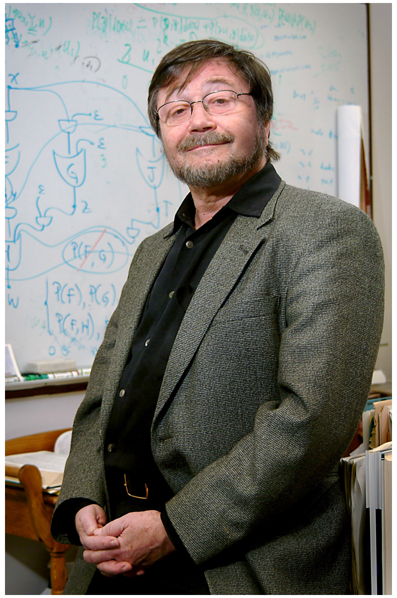
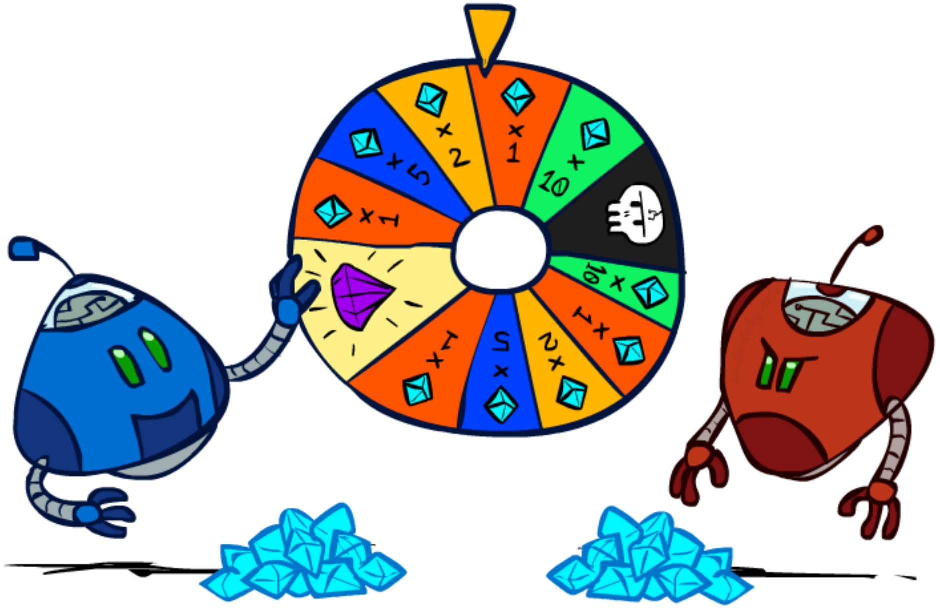


Artificial Intelligence



Adversarial Search with Uncertainty

CS 444 – Spring 2021

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Much of this lecture is taken from
Dan Klein and Pieter Abbeel AI class at UC Berkeley

Today

Expand search to address games with uncertain outcomes.

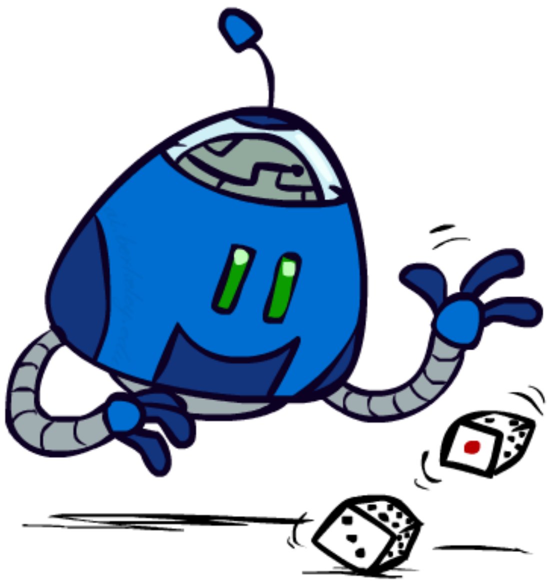
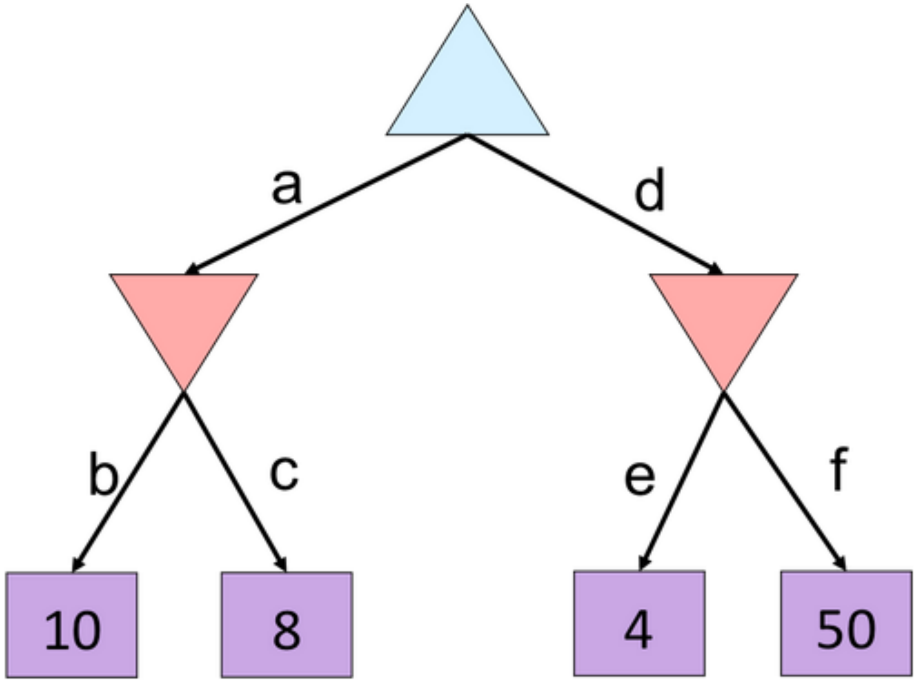


