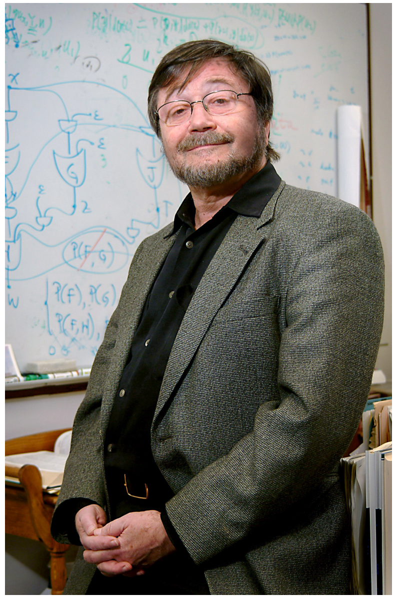
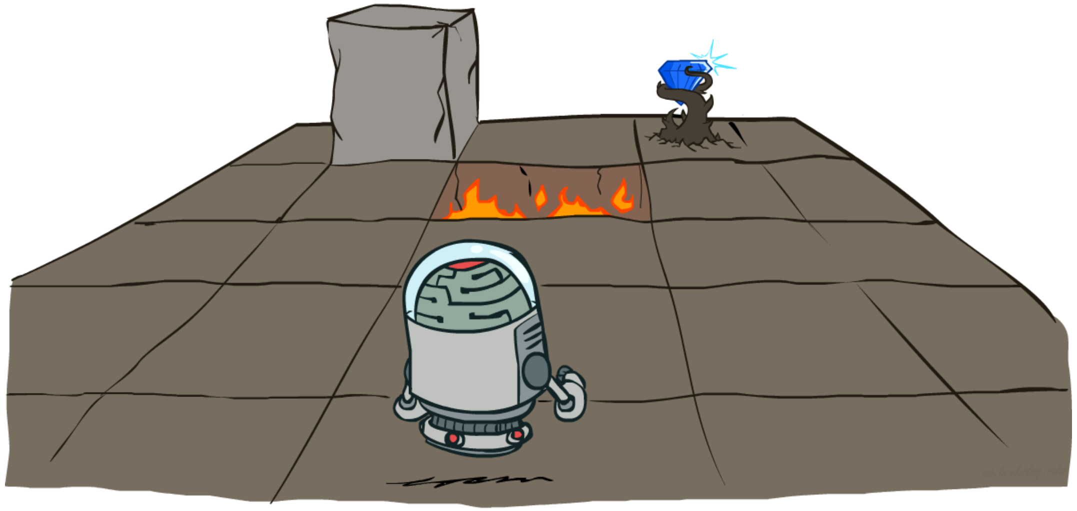


Artificial Intelligence



Markov Decision Processes (Part 2)

CS 444 – Spring 2021

Dr. Kevin Molloy

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Much of this lecture is taken from
Dan Klein and Pieter Abbeel AI class at UC Berkeley

Announcements

- HW 6 is will be release tomorrow. Due next Tuesday evening.

- Quiz 3a will be published Tuesday after class and due **before** class on Thursday (March 11^{th}) (so, March 10^{th} in reality).

Example: Grid World

- A maze-like problem
  - The agent lives in a grid
  - Walls block the agent's path
- Noisy movement: actions do not always go as planned
  - 80% of the time, the action North takes the agent North (if there is no wall there)
  - 10% of the time, North takes the agent West; 10% East
  - If there is a wall in the direction the agent would have been taken, the agent stays put
- The agent receives rewards each time step
  - Small "living" reward each step (can be negative)
  - Big rewards come at the end (good or bad)
- Goal: maximize sum of rewards

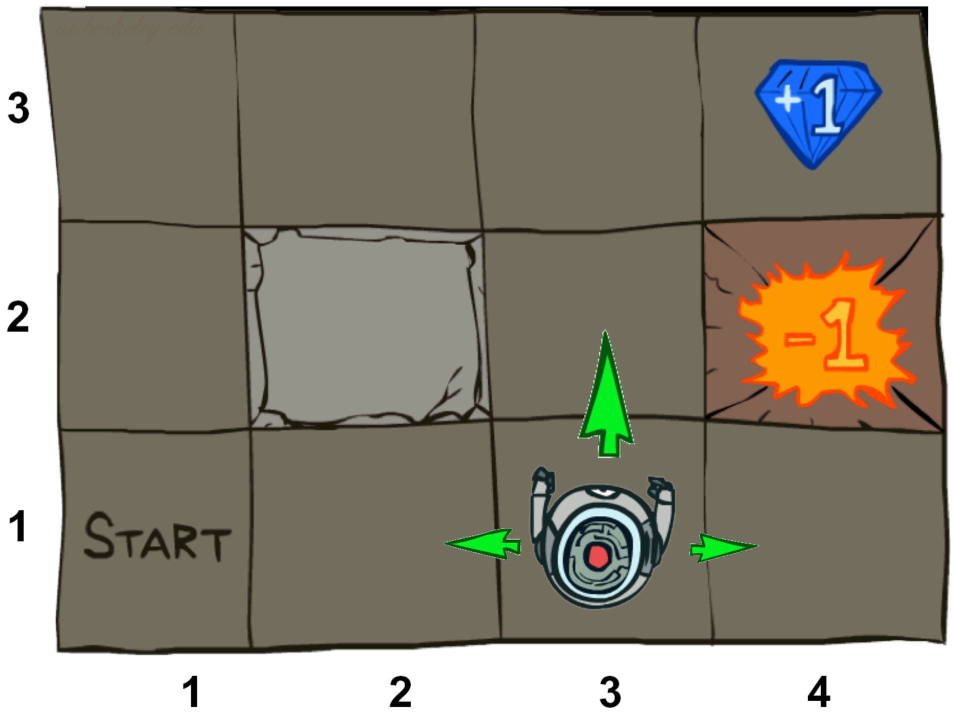
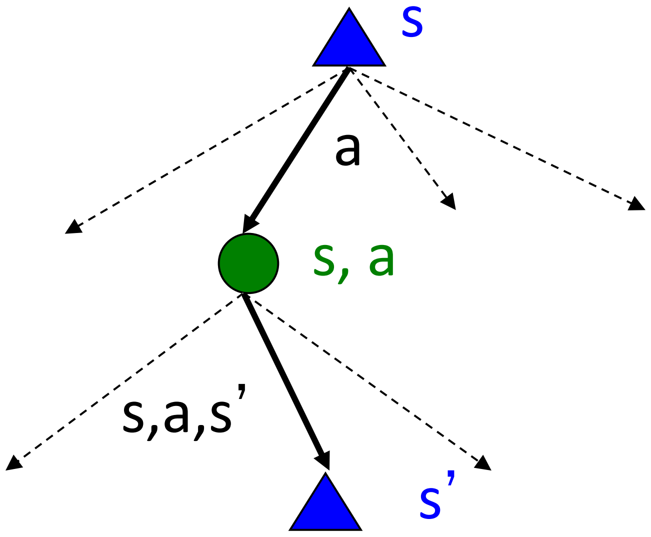


Figure from Berkley AI

Recap: MDPs

- Markov decision processes:
  - Set of states S
  - Start state s_0
  - Set of actions A
  - Transitions P(s’|s,a) (or T(s,a,s’))
  - Rewards R(s,a,s’) (and discount γ)



- MDP quantities so far:
  - Policy = Choice of action for each state
  - Utility = sum of (discounted) rewards

Figure from Berkley AI

Optimal Quantities

- The value (utility) of a state s:

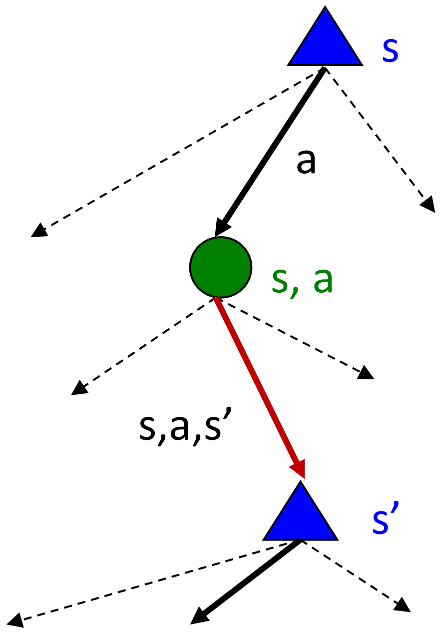
  $V^*(s)$ = expected utility starting in s and acting optimally

- The value (utility) of a q-state (s,a):

  $Q^*(s,a)$ = expected utility starting out having taken action a from state s and (thereafter) acting optimally

- The optimal policy:

  $\pi^*(s)$ = optimal action from state s



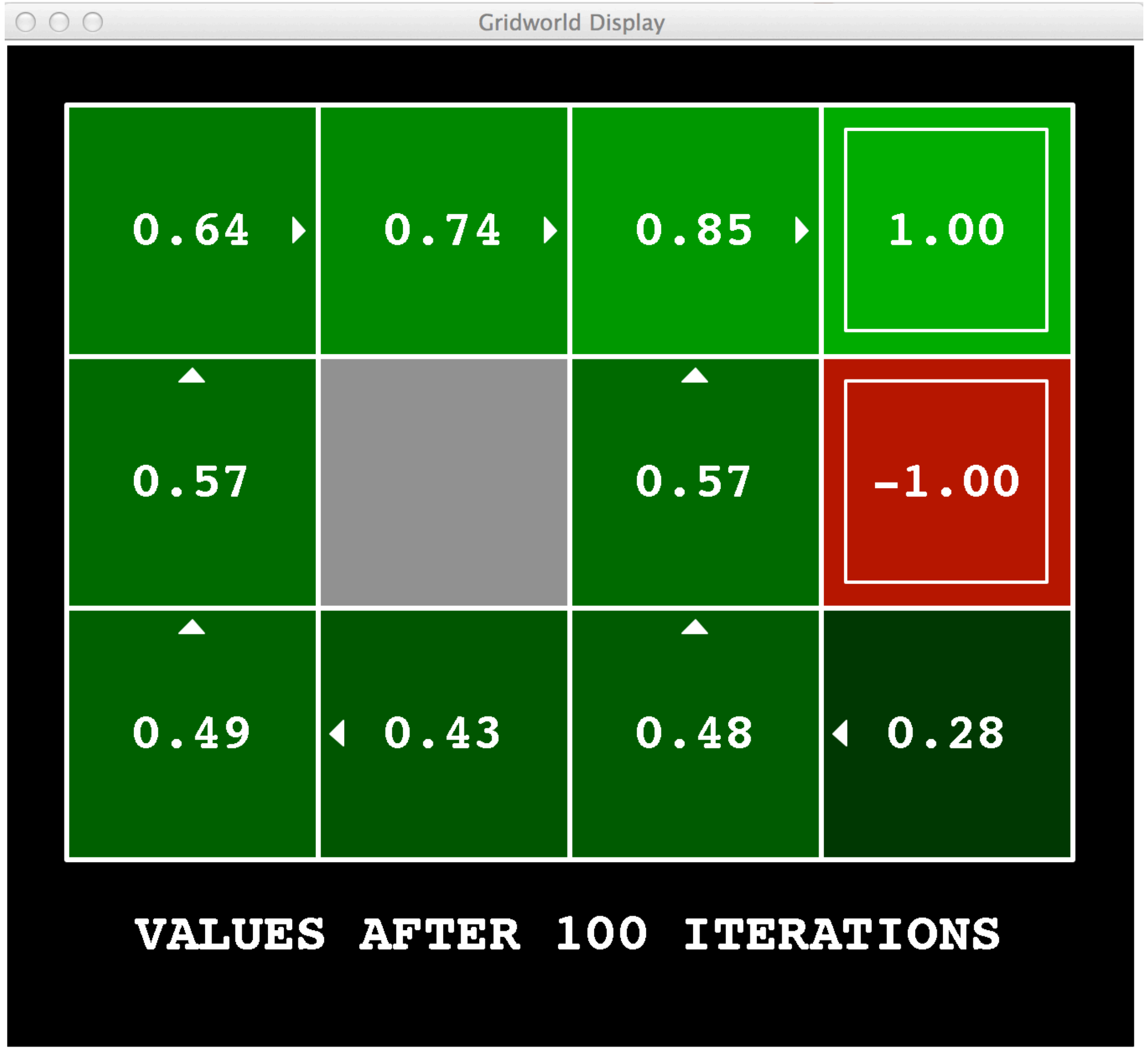
s is a *state*

(s, a) is a *q-state*

(s,a,s’) is a *transition*

Figure from Berkley AI

Gridworld Values V* and Q*



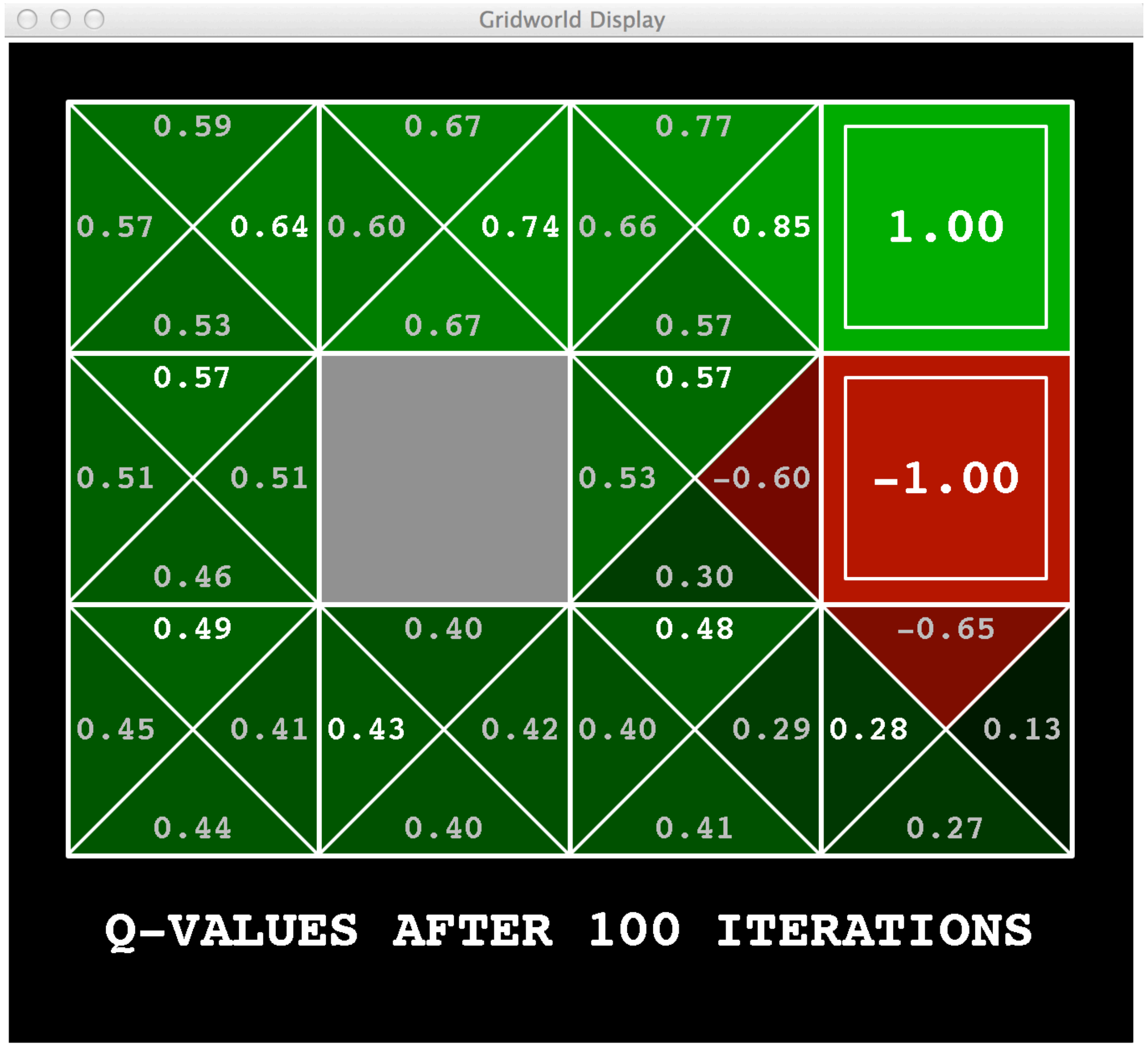


Figure from Berkley AI

The Bellman Equations

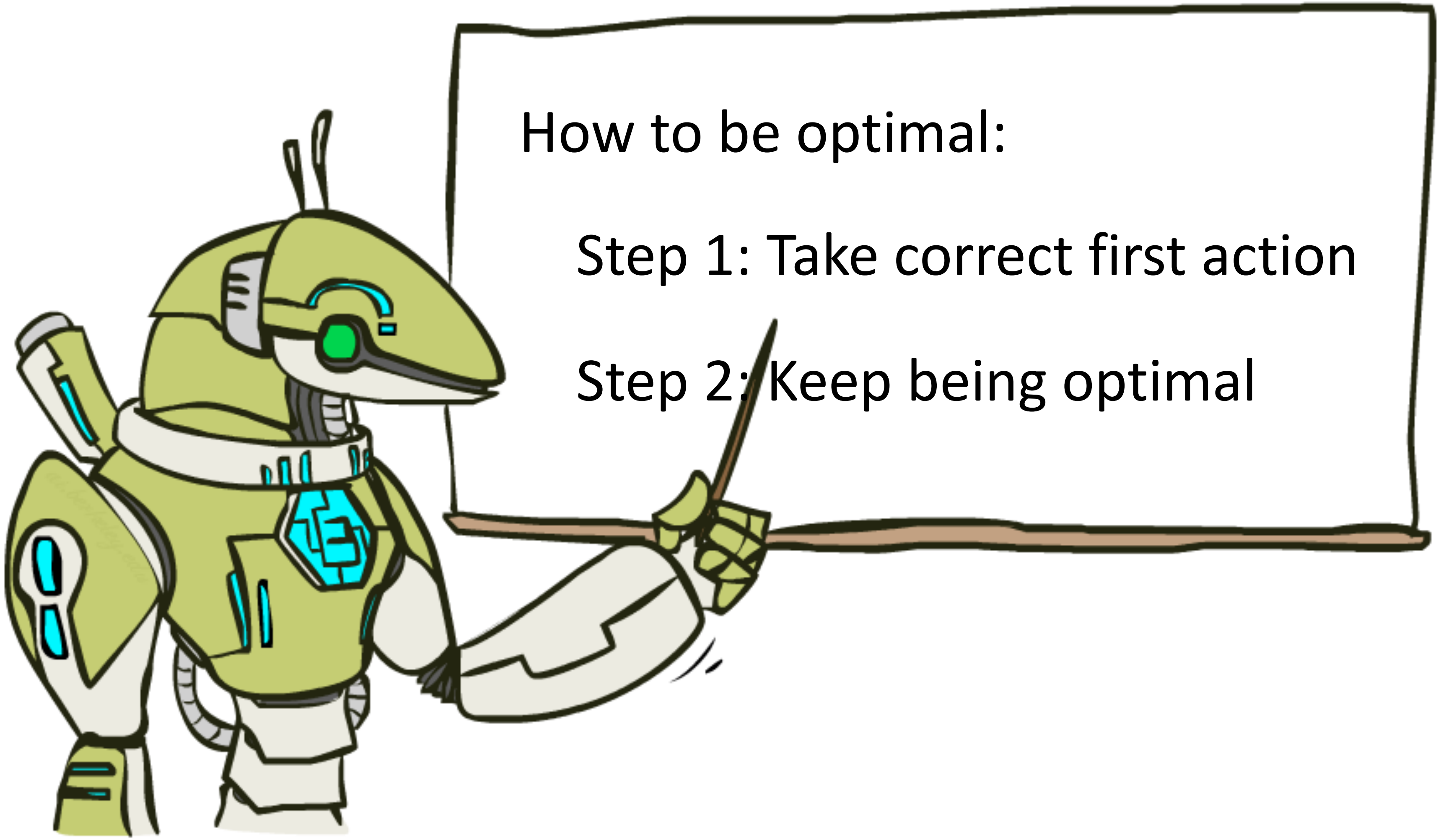


Figure from Berkley AI

The Bellman Equations

- Definition of "optimal utility" via expectimax recurrence gives a simple one-step lookahead relationship amongst optimal utility values

$$V^*(s) = \max_a Q^*(s,a)$$

$$Q^*(s,a) = \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$

- These are the **Bellman equations**, and they characterize optimal values in a way we'll use over and over

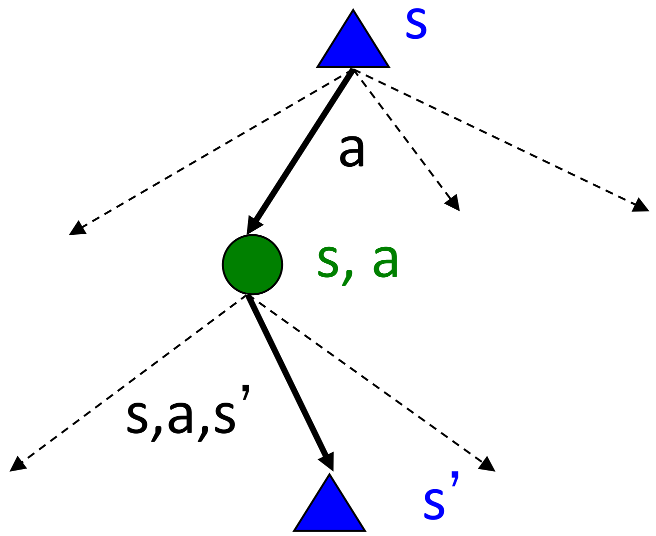


Figure from Berkley AI

Value Iteration

- Bellman equations characterize the optimal values:

$$V^*(s) = \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V^*(s')\right]$$

- Value iteration computes them:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s')\right]$$

- Value iteration is just a fixed point solution method
  - … though the V_k vectors are also interpretable as time-limited values

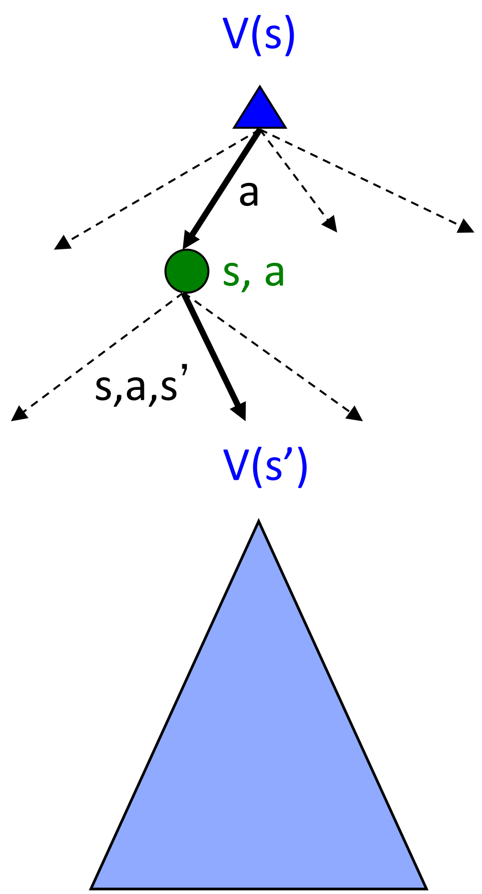


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Figure from Berkley AI

Convergence*

- How do we know the V_k vectors are going to converge?
- Case 1: If the tree has maximum depth M, then V_M holds the actual untruncated values
- Case 2: If the discount is less than 1
  - Sketch: For any state V_k and V_{k+1} can be viewed as depth k+1 expectimax results in nearly identical search trees
  - The difference is that on the bottom layer, V_{k+1} has actual rewards while V_k has zeros
  - That last layer is at best all R_{MAX}
  - It is at worst R_{MIN}
  - But everything is discounted by γ^k that far out
  - So V_k and V_{k+1} are at most $\gamma^k \max|R|$ different
  - So as k increases, the values converge

