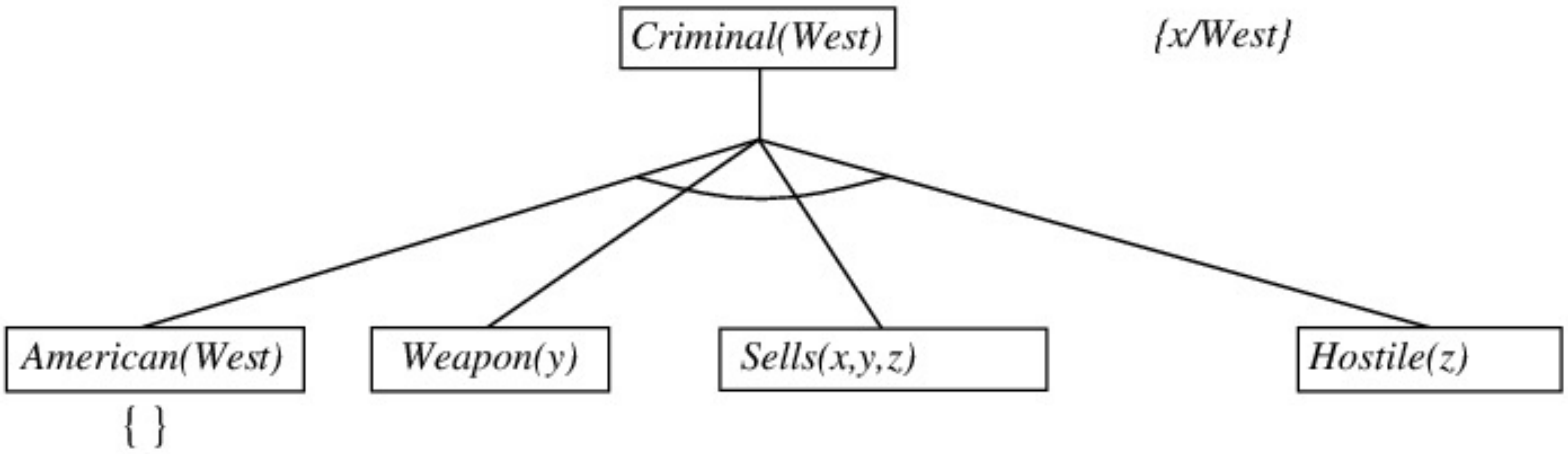
Backward Chaining Examples

Criminal(West)