Figure from Berkley AI

Alpha Beta Quiz

```
alpha= -np.inf
beta = np.inf
best_action = None
for a in game.actions(state):
    v = min_value(game.result(state, a),alpha, beta)
    if v > alpha:
        alpha = v
        best_action = a
return best_action
```

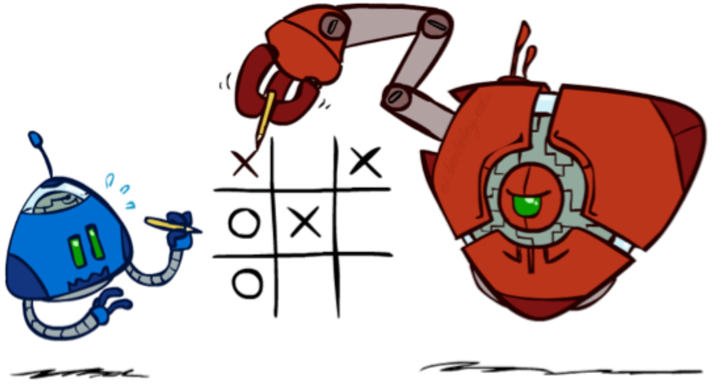
```
def max_value(state, alpha, beta):
    if game.terminal_test(state):
        return game.utility(state, player)
    v = -np.inf
    for a in game.actions(state):
        v = max(v, min_value(game.result(state, a), alpha, beta))
        if v >= beta:
            return v
        alpha = max(alpha, v)
    return v
```

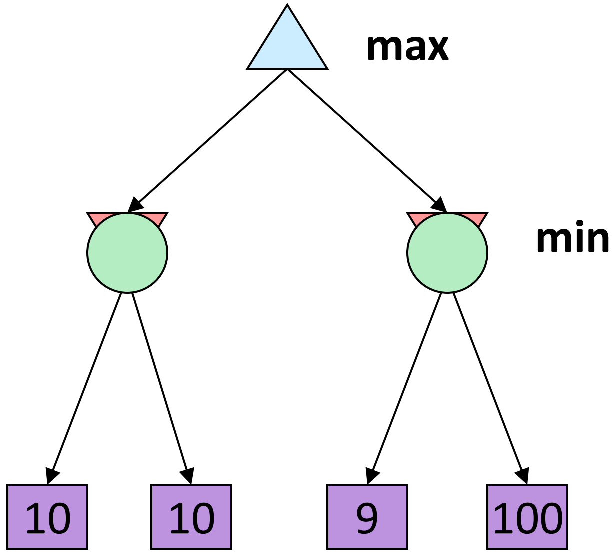


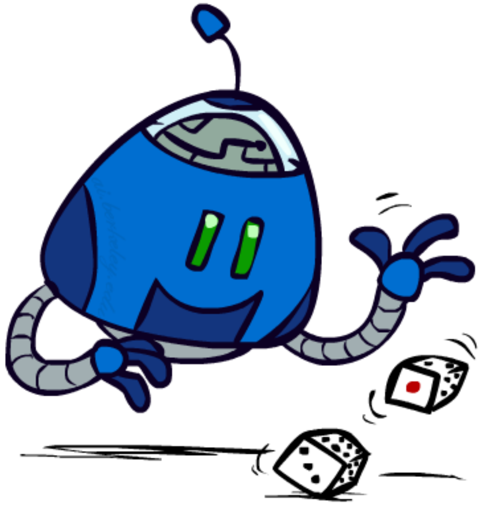

```
def min_value(state, alpha, beta):
    if game.terminal_test(state):
        return game.utility(state, player)
    v = np.inf
    for a in game.actions(state):
        v = min(v, max_value(game.result(state, a), alpha, beta))
        if v <= alpha:
            return v
        beta = min(beta, v)
    return v
```


Figure from Berkley AI

Worst-Case vs. Average Case







Idea: Uncertain outcomes controlled by chance, not an adversary!

Figure from Berkley AI

Expectimax Search

- Why wouldn't we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the ghosts respond randomly
  - Actions can fail: when moving a robot, wheels might slip
- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes
- Expectimax search: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities
  - I.e. take weighted average (expectation) of children
- Later, we'll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes