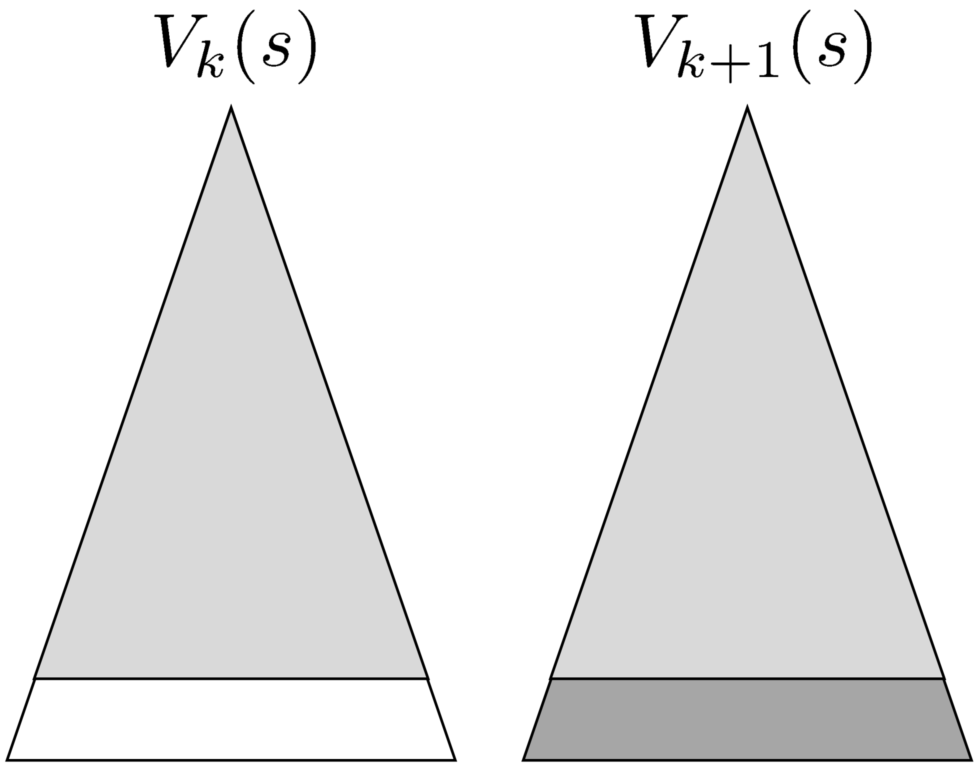


Figure from Berkley AI

Policy Methods

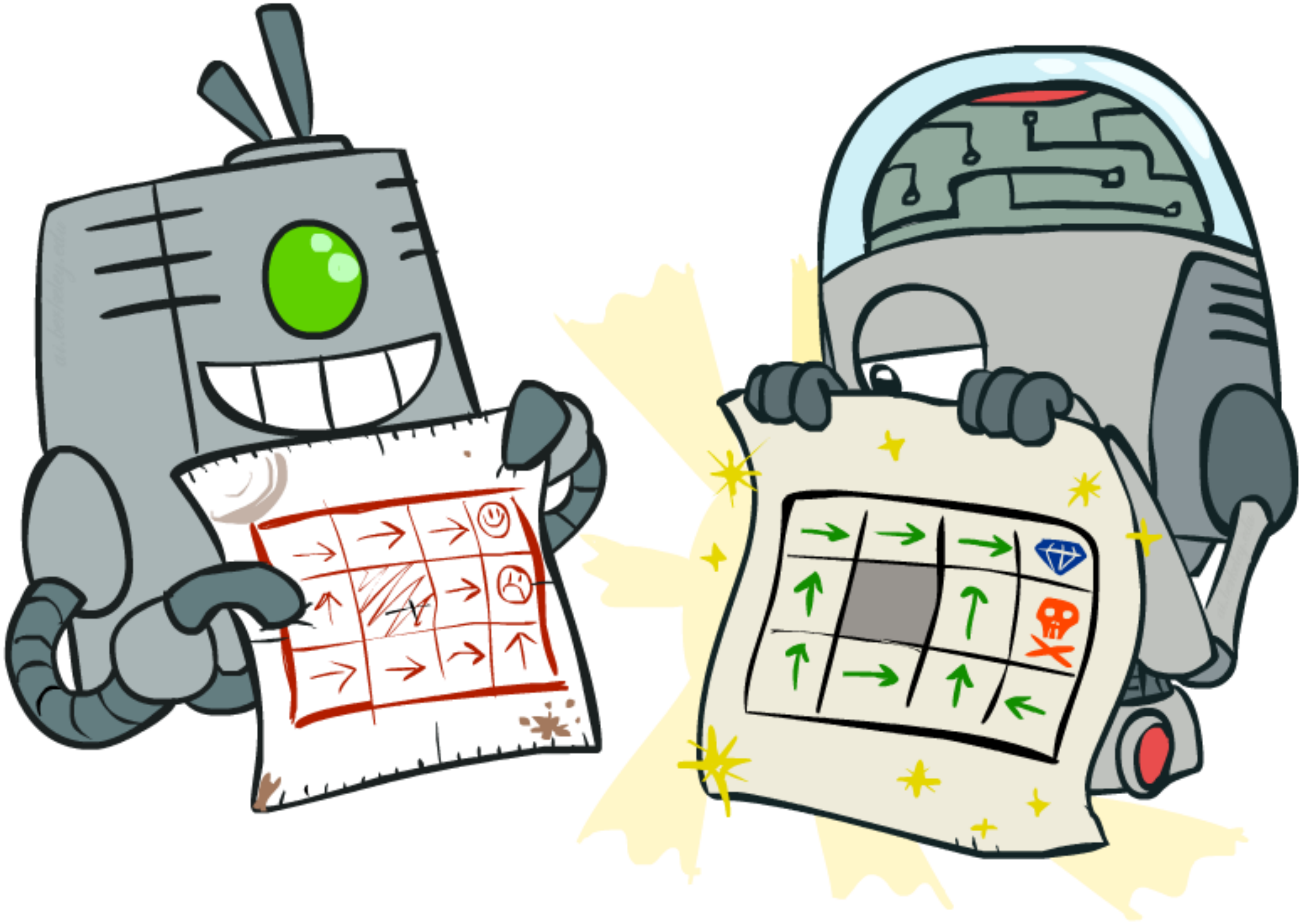


Figure from Berkley AI

Policy Evaluation

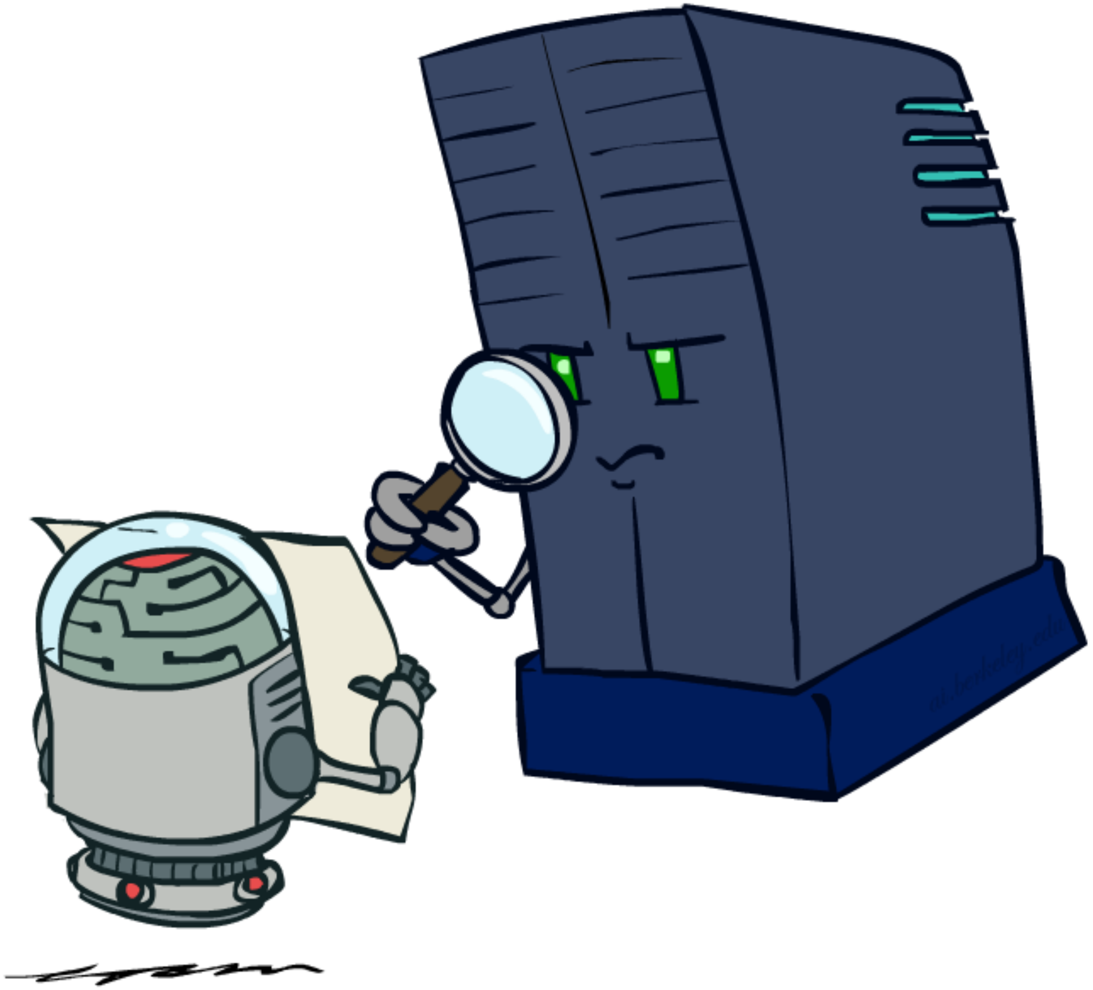
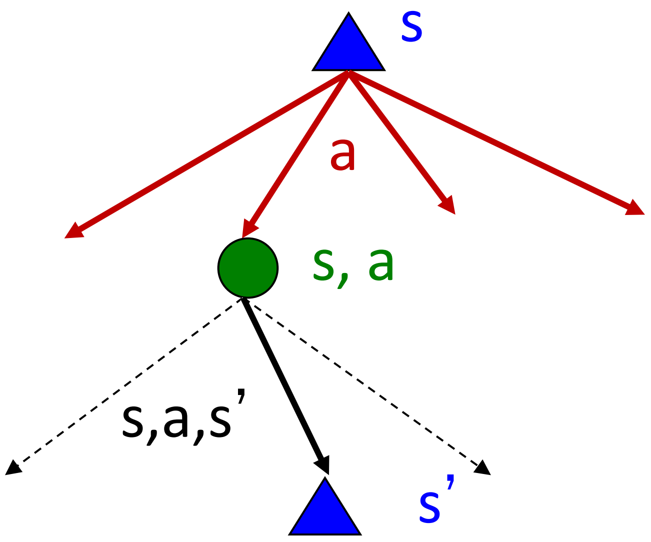


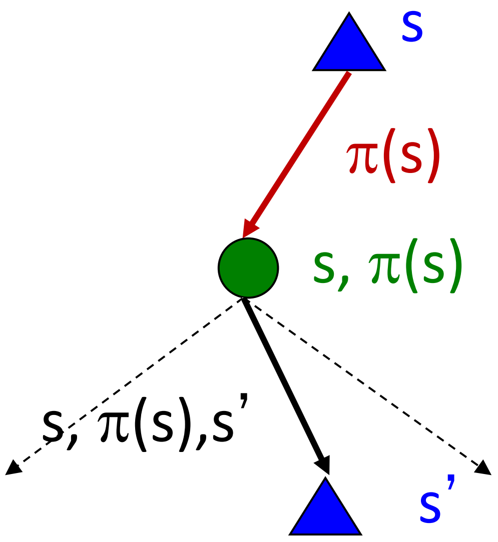
Figure from Berkley AI

Fixed Policies

Do the optimal action



Do what π says to do



- Expectimax trees max over all actions to compute the optimal values
- If we fixed some policy $\pi(s)$, then the tree would be simpler – only one action per state
  - … though the tree's value would depend on which policy we fixed

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Figure from Berkley AI

Utilities for a Fixed Policy

- Another basic operation: compute the utility of a state s under a fixed (generally non-optimal) policy

- Define the utility of a state s, under a fixed policy π:

  $V^{\pi}(s)$ = expected total discounted rewards starting in s and following π

- Recursive relation (one-step look-ahead / Bellman equation):

$$V^{\pi}(s) = \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V^{\pi}(s')]$$

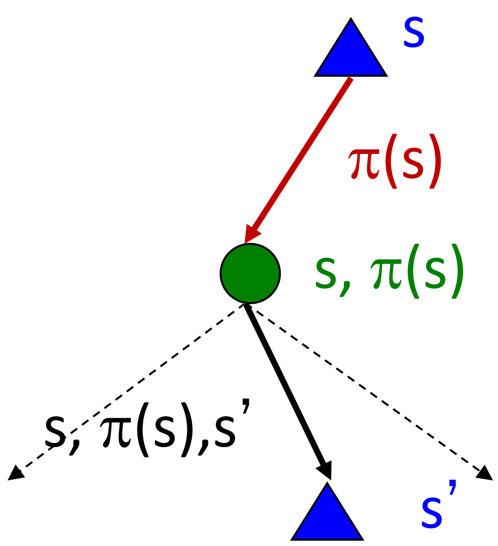
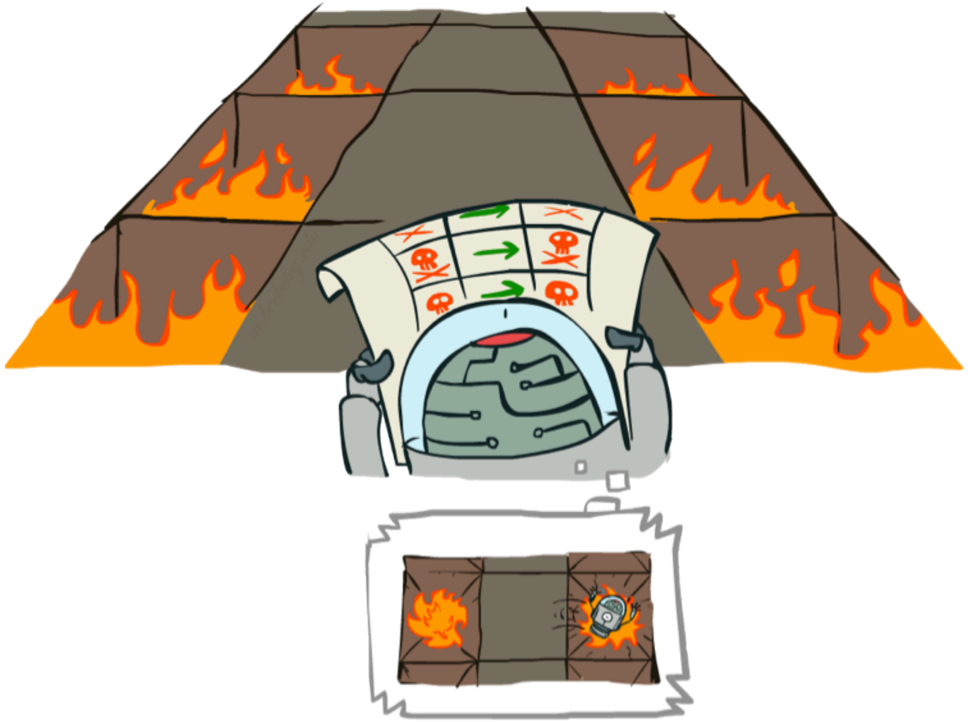


Figure from Berkley AI

Example: Policy Evaluation

Always Go Right



Always Go Forward

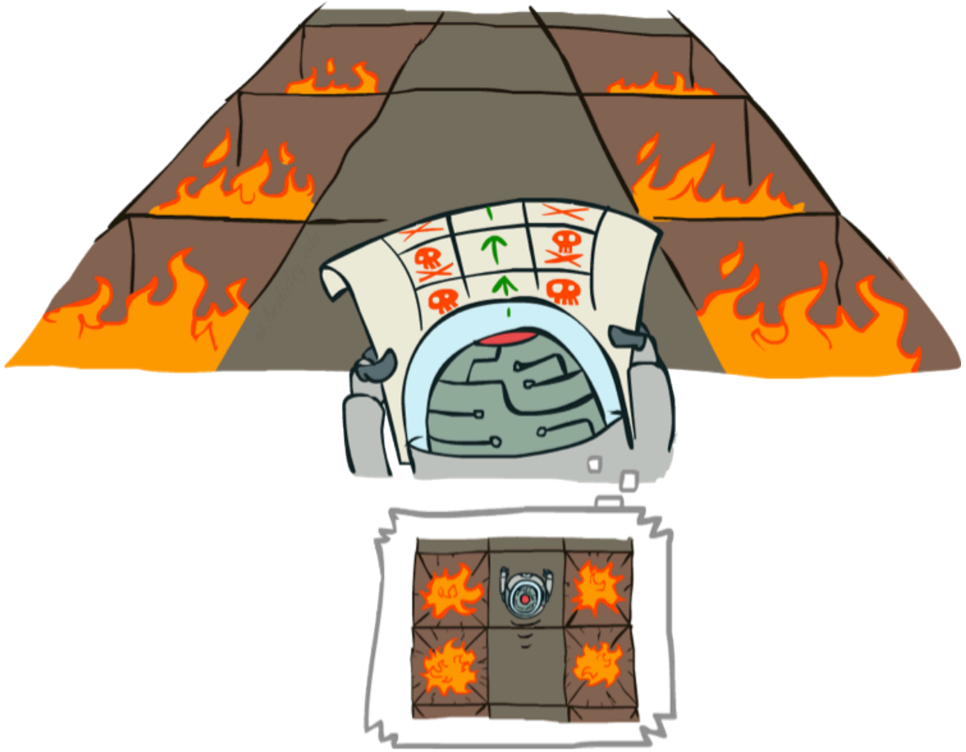


Figure from Berkley AI

Example: Policy Evaluation

Always Go Right

Always Go Forward

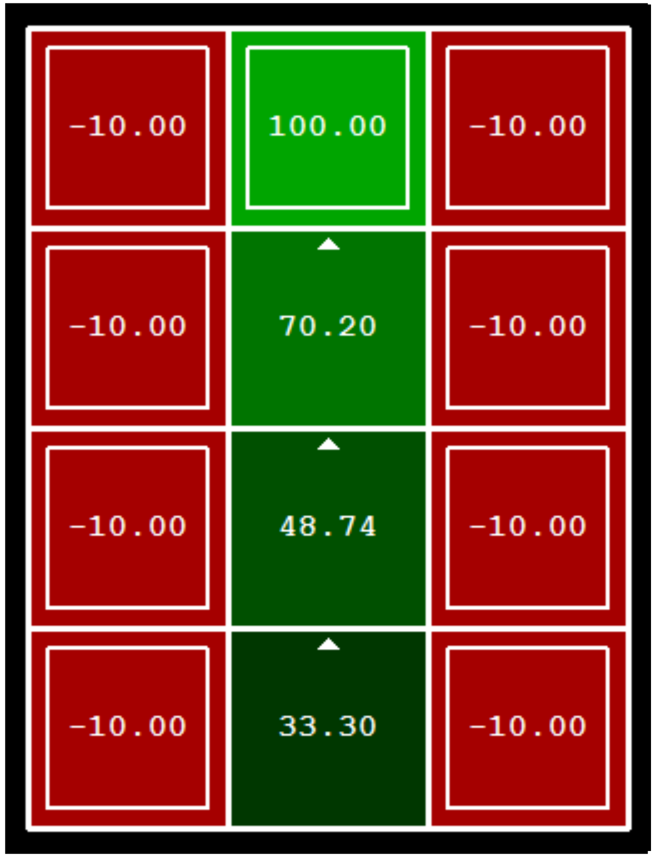


Figure from Berkley AI

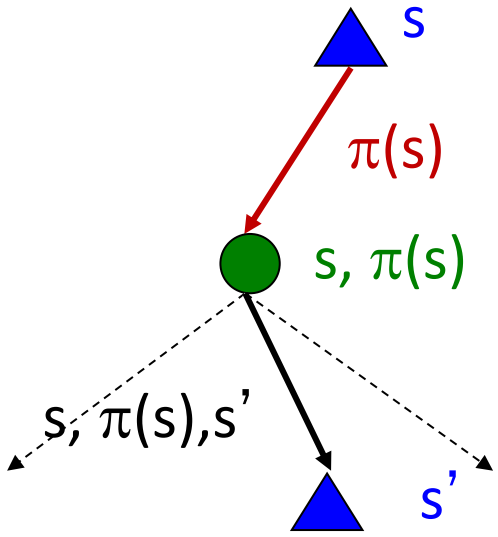
Policy Evaluation

- How do we calculate the V's for a fixed policy π?