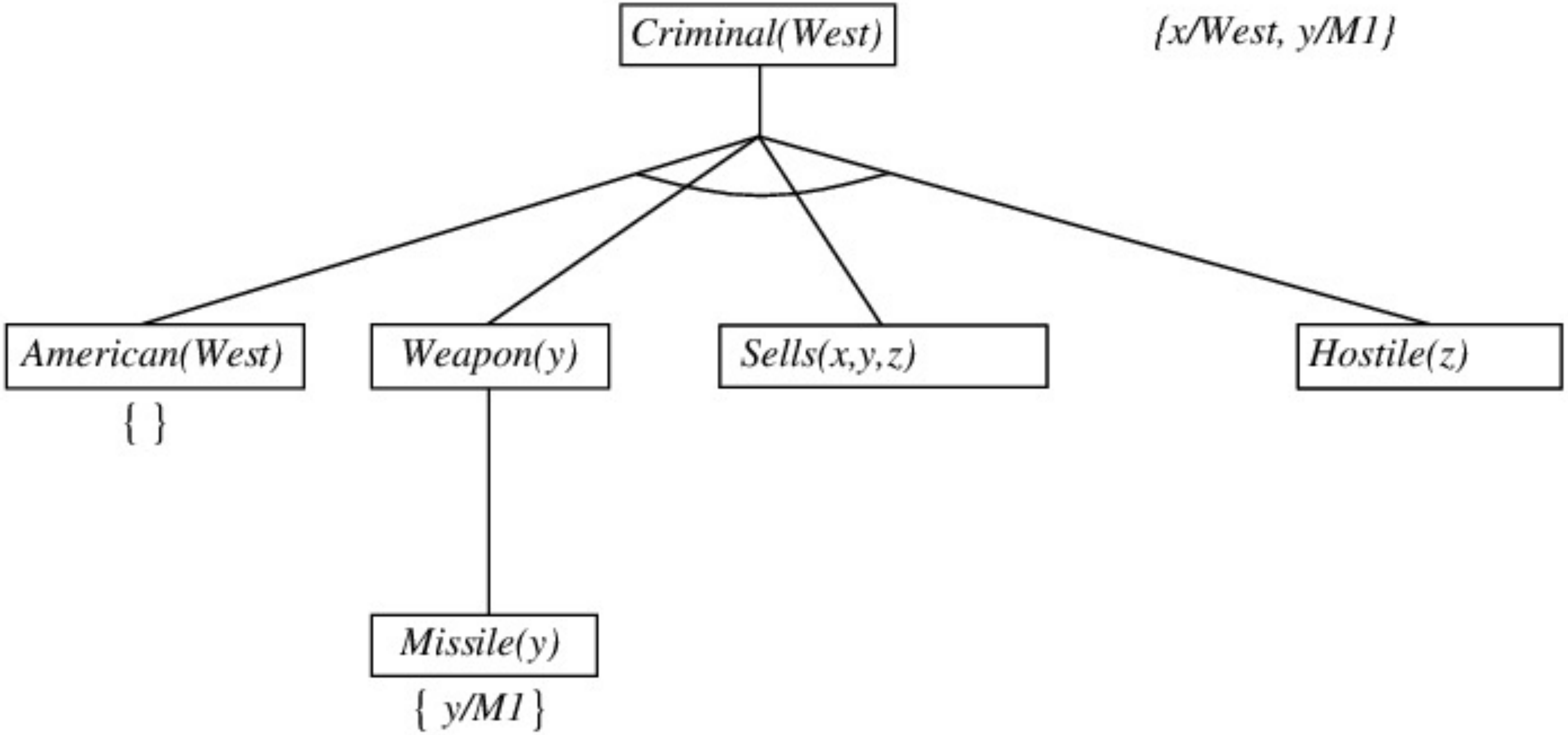
Backward Chaining Examples



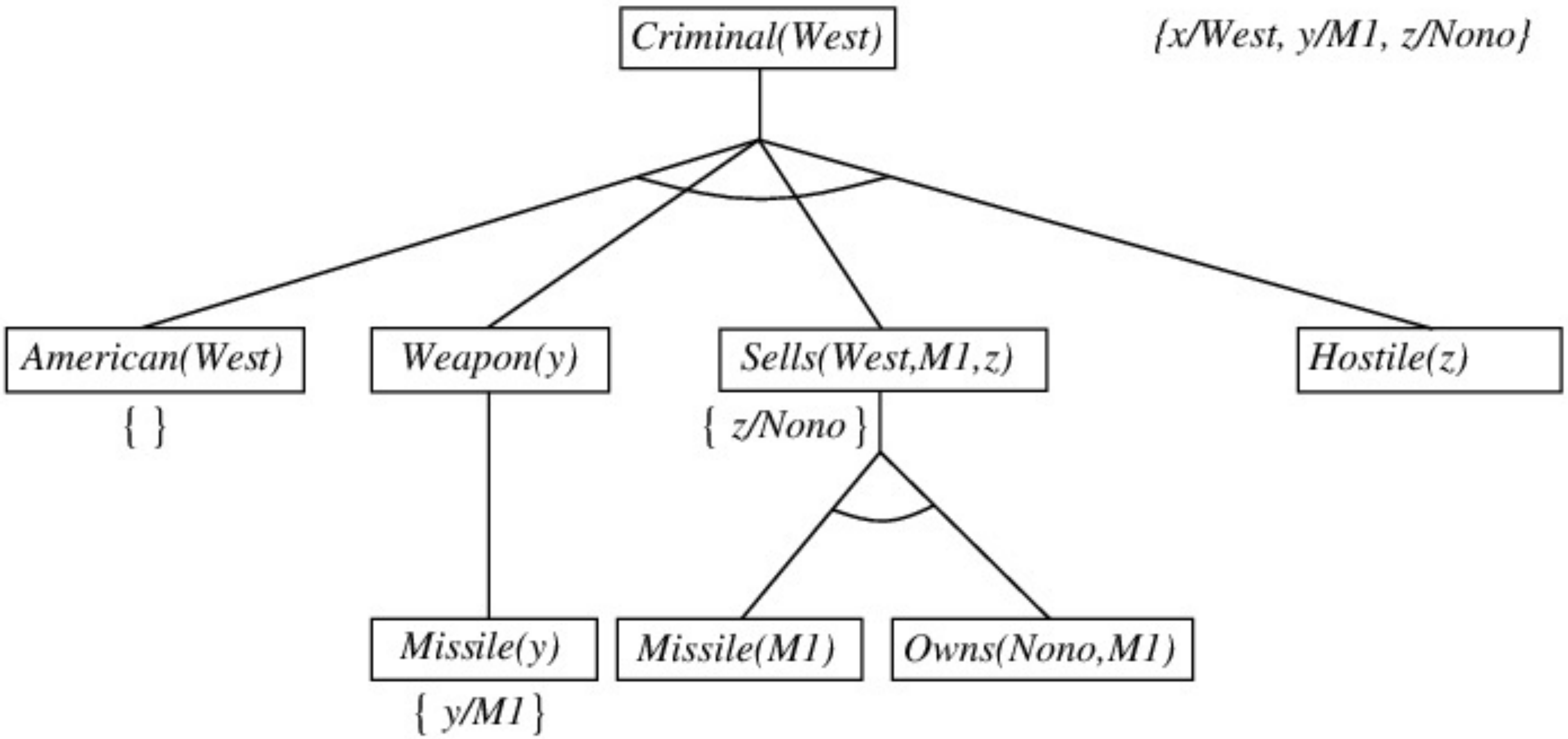
Backward Chaining Examples



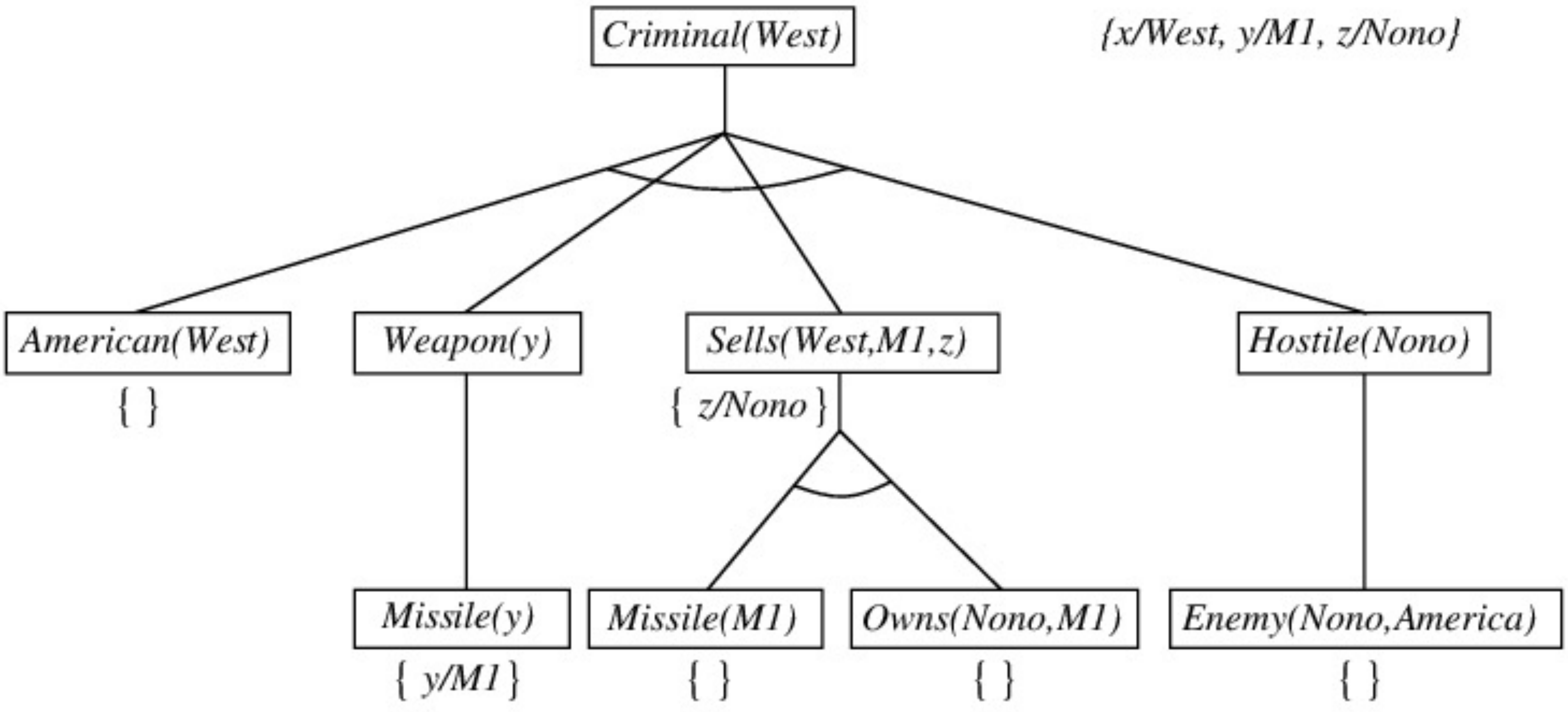
Similar Problems



Backward Chaining Examples



Backward Chaining Examples

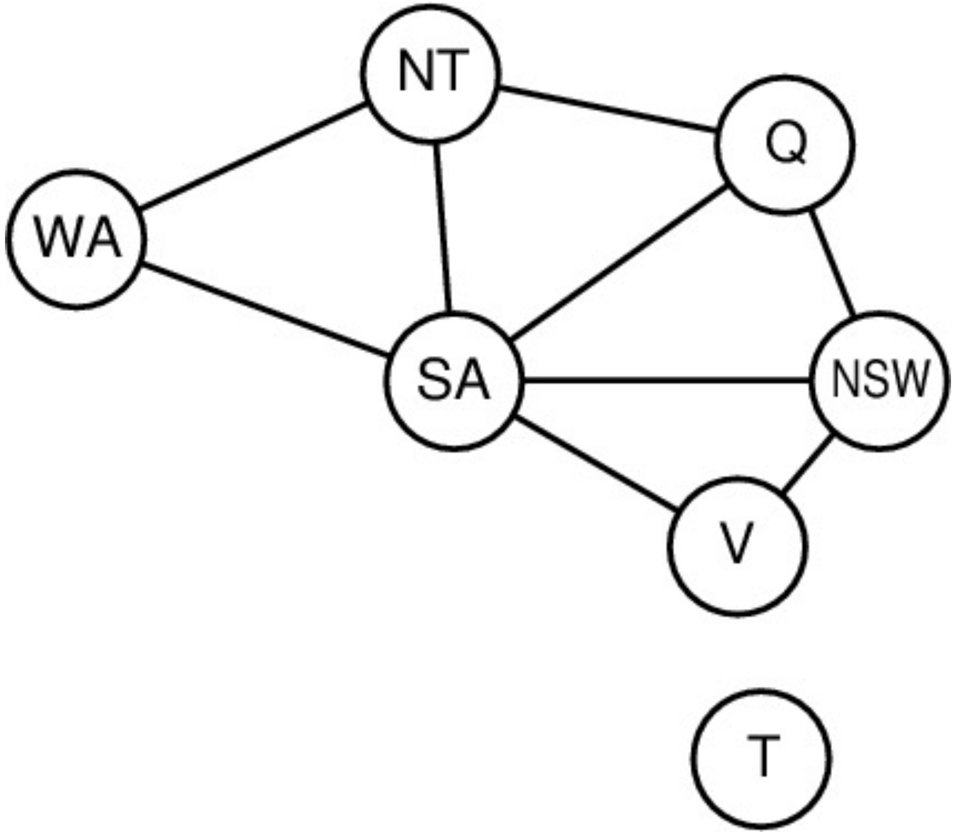


Backward-Chaining

Depth-first recursive proof search: space is linear in size of proof (good)

But, incomplete because of infinite loops (can be fixed) as we will see soon.

Similar Problems



Diff(wa, nt) ∧ Diff(wa, sa) ∧ Diff(nt, q) ∧ Diff(nt, sa) ∧ Diff(q, nsw) ∧ Diff(q, sa) ∧ Diff(nsw, v) ∧ Diff(nsw, sa) ∧ Diff(v, sa)

$\Longrightarrow$ Colorable()

Colorable() is inferred iff the CSP has a solution. CSPs include 3SAT as a special case, hence, matching is NP-Hard.

Resolution – Removing UI (∀)

We need to convert our sentences to CNF (just like propositional logic).