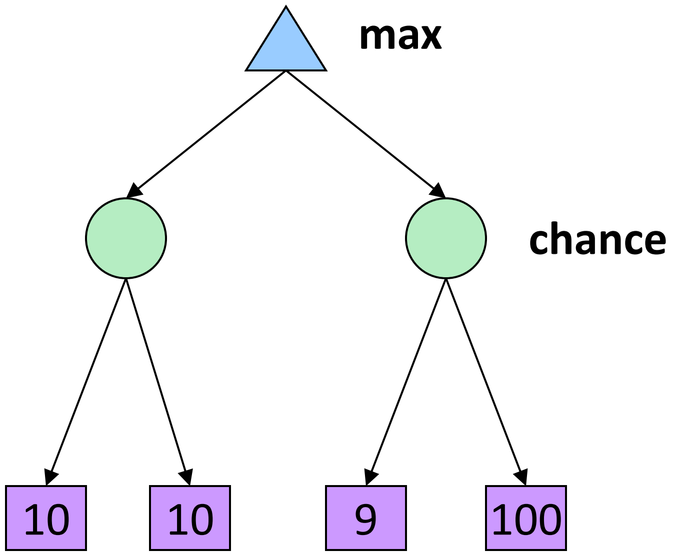
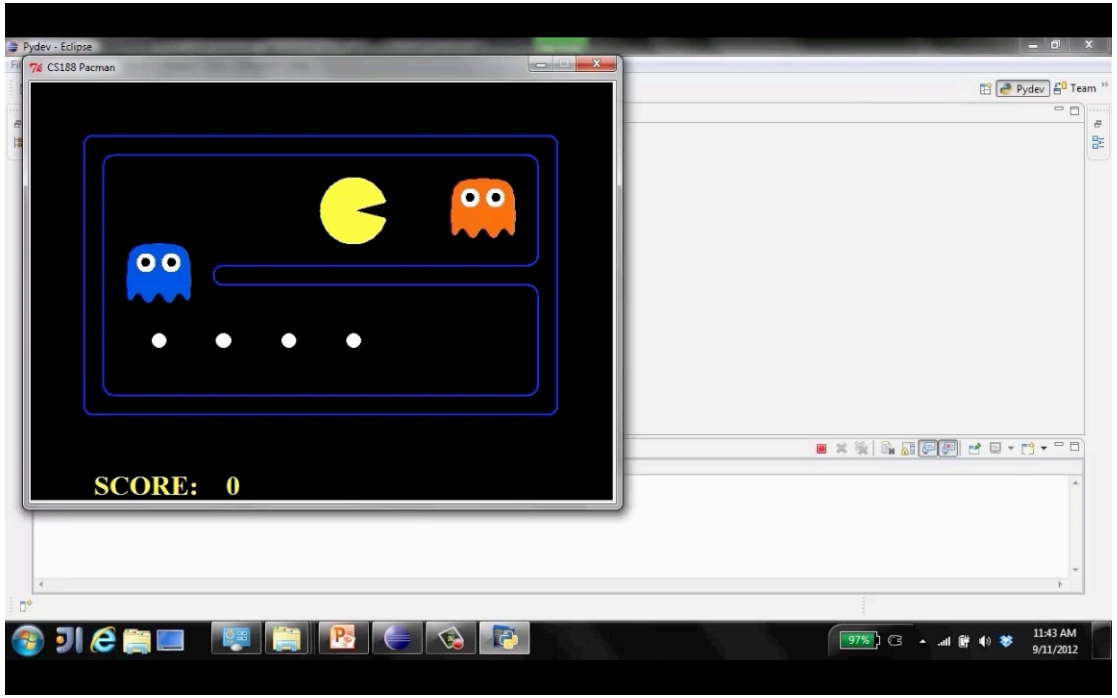