- **Idea 1:** Turn recursive Bellman equations into updates (like value iteration)

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



- Efficiency: $O(S^2)$ per iteration

- **Idea 2**: Without the maxes, the Bellman equations are just a linear system
  - Solve with Matlab (or your favorite linear system solver)

Figure from Berkley AI

Policy Extraction

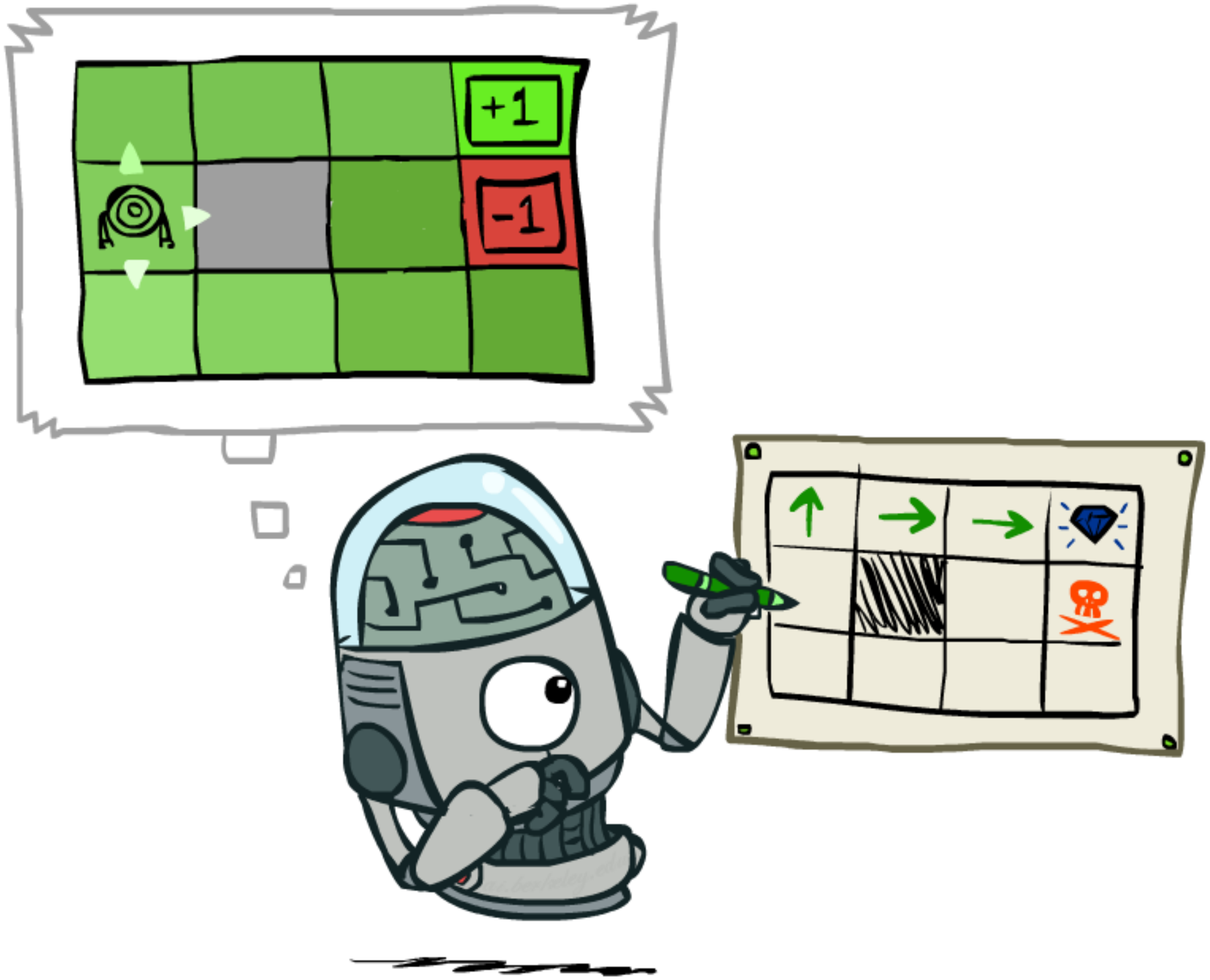


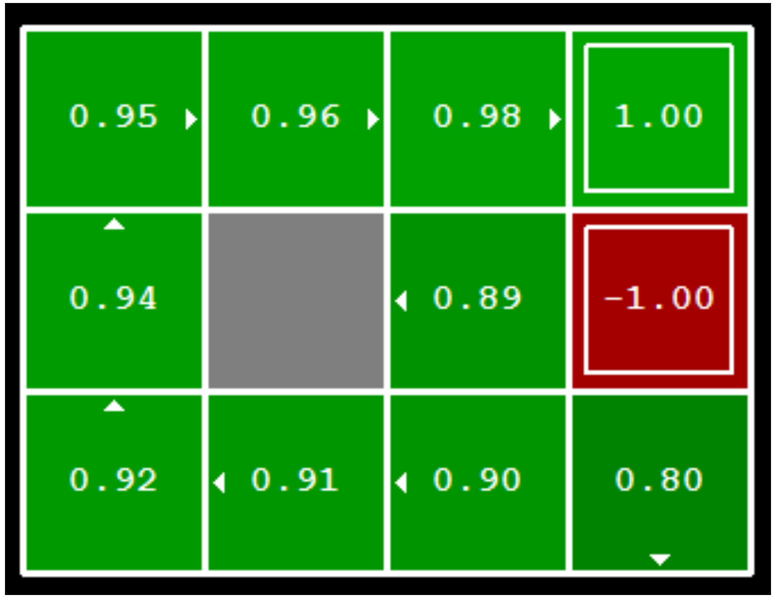
Figure from Berkley AI

Computing Actions from Values

- Let's imagine we have the optimal values V*(s)

- How should we act?
  - It's not obvious!

- We need to do a mini-expectimax (one step)

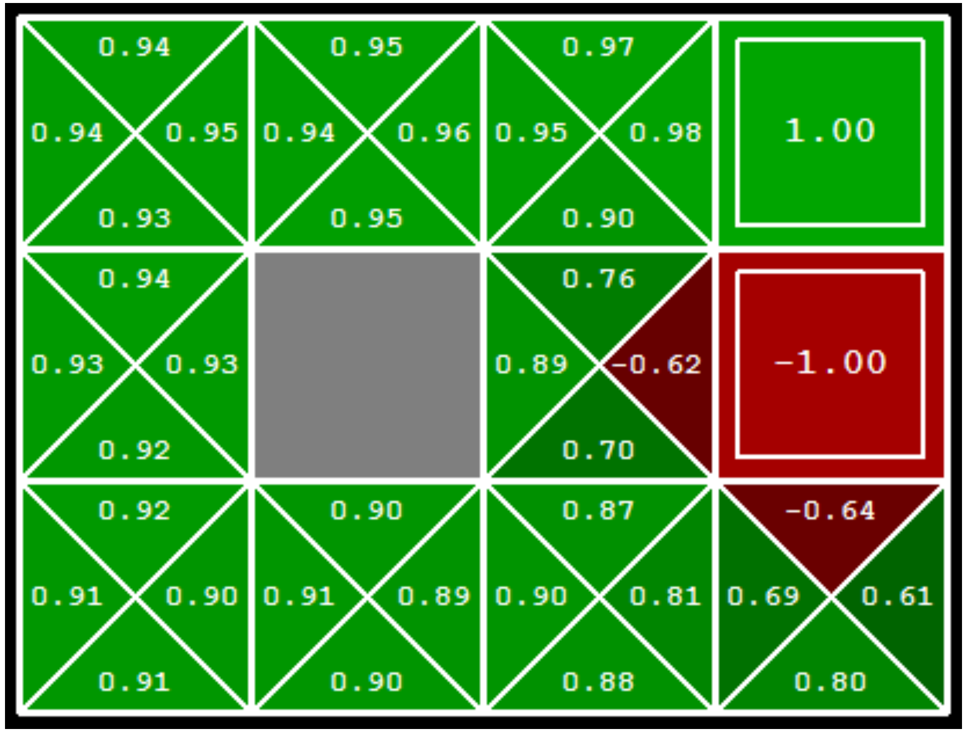


$$\pi^*(s) = \arg\max_a \sum_{s'} T(s,a,s')[R(s,a,s') + \gamma V^*(s')]$$

- This is called policy extraction, since it gets the policy implied by the values

Computing Actions from Q-Values

- Let's imagine we have the optimal q-values:



- How should we act?
  - Completely trivial to decide!

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

- Important lesson: actions are easier to select from q-values than values!

Figure from Berkley AI

Policy Iteration

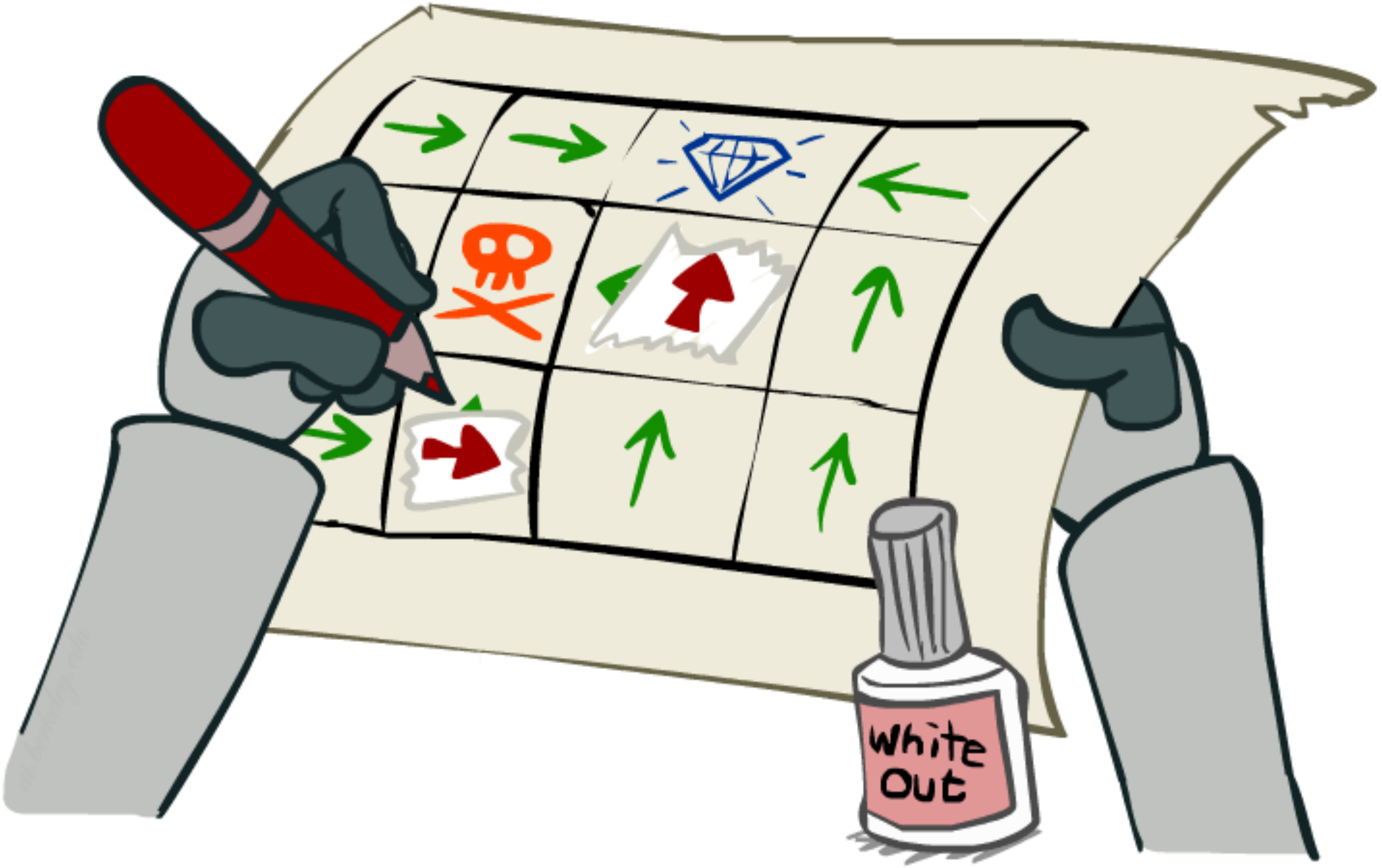
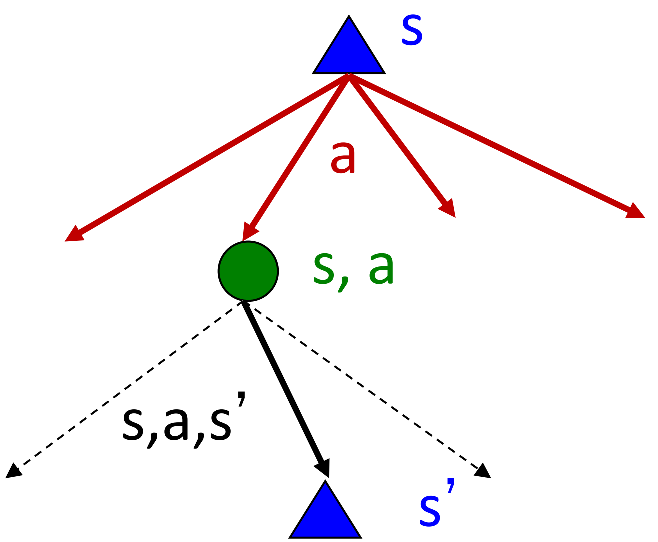


Figure from Berkley AI

Problems with Value Iteration

- Value iteration repeats the Bellman updates:

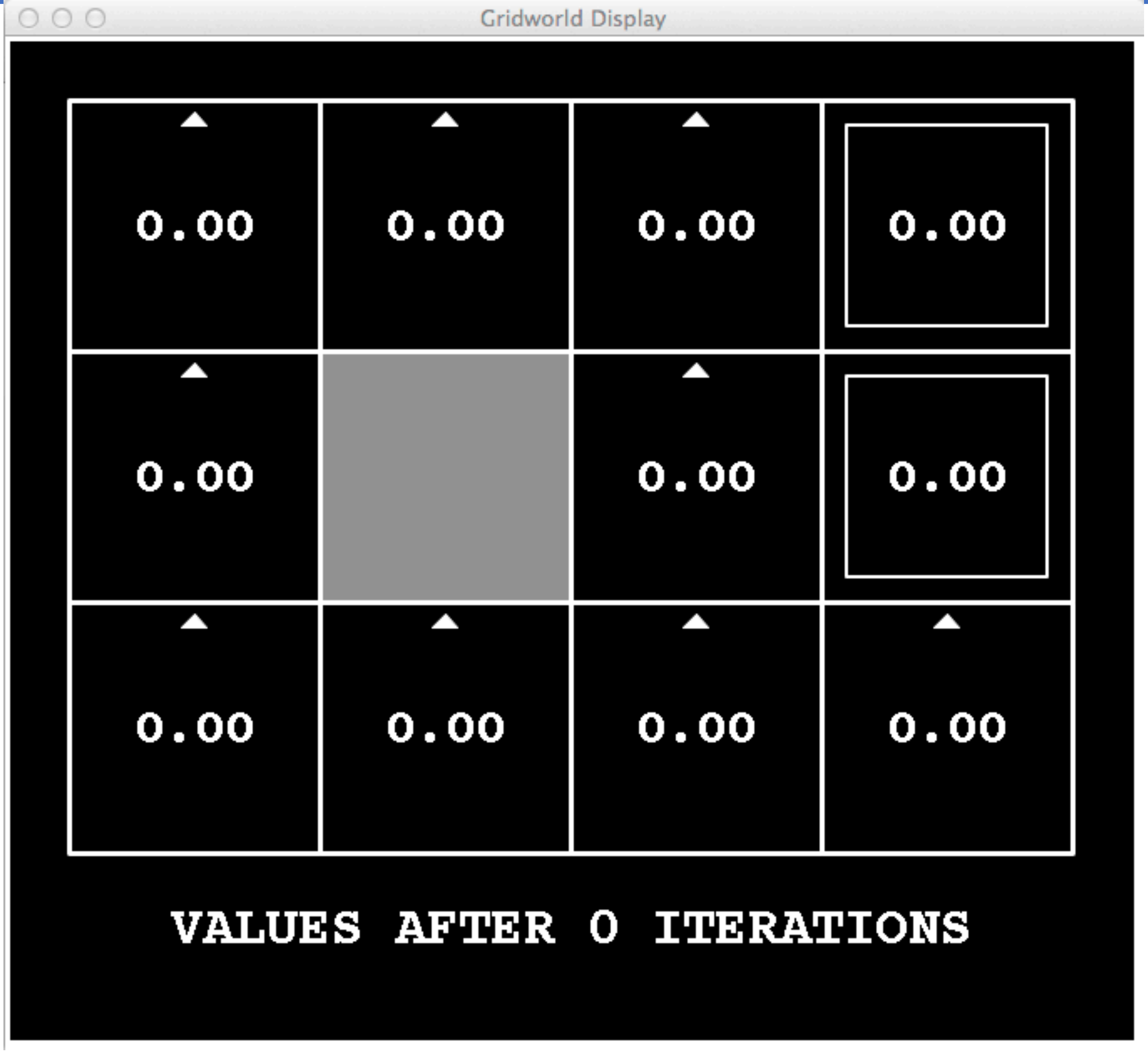
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma V_k(s') \right]$$



- Problem 1: It's slow – O(S^2A) per iteration
- Problem 2: The "max" at each state rarely changes
- Problem 3: The policy often converges long before the values

Figure from Berkley AI

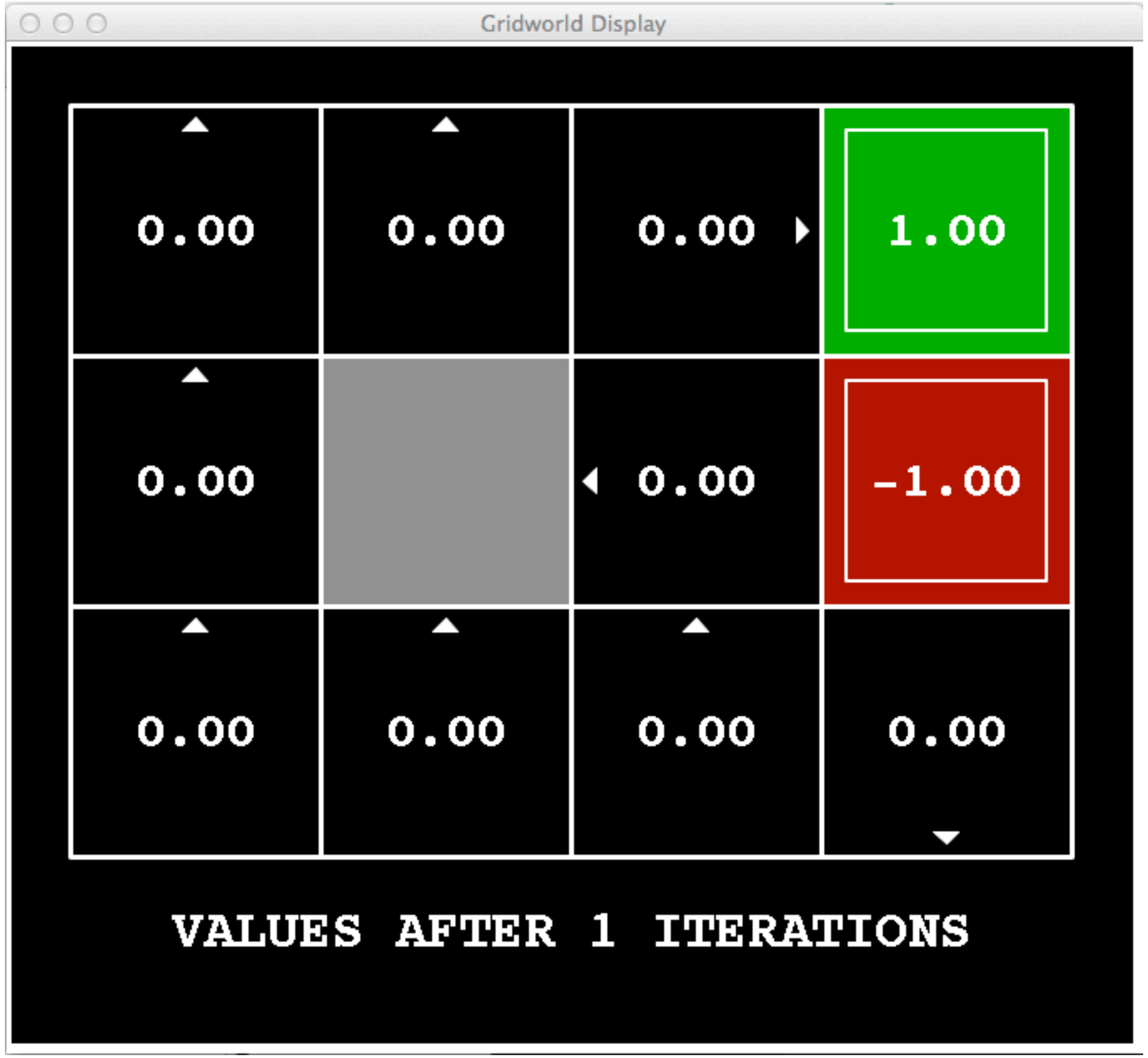
k = 0



Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

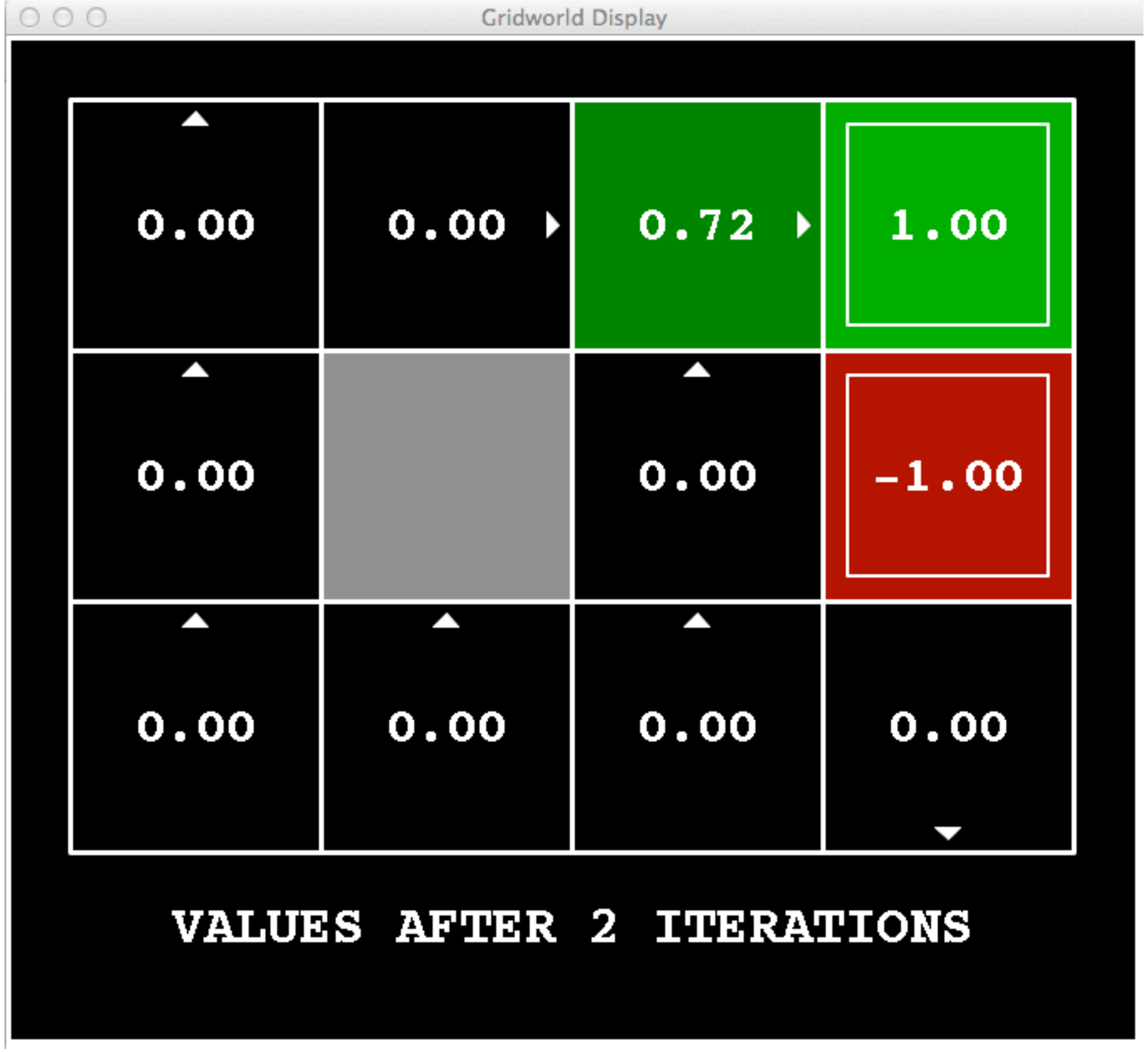
k = 1



Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

k = 2



Noise = 0.2
Discount = 0.9
Living reward = 0

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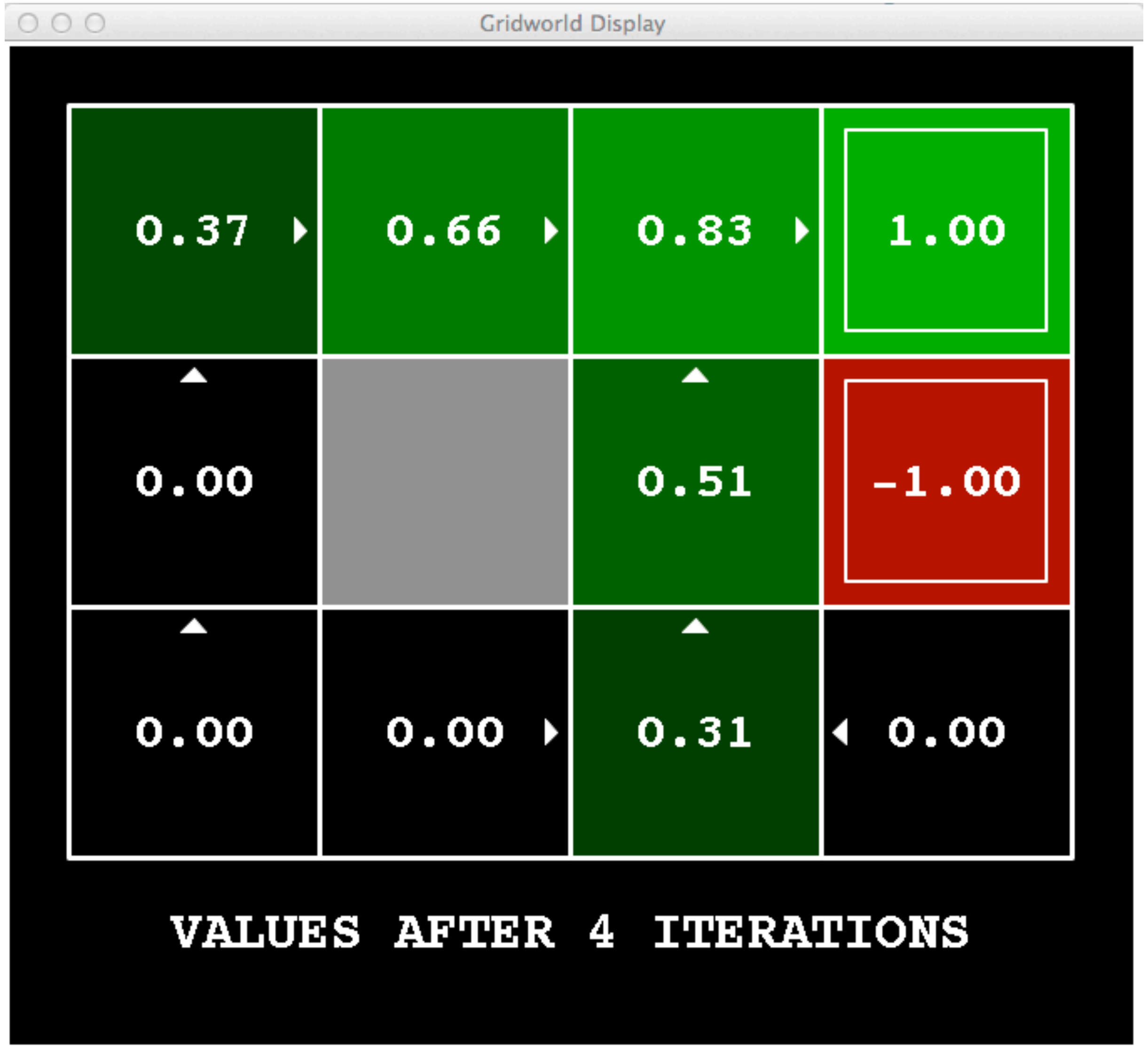
Figure from Berkley AI