How do we deal with **universal instantiation**:

$$\forall x\ American(x) \land Weapon(y) \land Sell(x, y, z) \land Hostile\ (z) \Longrightarrow Criminal(x)$$

Since we know how to do unification with variables, we can simply drop the UI terms.

$$American(x) \land Weapon(y) \land Sell(x, y, z) \land Hostile\ (z) \Longrightarrow Criminal(x)$$

Convert to CNF:

$$\neg(American(x) \land Weapon(y) \land Sell(x, y, z) \land Hostile\ (z)) \lor Criminal(x)$$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sell(x, y, z) \lor \neg Hostile\ (z) \lor Criminal(x)$$

And then use Unification to put in constants from the KB.

$$\neg American(West) \lor \neg Weapon(y) \lor \neg Sell(West, y, z) \lor \neg Hostile\ (z) \lor Criminal(West) \qquad \{x/West\}$$

Resolution -- Removing Existential Instantiation

We need to convert our sentences to CNF (just like propositional logic).

How do we deal with **universal instantiation**:

$\exists x$ Crown(x) $\wedge$ OnHead(x, John)

We know that some object exists that is the crown that is on John's head.

Thus, we can create a new constant, *k*, as long as *k* does not appear anywhere else in the knowledge base. Thus, we can get:

$$\text{Crown}(C_1) \wedge \text{OnHead}(C_1, \text{John})$$

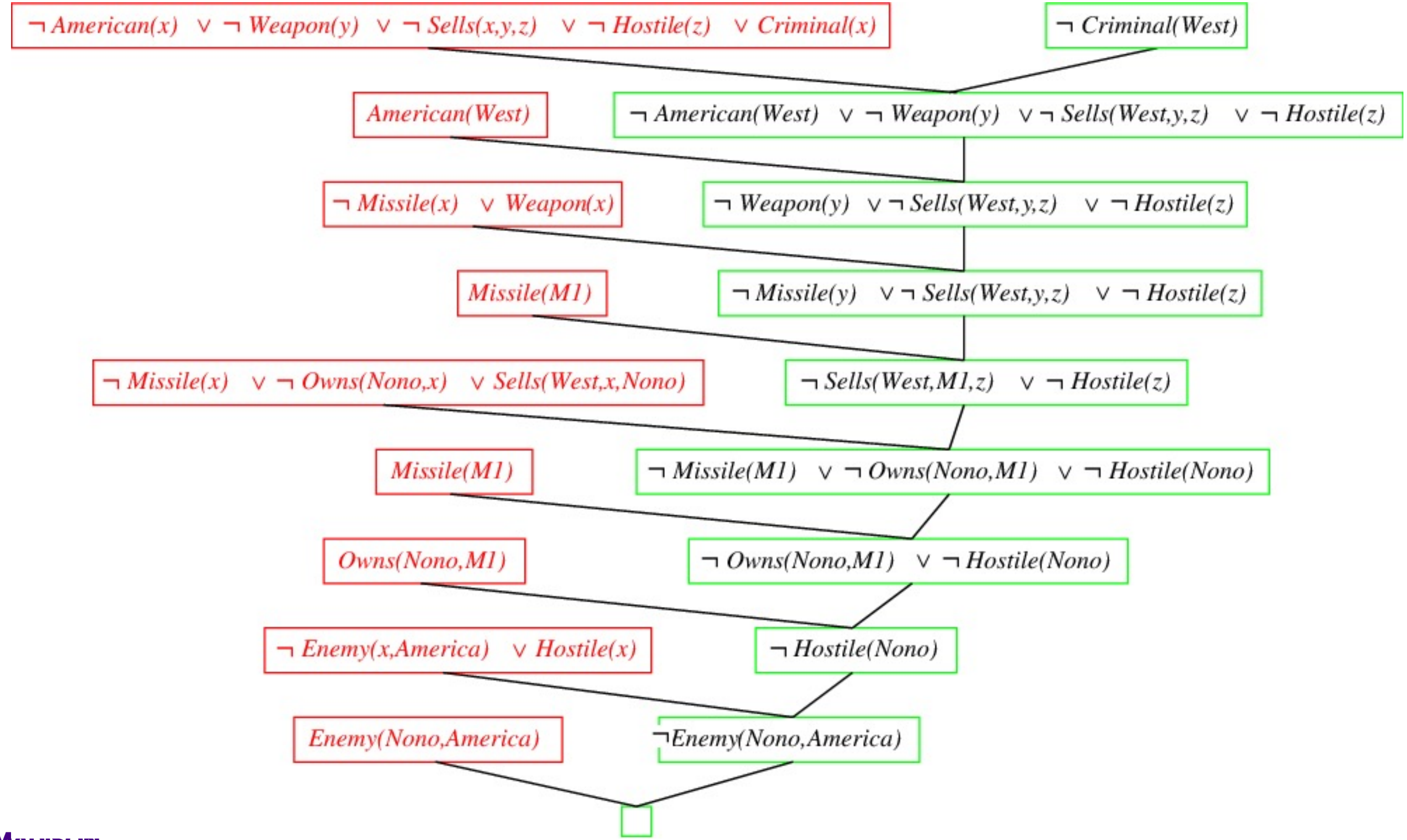
This process is called **Skolemization** (and the C_1 is a **skolem** constant).