Figure from Berkley AI

Video of Minimax vs Expectimax



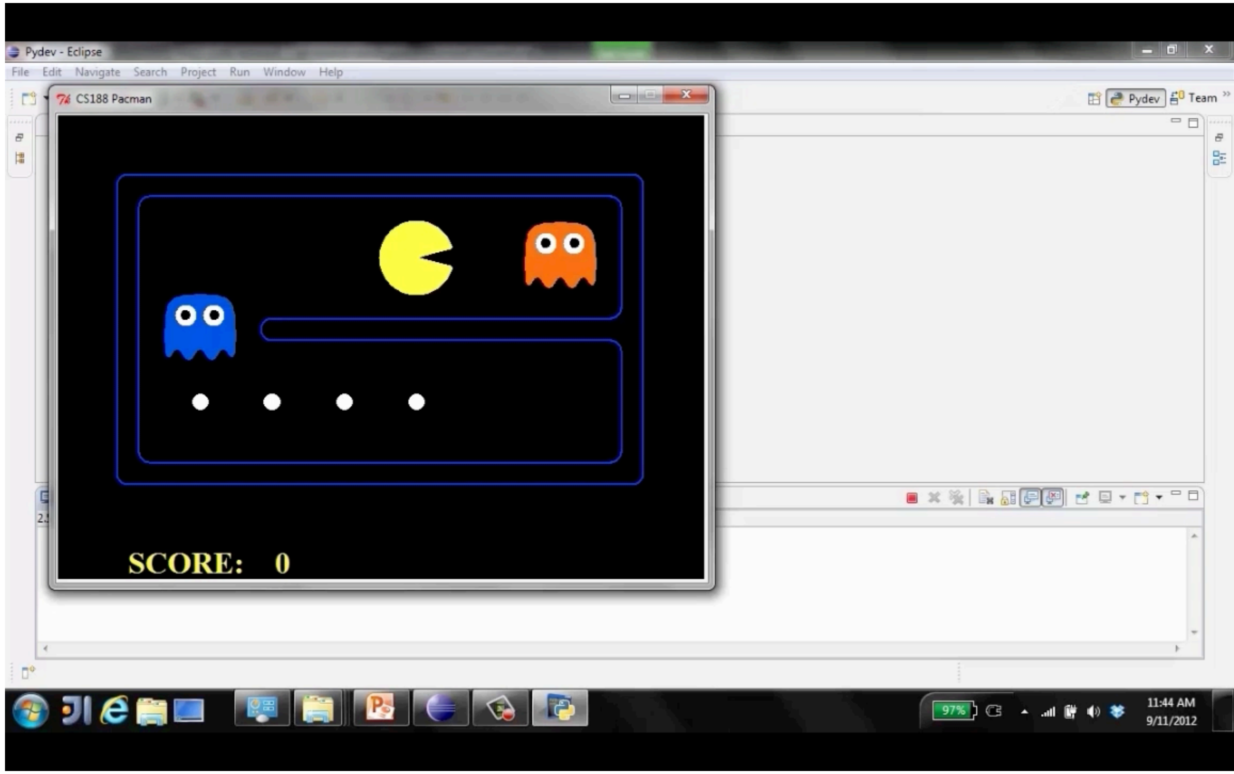


Figure from Berkley AI

Expectimax Pseudocode

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)
```

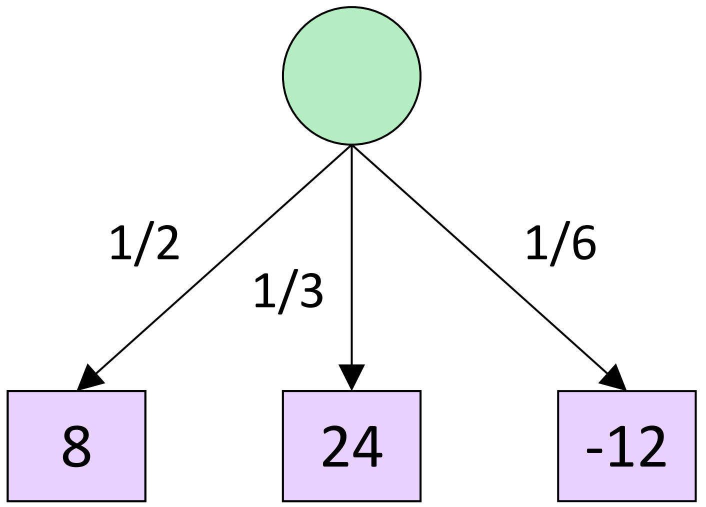
```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```


Figure from Berkley AI

Expectimax Pseudocode

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```



$$v = (1/2)\ (8) + (1/3)\ (24) + (1/6)\ (-12) = 10$$

Figure from Berkley AI

Expectimax Example

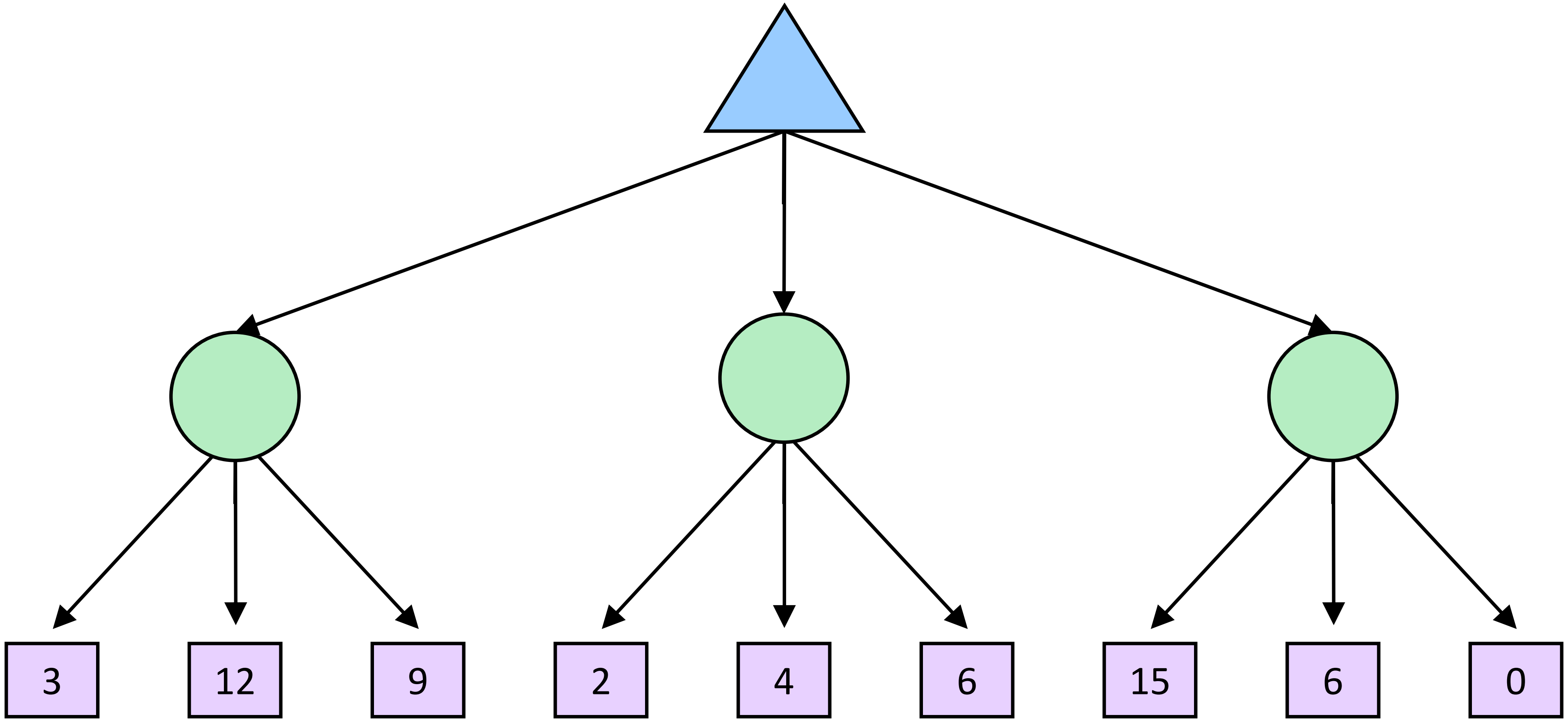
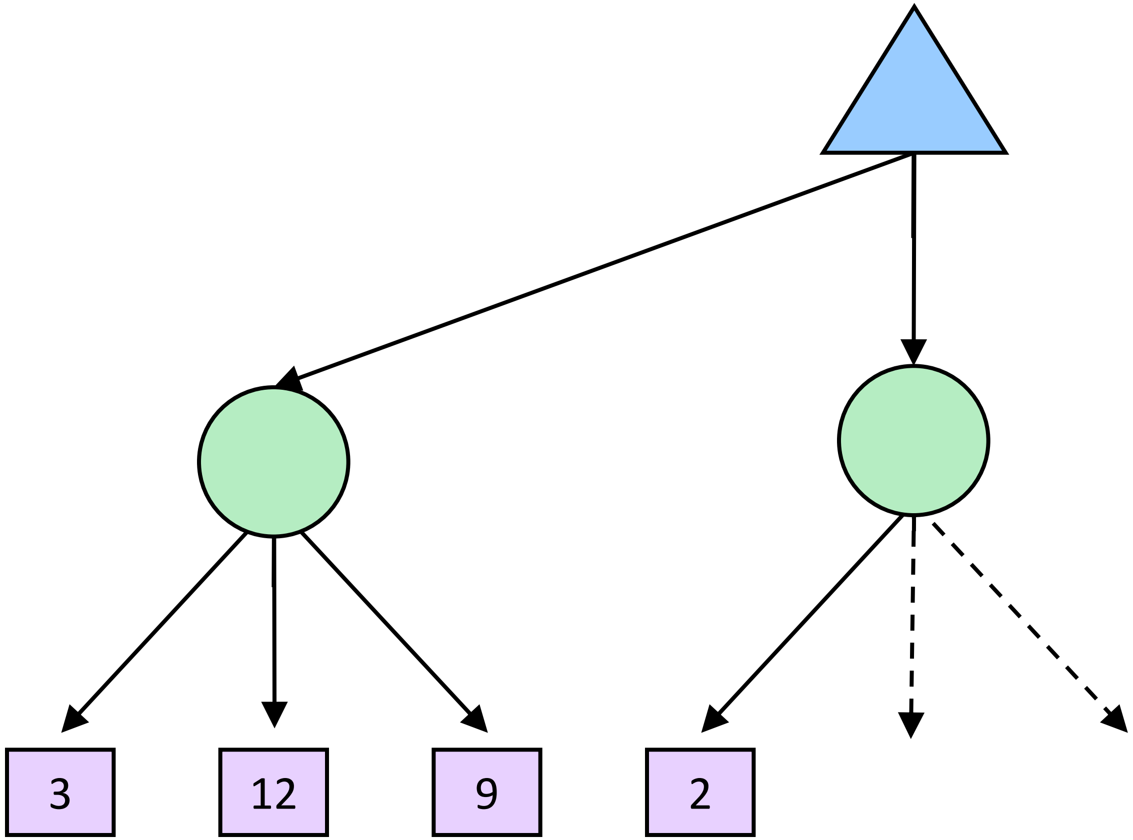