k = 3

Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

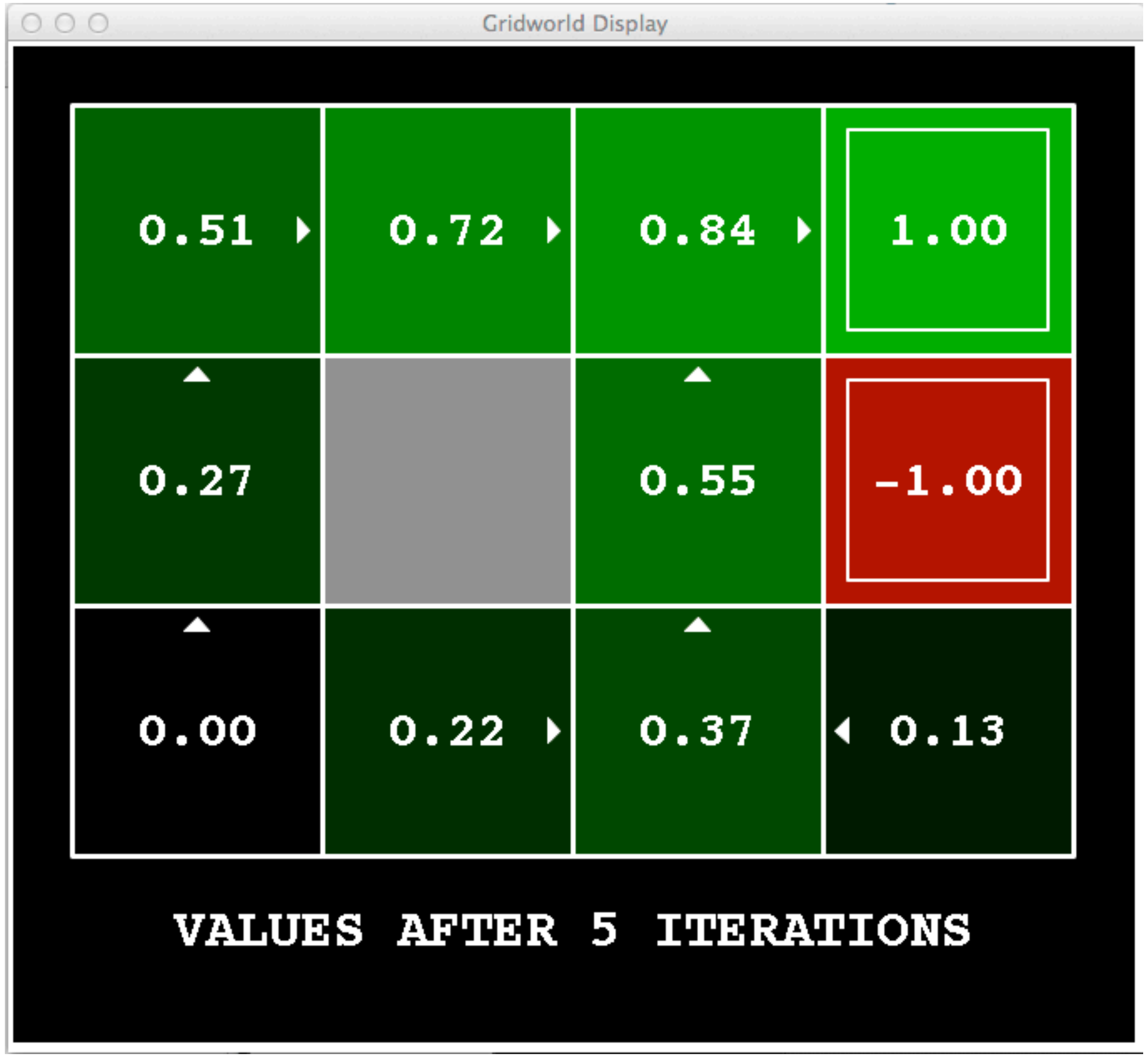
k = 4



Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

k = 5



Noise = 0.2
Discount = 0.9
Living reward = 0

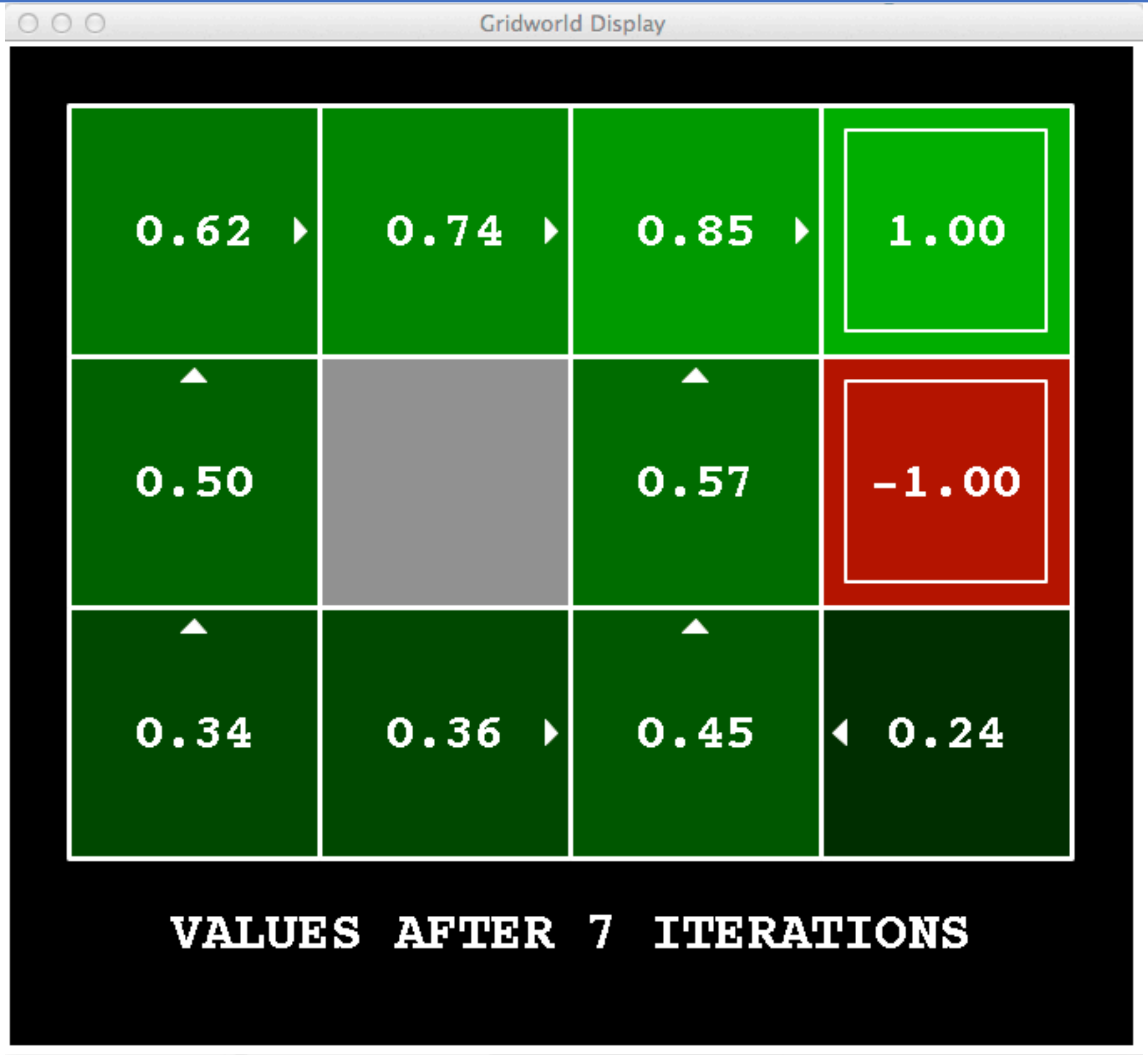
Figure from Berkley AI

k = 6

Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

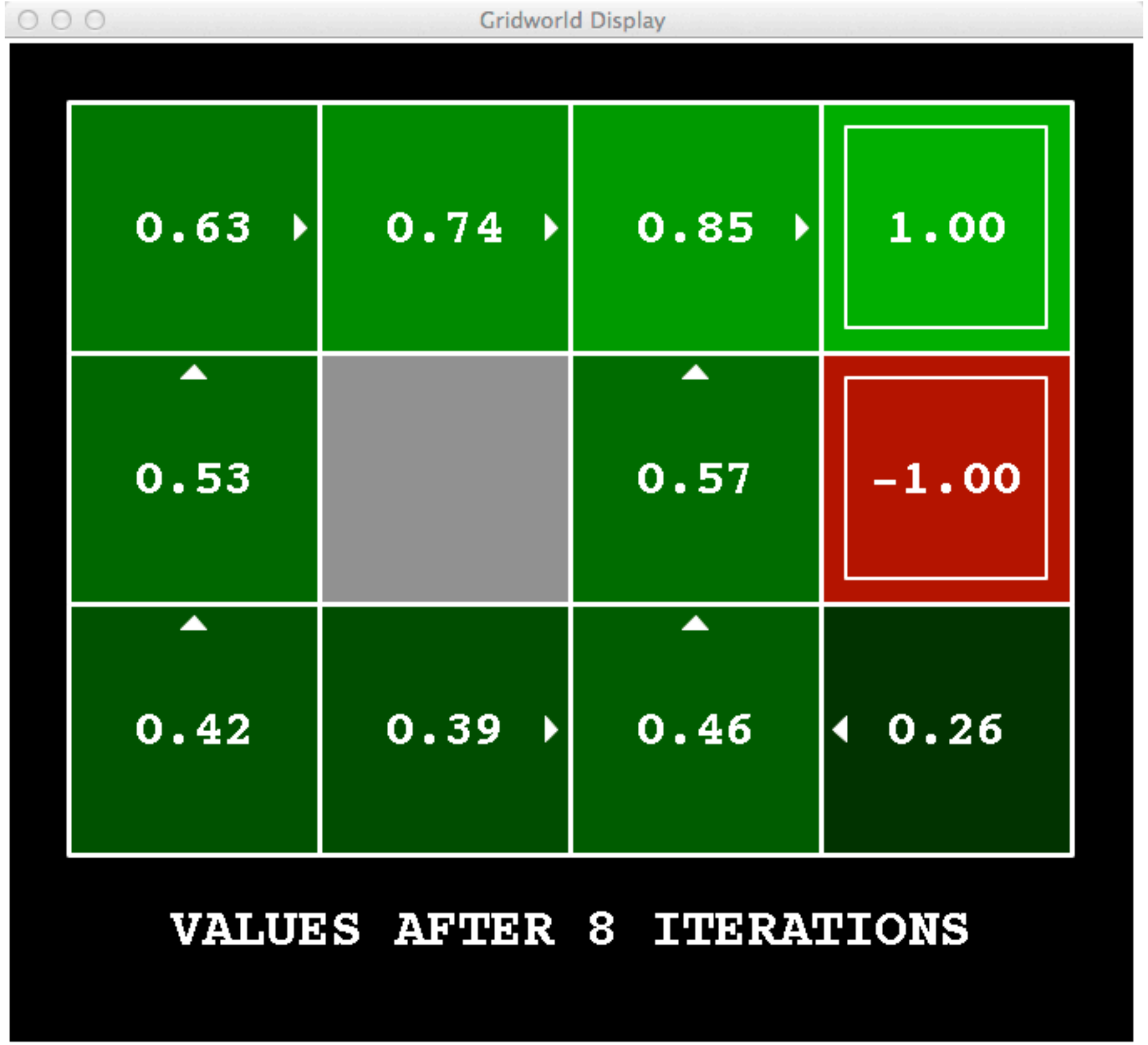
k = 7



Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

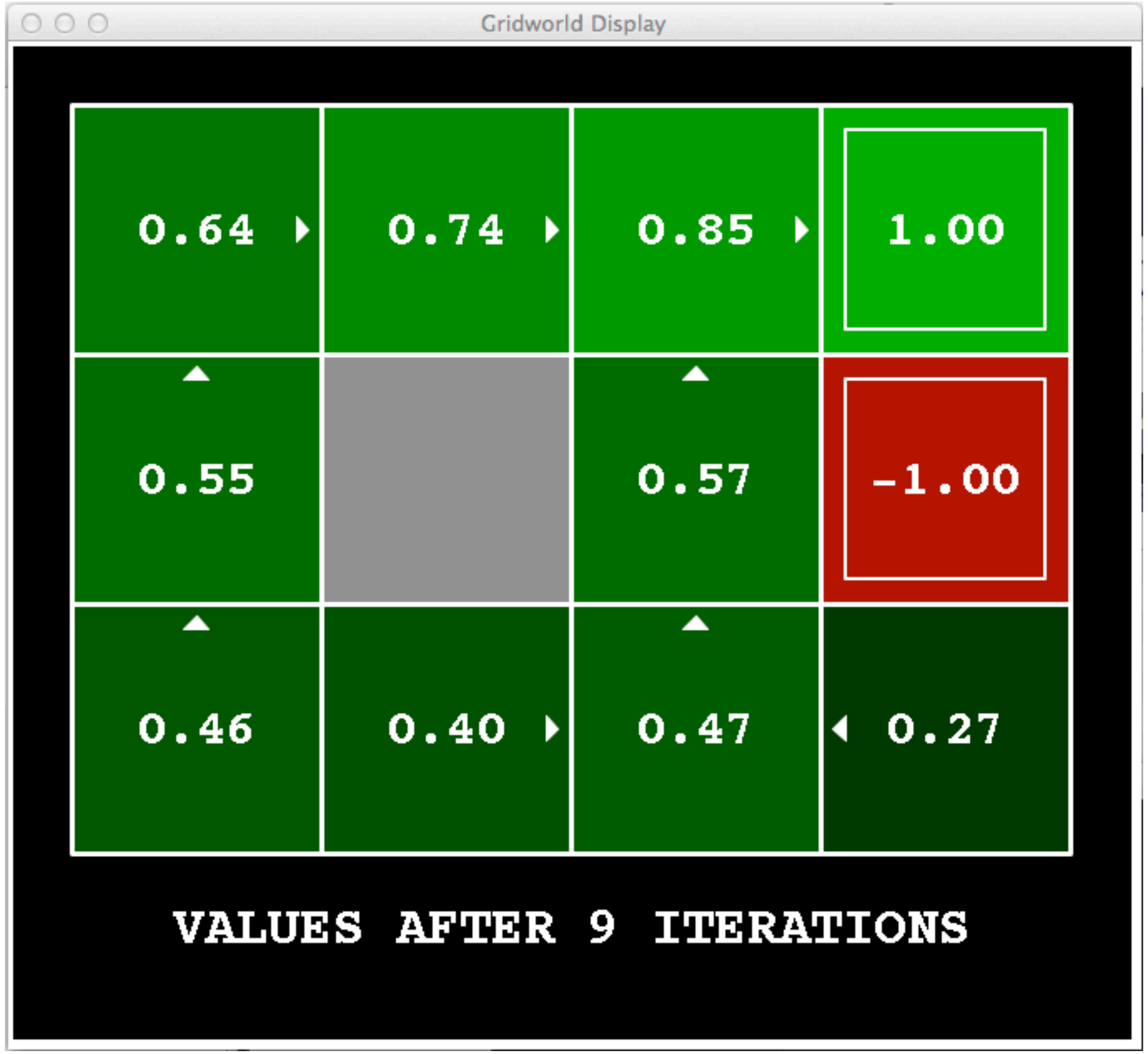
k = 8



Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

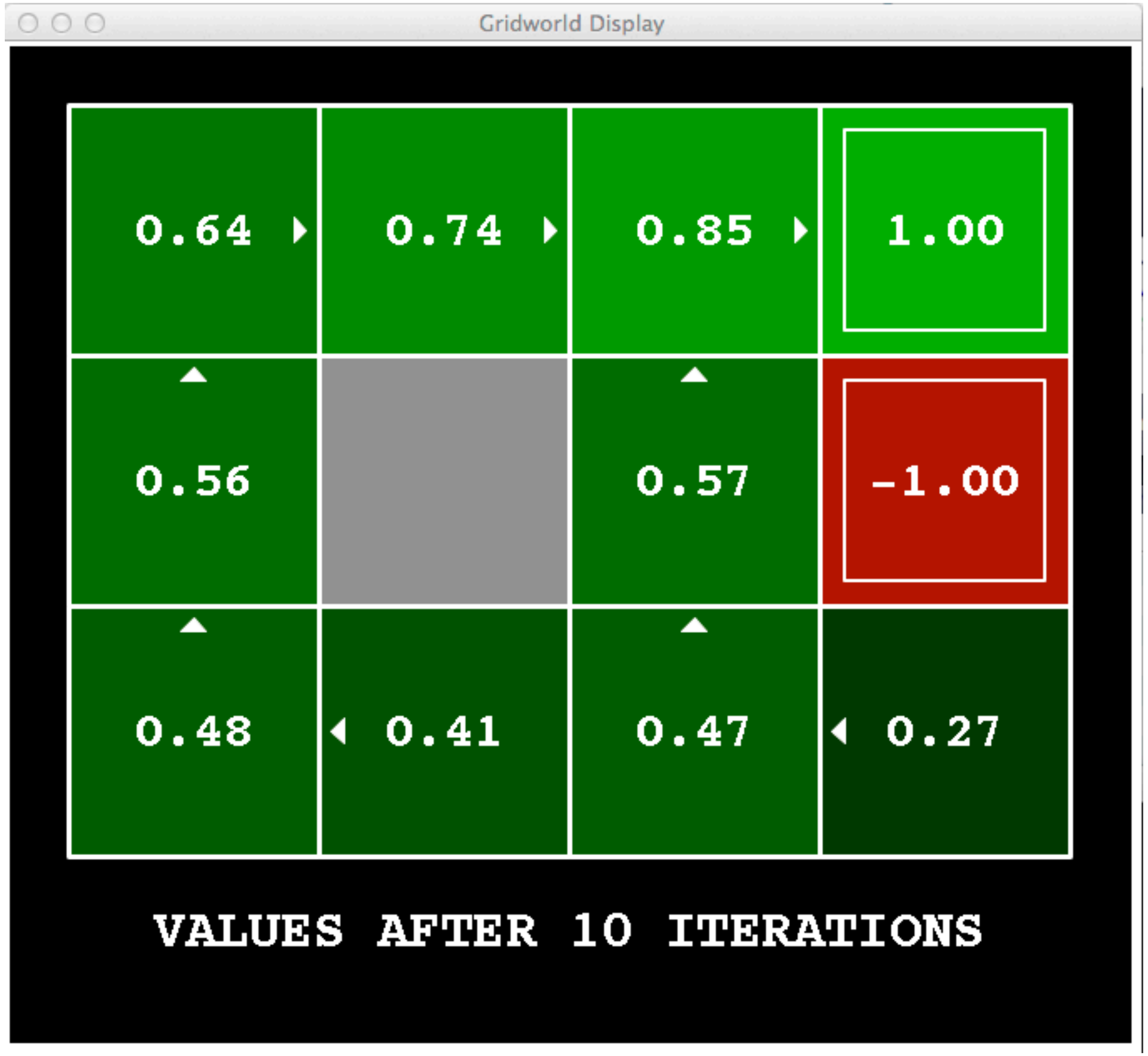
k = 9



Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

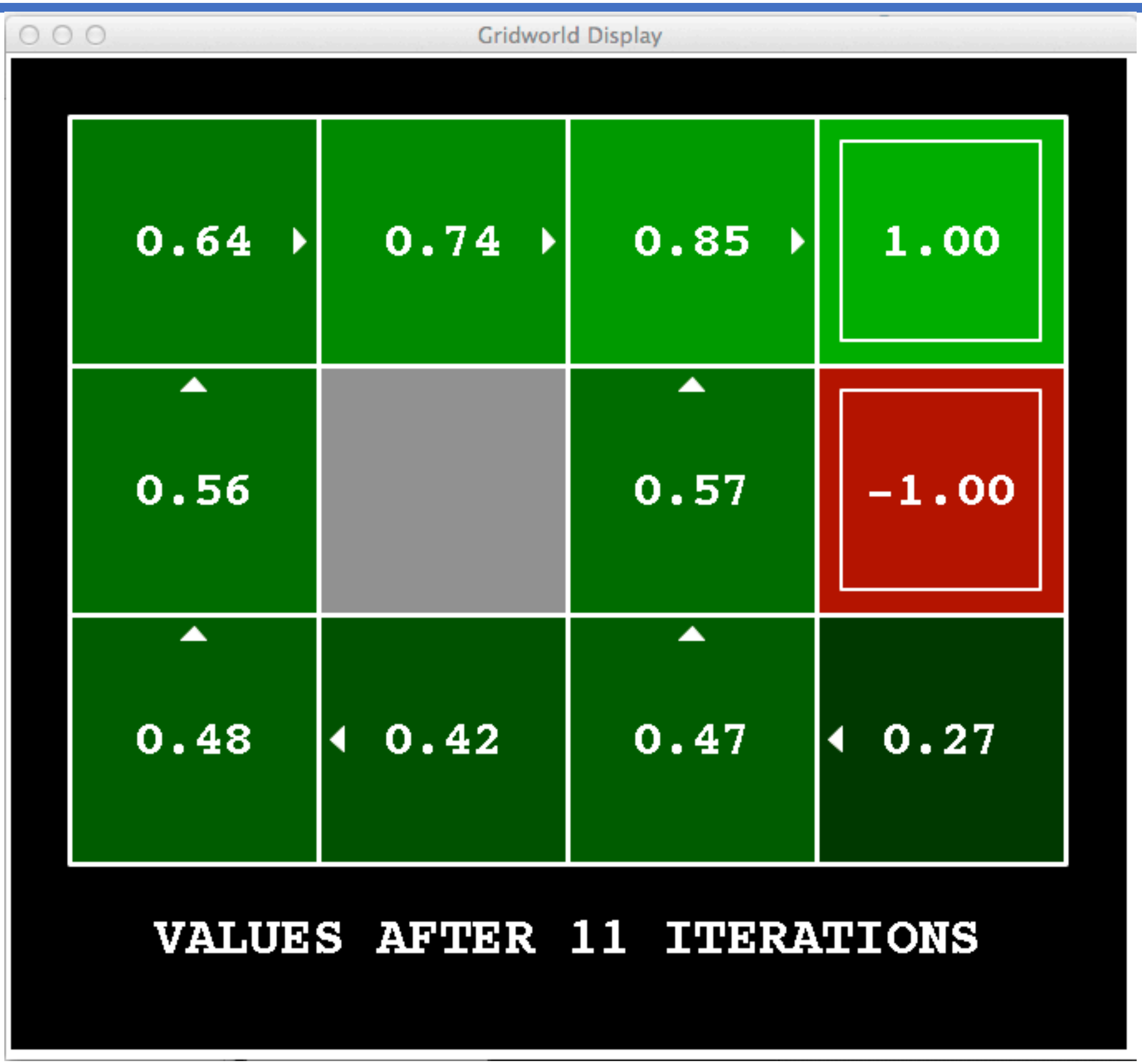
k = 10



Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

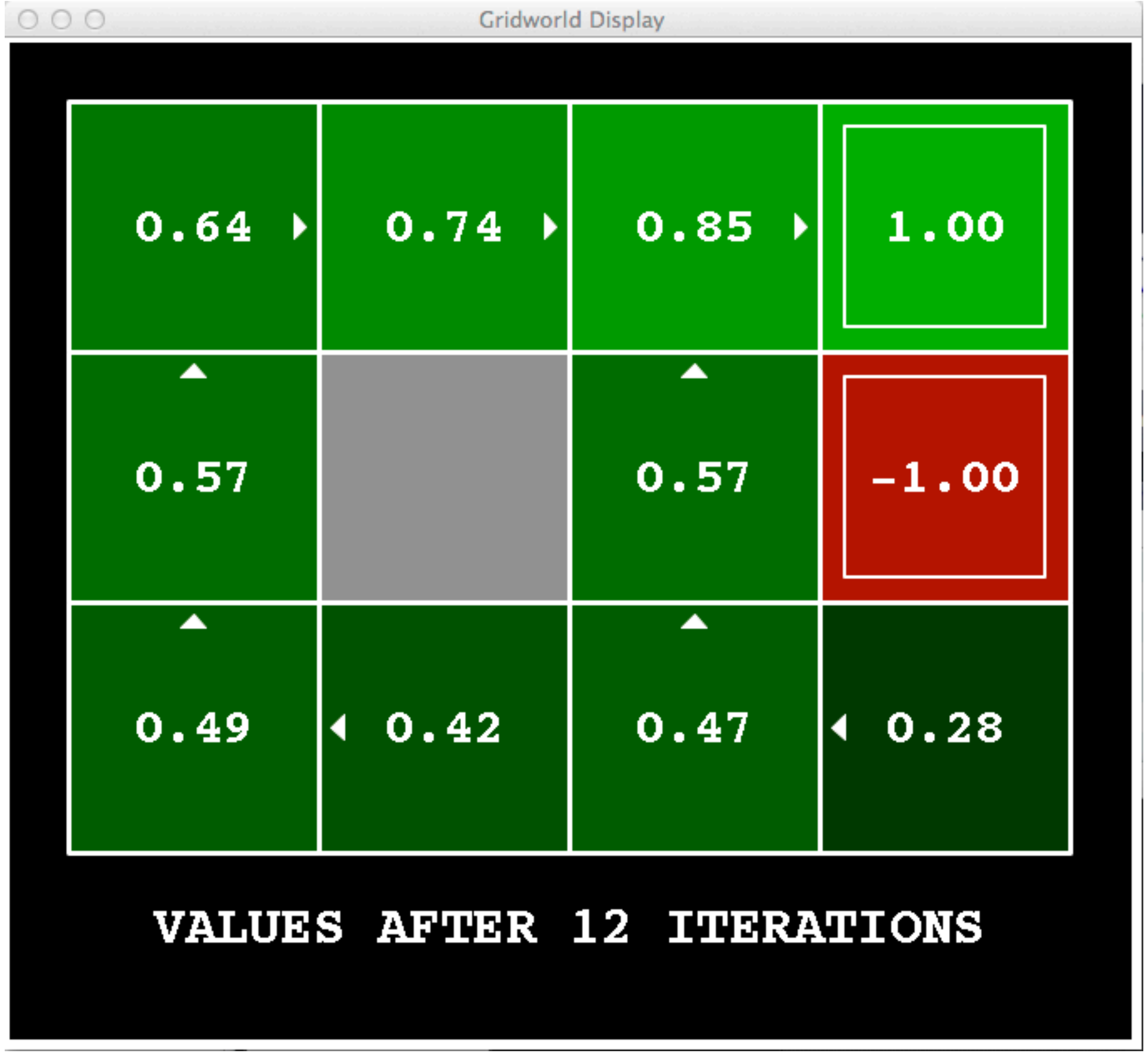
k = 11



Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

k = 12



Noise = 0.2
Discount = 0.9
Living reward = 0

Figure from Berkley AI

Policy Iteration

- Alternative approach for optimal values:
  - Step 1: Policy evaluation: calculate utilities for some fixed policy (not optimal utilities!) until convergence
  - Step 2: Policy improvement: update policy using one-step look-ahead with resulting converged (but not optimal!) utilities as future values
  - Repeat steps until policy converges

- This is policy iteration
  - It's still optimal!
  - Can converge (much) faster under some conditions

Comparison

- Evaluation: For fixed current policy π, find values with policy evaluation:
  - Iterate until values convee:

$$V_{k+1}^{\pi_i}(s) \leftarrow \sum_{s'} T(s, \pi_i(s), s') \left[R(s, \pi_i(s), s') + \gamma V_k^{\pi_i}(s') \right]$$

- Improvement: For fixed values, get a better policy using policy extraction
  - One-step look-ahead:

$$\pi_{i+1}(s) = \arg\max_a \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi_i}(s') \right]$$

Figure from Berkley AI

Comparison

- Both value iteration and policy iteration compute the same thing (all optimal values)

- In value iteration:
  - Every iteration updates both the values and (implicitly) the policy
  - We don’t track the policy, but taking the max over actions implicitly recomputes it

- In policy iteration:
  - We do several passes that update utilities with fixed policy (each pass is fast because we consider only one action, not all of them)
  - After the policy is evaluated, a new policy is chosen (slow like a value iteration pass)
  - The new policy will be better (or we’re done)

- Both are dynamic programs for solving MDPs

Summary: MDP Algorithms

- So you want to….
  - Compute optimal values: use value iteration or policy iteration
  - Compute values for a particular policy: use policy evaluation
  - Turn your values into a policy: use policy extraction (one-step lookahead)

- These all look the same!
  - They basically are – they are all variations of Bellman updates
  - They all use one-step lookahead expectimax fragments
  - They differ only in whether we plug in a fixed policy or max over actions