Example: Nono .. Has some missiles

$\exists x$ Owns(Nono, x) $\wedge$ Missile (x):

Owns(Nono, M_1) and Missle(M_1)

Example of Resolution



Example of Resolution Using a Table

Original	CNF
∀x American (x) ∧ Weapon(y) ∧ Sells(x, y, z) ∧ Hostile(z) ⟹ Criminal(x)	¬(American (x) ∧ Weapon(y) ∧ Sells(x, y, z) ∧ Hostile(z)) ∨ Criminal(x) (drop universal instantiation and perform implication elimination)
	¬American (x) ∨ ¬Weapon(y) ∨ ¬Sells(x, y, z) ∨ ¬Hostile(z) ∨ Criminal(x) (deMorgans)
∃x Owns(Nono, x) ∧ Missile (x)	Owns(Nono, M_1) ∧ Missle(M_1) (existential elimination via skolem constant)
	Owns(Nono, M_1) (and-elimination for the next 2 lines)
	Missle(M_1)
∀x Owns(Nono, x) ∧ Missile (x) ⟹ Sells(West, x, Nono)	¬(Owns(Nono, x) ∧ Missile (x)) ∨ Sells(West, x, Nono) (drop universal instantiation and perform implication elimination)
	¬Owns(Nono, x) ∨ ¬(Missile (x) ∨ Sells(West, x, Nono) (deMorgans)
∀x Missile (x) ⟹ Weapon(x)	¬ Missile (x) ∨ Weapon(x) (drop universal instantiation and perform implication elimination)
∀ x Enemy(x, America) ⟹ Hostile(x)	¬ (Enemy(x, America) ∨ Hostile (x) (drop universal instantiation and perform implication elimination)
American(West)	American(West)
Enemy(Nono, America)	Enemy(Nono, America)
Prove Criminal(West), so α = Criminal(West)	¬Criminal(West)

Example of Resolution using a Table

Line #	Clause	Lines and rule
1	¬American (x) ∨ ¬Weapon(y) ∨ ¬Sells(x, y, z) ∨ ¬Hostile(z) ∨ Criminal(x)	Given
2	Missle(M_1)	
3	Owns(Nono, M_1)	
4	¬Owns(Nono, x) ∨ ¬Missile (x) ∨ Sells(West, x, Nono)	
5	¬ Missile (x) ∨ Weapon(x)	
6	¬ Enemy(x, America) ∨ Hostile (x)	
7	American(West)	
8	Enemy(Nono, America)	
9	¬Criminal(West)	$\neg\alpha$
10	¬American (West) ∨ ¬Weapon(y) ∨ ¬Sells(West, y, z) ∨ ¬Hostile(z)	1/9 {x/West}
11	Hostile(Nono)	6/8 {x/Nono}

Example of Resolution using a Table

Line #	Clause	Lines and rule
1	¬American (x) ∨ ¬Weapon(y) ∨ ¬Sells(x, y, z) ∨ ¬Hostile(z) ∨ Criminal(x)	Given
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5	¬ Missile (x) ∨ Weapon(x)	
6	¬ Enemy(x, America) ∨ Hostile (x)	
7	American(West)	
8	Enemy(Nono, America)	
9	¬Criminal(West)	$\neg\alpha$
10	¬American (West) ∨ ¬Weapon(y) ∨ ¬Sells(West, y, z) ∨ ¬Hostile(z)	1/9 {x/West}
11	Hostile(Nono)	6/8 {x/Nono}
12	¬American (West) ∨ ¬Weapon(y) ∨ ¬Sells(West, y, Nono)	10/11 {z/Nono}