Figure from Berkley AI

Expectimax Pruning?



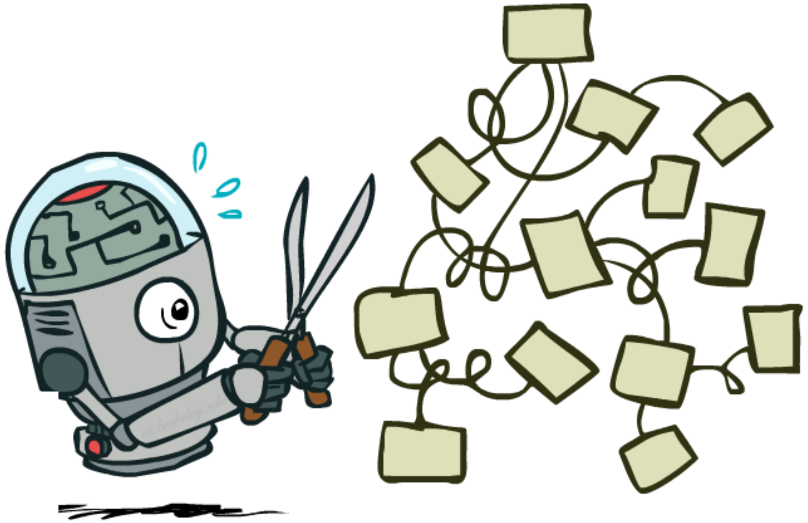
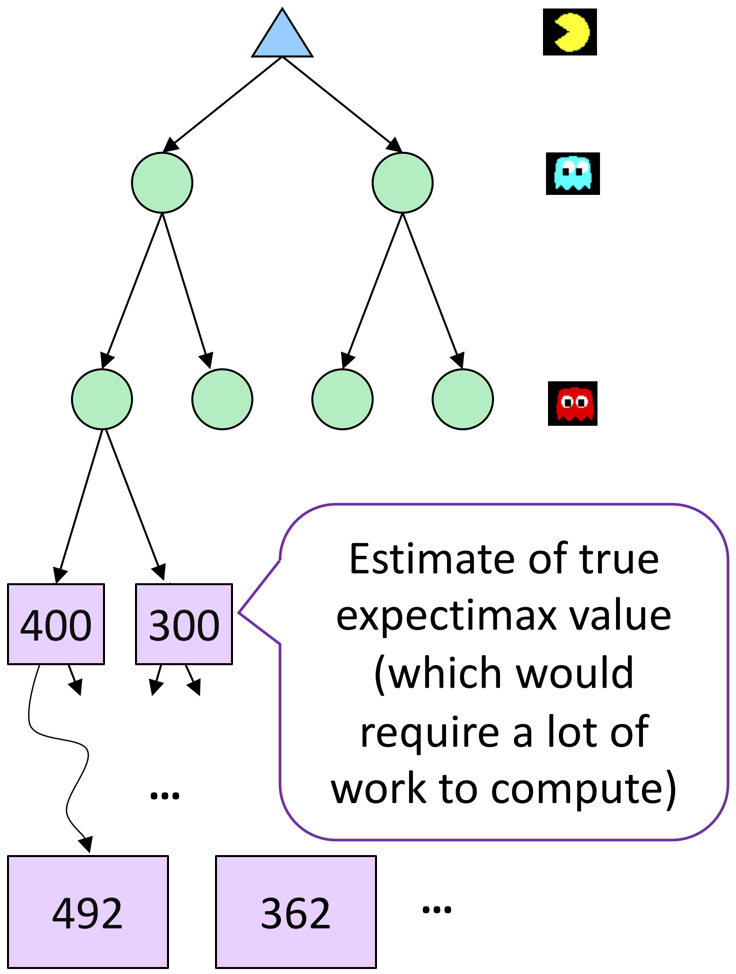