Double Bandits

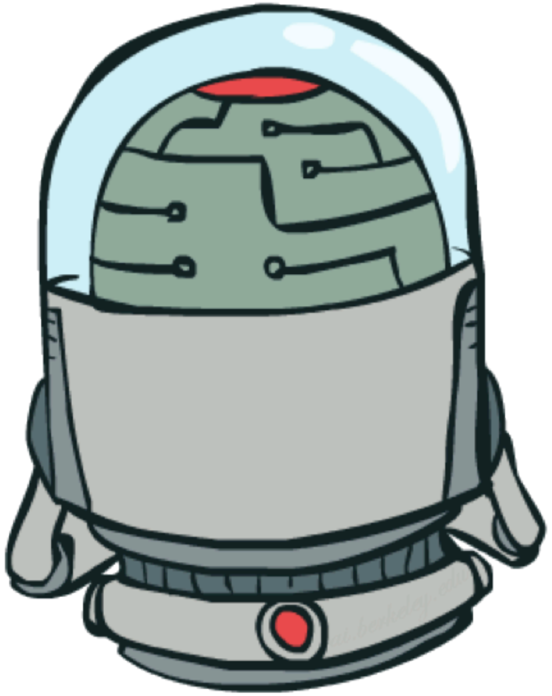


Figure from Berkley AI

Double-Bandit MDP

- Actions: *Blue*, *Red*
- States: Win, Lose

No discount
100 time steps
Both states have the same value

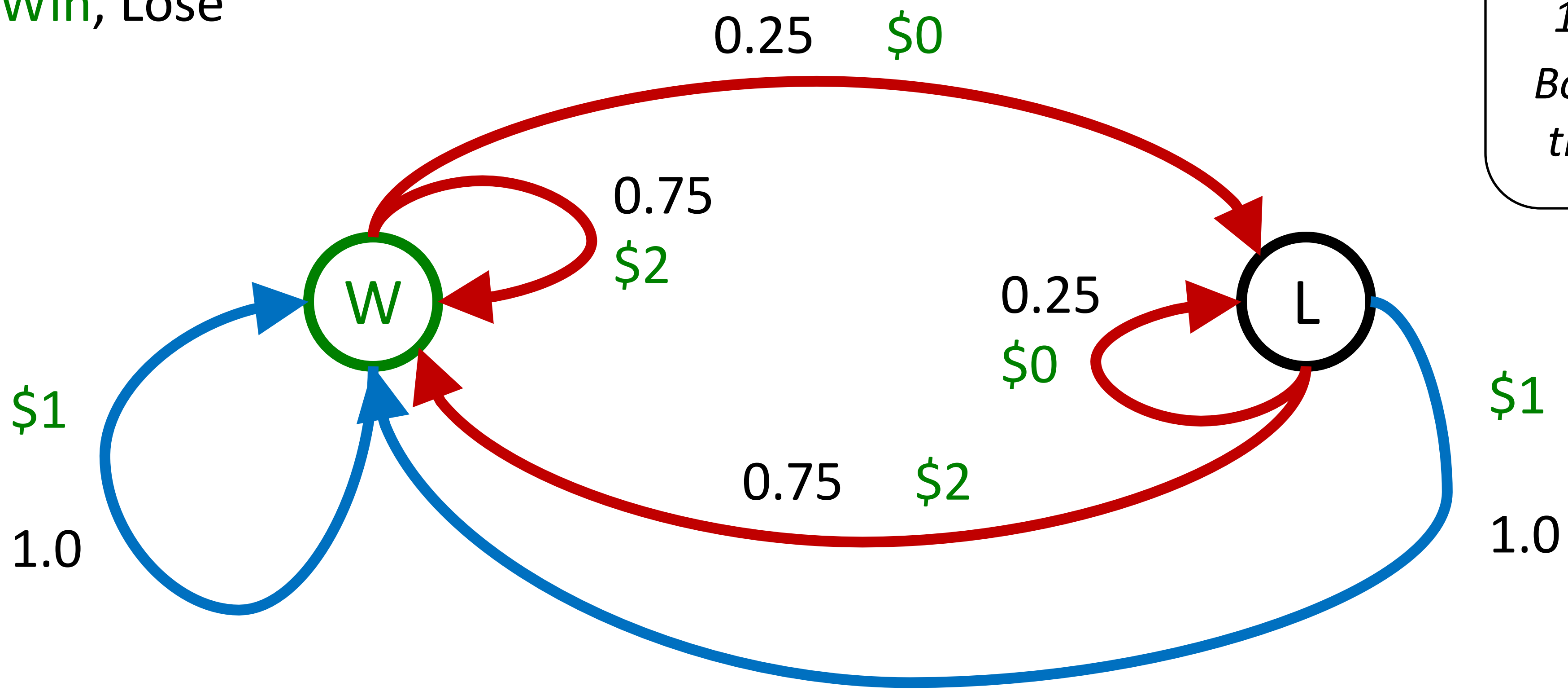


Figure from Berkley AI

Offline Planning

- Solving MDPs is offline planning
  - You determine all quantities through computation
  - You need to know the details of the MDP
  - You do not actually play the game!

No discount
100 time steps
Both states have the same value

	Value
Play Red	150
Play Blue	100

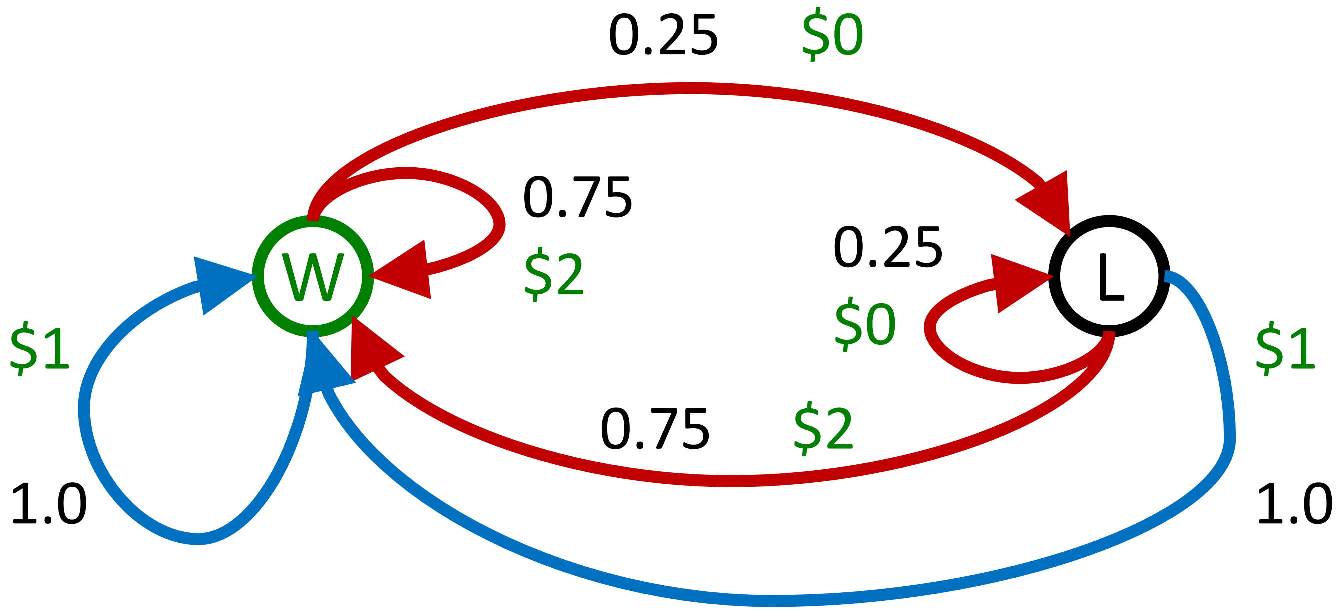


Figure from Berkley AI

Let's Play!

$2 $2 $0 $2 $2

$2 $2 $0 $0 $0

Figure from Berkley AI

Online Planning

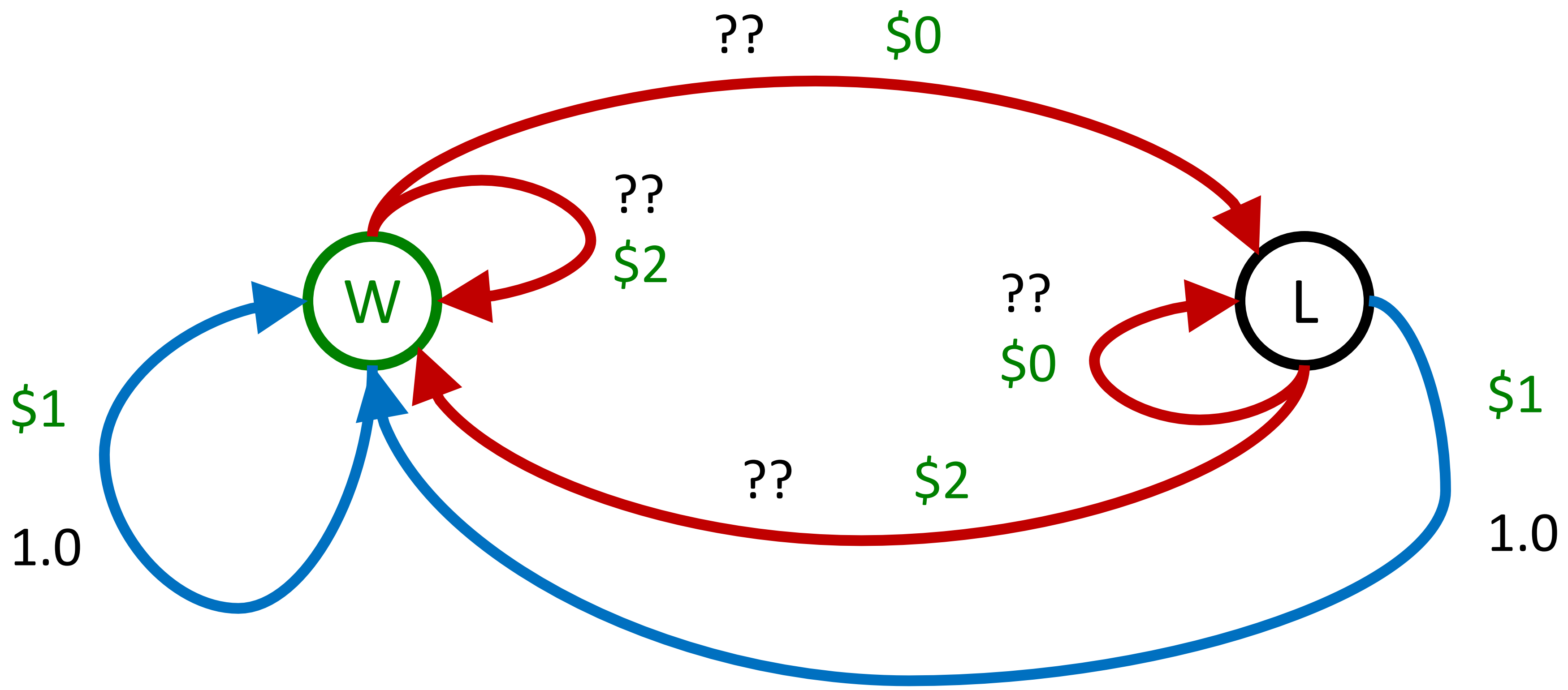


Figure from Berkley AI

Let's Play!

$0 $0 $0 $2 $0

$2 $0 $0 $0 $0

Figure from Berkley AI

What Just Happened?

- That wasn't planning, it was learning!
  - Specifically, reinforcement learning
  - There was an MDP, but you couldn't solve it with just computation
  - You needed to actually act to figure it out

- Important ideas in reinforcement learning that came up
  - Exploration: you have to try unknown actions to get information
  - Exploitation: eventually, you have to use what you know
  - Regret: even if you learn intelligently, you make mistakes
  - Sampling: because of chance, you have to try things repeatedly
  - Difficulty: learning can be much harder than solving a known MDP

Figure from Berkley AI

Next Time: Reinforcement Learning!

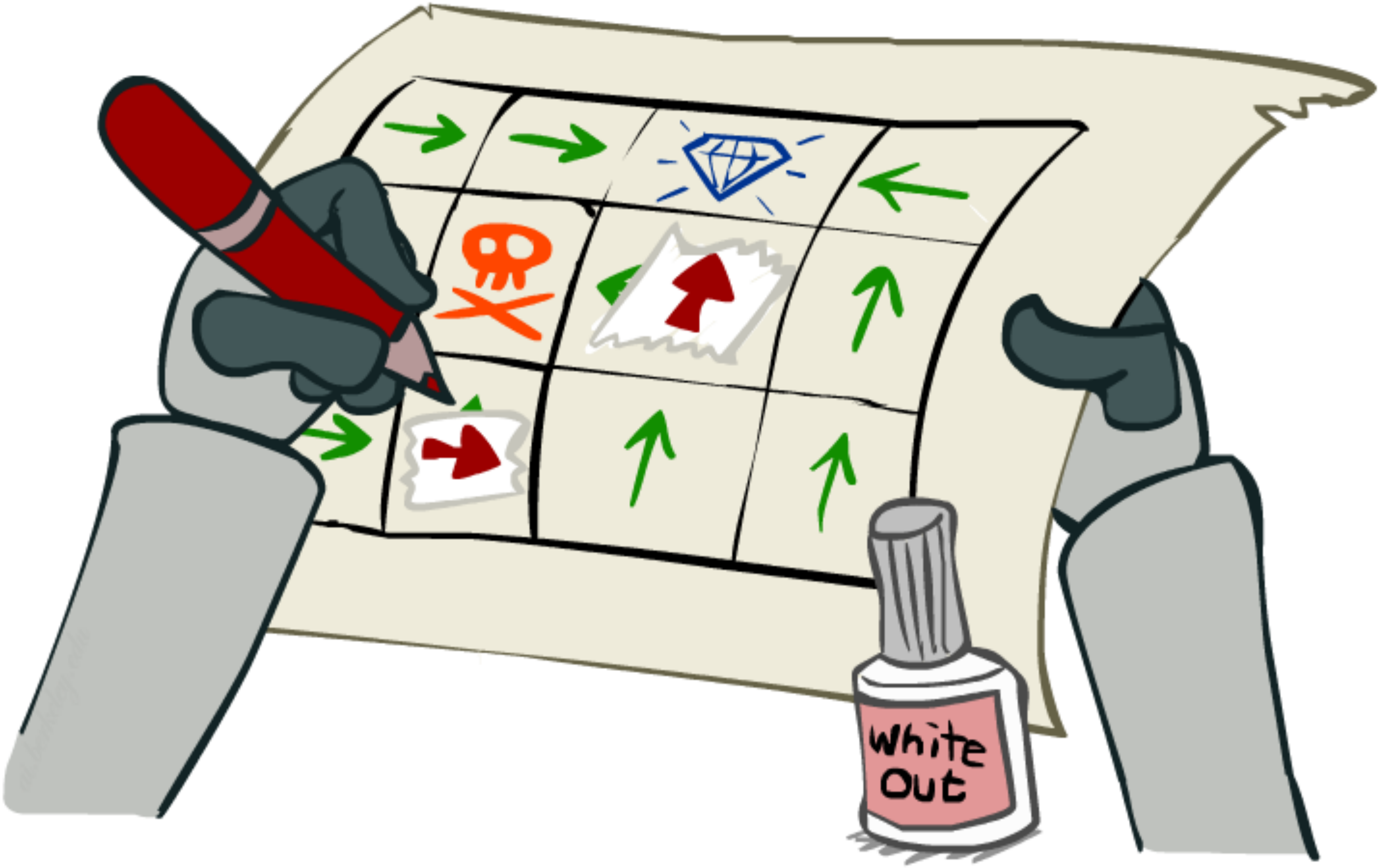


Figure from Berkley AI