Example of Resolution using a Table

Line #	Clause	Lines and rule
1	¬American (x) ∨ ¬Weapon(y) ∨ ¬Sells(x, y, z) ∨ ¬Hostile(z) ∨ Criminal(x)	Given
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11	Hostile(Nono)	6/8 {x/Nono}
12	¬American (West) ∨ ¬Weapon(y) ∨ ¬Sells(West, y, Nono)	10/11 {z/Nono}
13	¬Weapon(y) ∨ ¬Sells(West, y, Nono)	7/12

Example of Resolution using a Table

Line #	Clause	Lines and rule
1	¬American (x) ∨ ¬Weapon(y) ∨ ¬Sells(x, y, z) ∨ ¬Hostile(z) ∨ Criminal(x)	Given
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11	Hostile(Nono)	6/8 {x/Nono}
12	¬American (West) ∨ ¬Weapon(y) ∨ ¬Sells(West, y, Nono)	10/11 {z/Nono}
13	¬Weapon(y) ∨ ¬Sells(West, y, Nono)	7/12
14	¬Owns(Nono, M_1) ∨ Sells(West, M_1, Nono)	2/4 {x/M_1}

Example of Resolution using a Table

Line #	Clause	Lines and rule
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5	¬ Missile (x) ∨ Weapon(x)	
6	¬ Enemy(x, America) ∨ Hostile (x)	
7	American(West)	
8	Enemy(Nono, America)	
9	¬Criminal(West)	$\neg\alpha$
10	¬American (West) ∨ ¬Weapon(y) ∨ ¬Sells(West, y, z) ∨ ¬Hostile(z)	1/9 {x/West}
11	Hostile(Nono)	6/8 {x/Nono}
12	¬American (West) ∨ ¬Weapon(y) ∨ ¬Sells(West, y, Nono)	10/11 {z/Nono}
13	¬Weapon(y) ∨ ¬Sells(West, y, Nono)	7/12
14	¬Owns(Nono, M_1) ∨ Sells(West, M_1, Nono)	2/4 {x/M_1}
15	Sells(West, M_1, Nono)	3/14

Example of Resolution using a Table

Line #	Clause	Lines and rule
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13	¬Weapon(y) ∨ ¬Sells(West, y, Nono)	7/12
14	¬Owns(Nono, M_1) ∨ Sells(West, M_1, Nono)	2/4 {x/M_1}
15	Sells(West, M_1, Nono)	3/14
16	Weapon(M_1)	2/5 {x/ M_1}

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Example of Resolution using a Table

Line #	Clause	Lines and rule
1	¬American (x) ∨ ¬Weapon(y) ∨ ¬Sells(x, y, z) ∨ ¬Hostile(z) ∨ Criminal(x)	Given
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8	Enemy(Nono, America)	
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11	Hostile(Nono)	6/8 {x/Nono}
12	¬American (West) ∨ ¬Weapon(y) ∨ ¬Sells(West, y, Nono)	10/11 {z/Nono}
13	¬Weapon(y) ∨ ¬Sells(West, y, Nono)	7/12
14	¬Owns(Nono, M_1) ∨ Sells(West, M_1, Nono)	2/4 {x/M_1}
15	Sells(West, M_1, Nono)	3/14
16	Weapon(M_1)	2/5 {x/ M_1}

Example of Resolution using a Table

Line #	Clause	Lines and rule
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14	¬Owns(Nono, M_1) ∨ Sells(West, M_1, Nono)	2/4 {x/M_1}
15	Sells(West, M_1, Nono)	3/14
16	Weapon(M_1)	2/5 {x/ M_1}
17	¬Sells(West, M_1, Nono)	12/16 {y/M_1}

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Example of Resolution using a Table

Line #	Clause	Lines and rule
1	¬American (x) ∨ ¬Weapon(y) ∨ ¬Sells(x, y, z) ∨ ¬Hostile(z) ∨ Criminal(x)	Given
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15	Sells(West, M_1, Nono)	3/14
16	Weapon(M_1)	2/5 {x/ M_1}
17	¬Sells(West, M_1, Nono)	12/16 {y/M_1}
18	▢	15/17

Summary of FOL