Figure from Berkley AI

Depth-Limited Expectimax



Probabilities

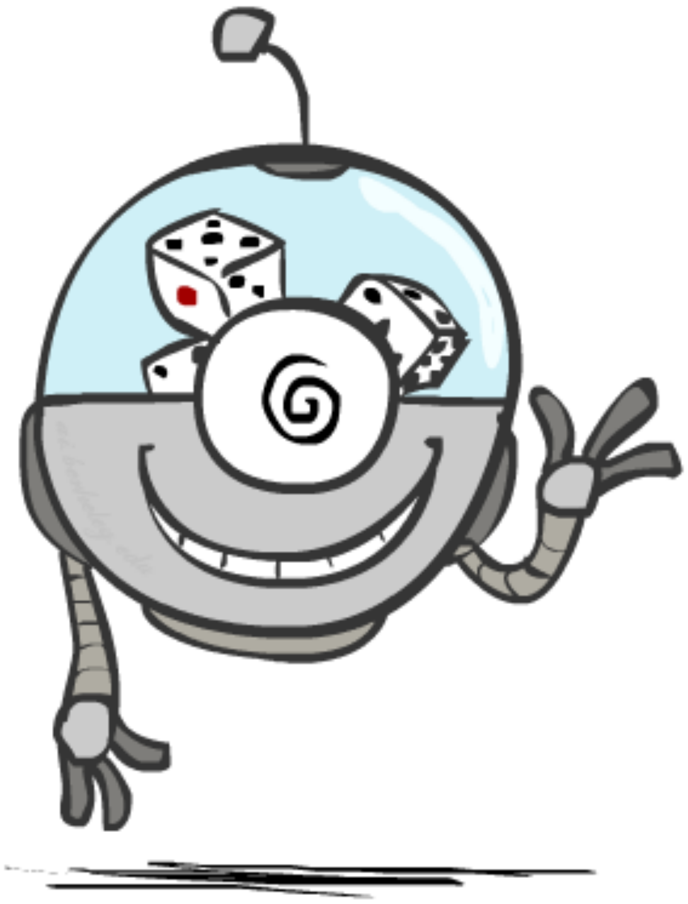


Figure from Berkley AI

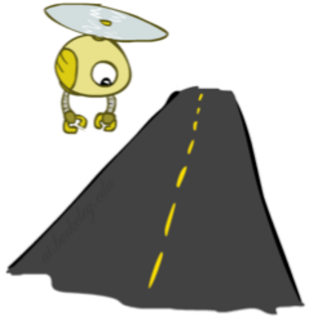
Probability Review

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes

- Example: Traffic on freeway
  - Random variable: T = whether there’s traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25

- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one

- As we get more evidence, probabilities may change:
  - P(T=heavy) = 0.25, P(T=heavy | Hour=8am) = 0.60
  - We’ll talk about methods for reasoning and updating probabilities later



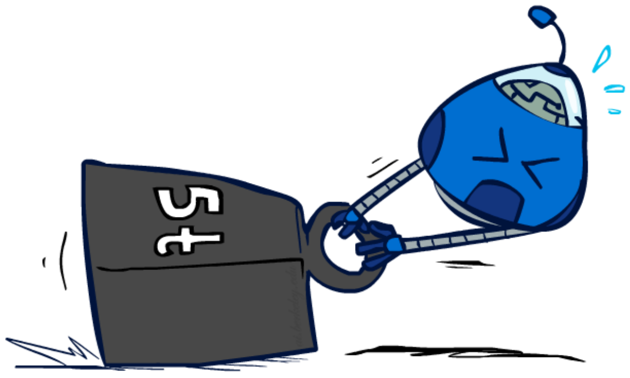
0.25

0.50

0.25

Figure from Berkley AI

Expectations Review



- The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes

- Example: How long to get to the airport?

Time:	20 min		30 min		60 min		
	x	+	x	+	x	⇒	35 min
Probability:	0.25		0.50		0.25		

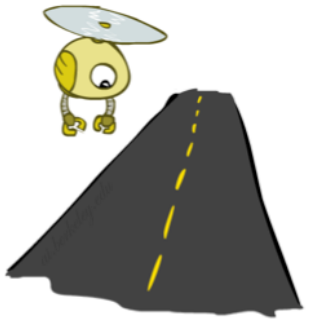


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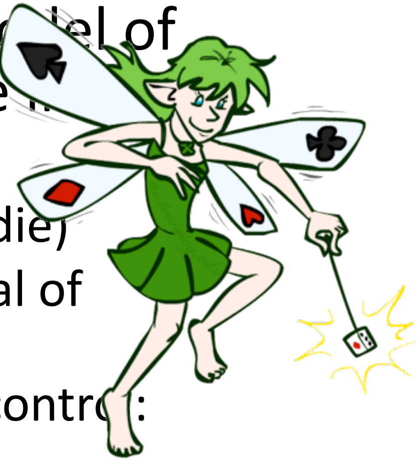
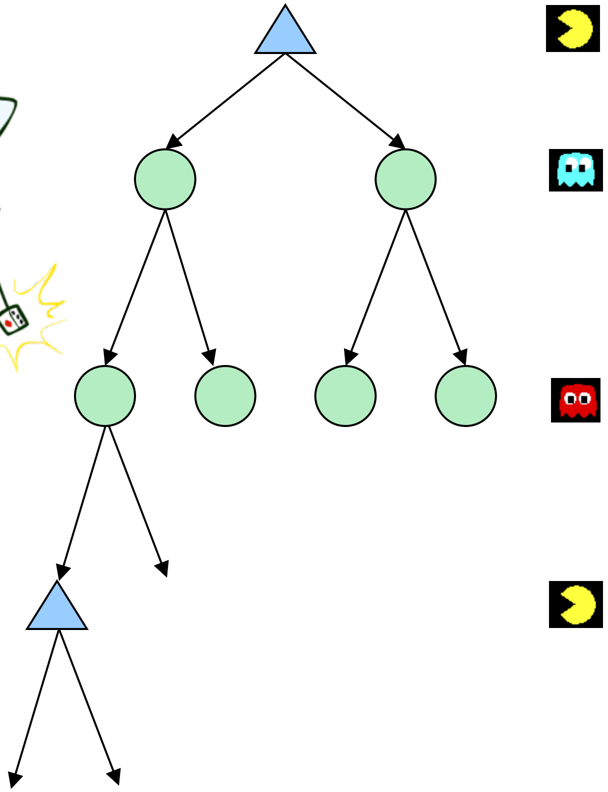
Figure from Berkley AI

What Probabilities to Use?

- In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a chance node for any outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!



- For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes

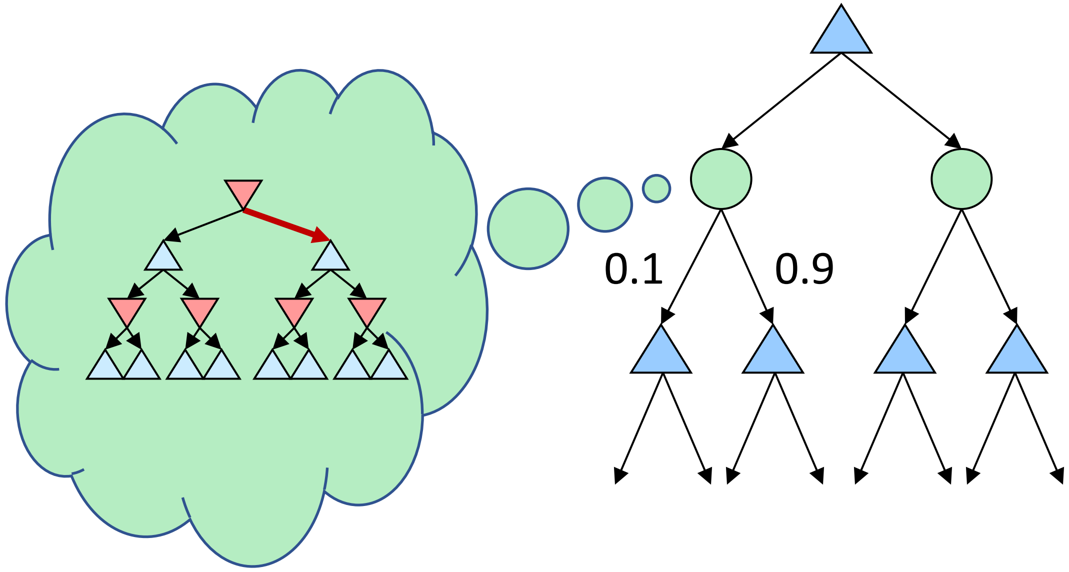


Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!

Figure from Berkley AI

Quiz: Informed Probabilities

- Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise
- Question: What tree search should you use?



- Answer: Expectimax!
  - To figure out EACH chance node's probabilities, you have to run a simulation of your opponent
  - This kind of thing gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you…
  - … except for minimax, which has the nice property that it all collapses into one game tree

The Dangers of Optimism and Pessimism

Dangerous Optimism

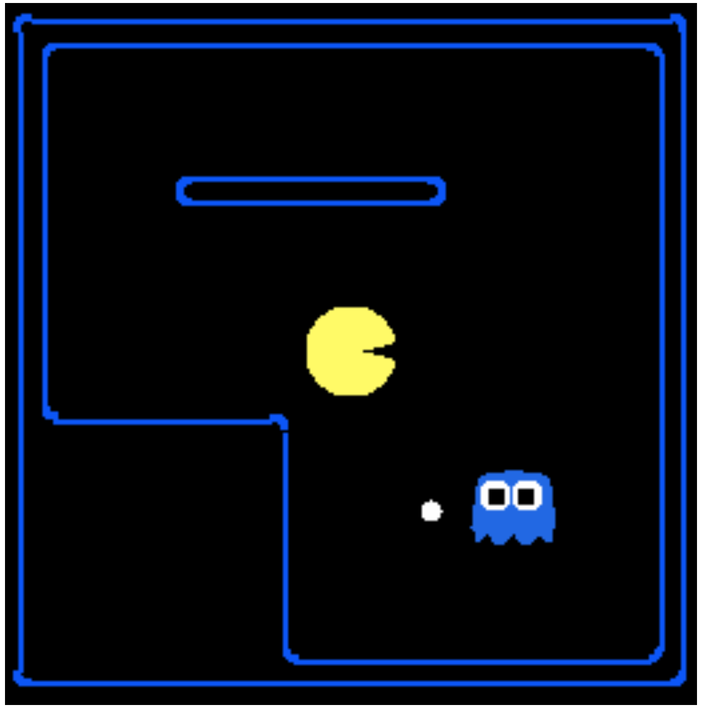
Assuming chance when the world is adversarial

Dangerous Pessimism

Assuming the worst case when it’s not likely

Figure from Berkley AI

Assumptions vs. Reality



	Adversarial Ghost	Random Ghost
Minimax Pacman		
Expectimax Pacman		

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

World Assumption Demo – Random Ghost Expectimax Pacman

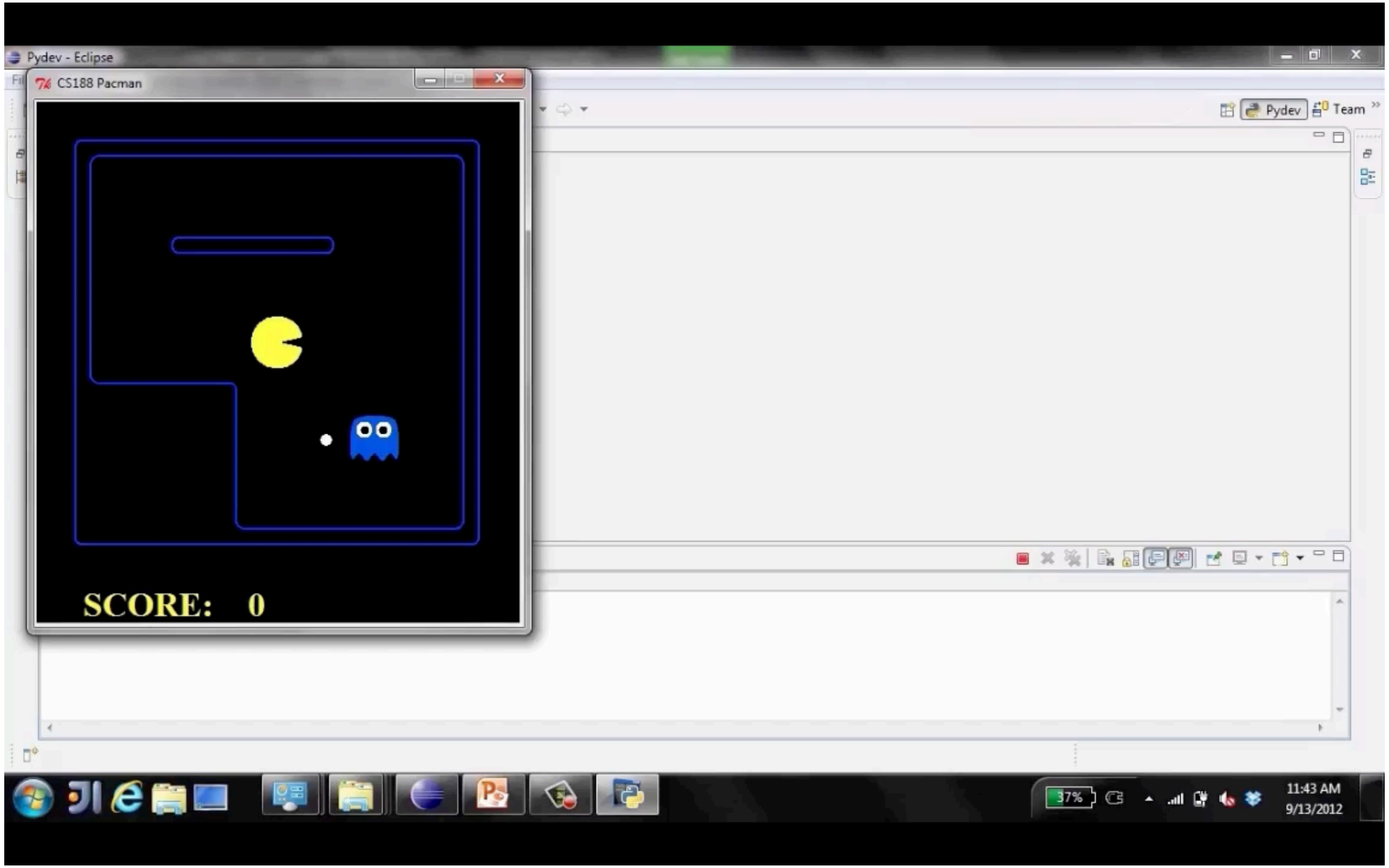


Figure from Berkley AI

World Assumption Demo – Adversarial Ghost Minimax Pacman

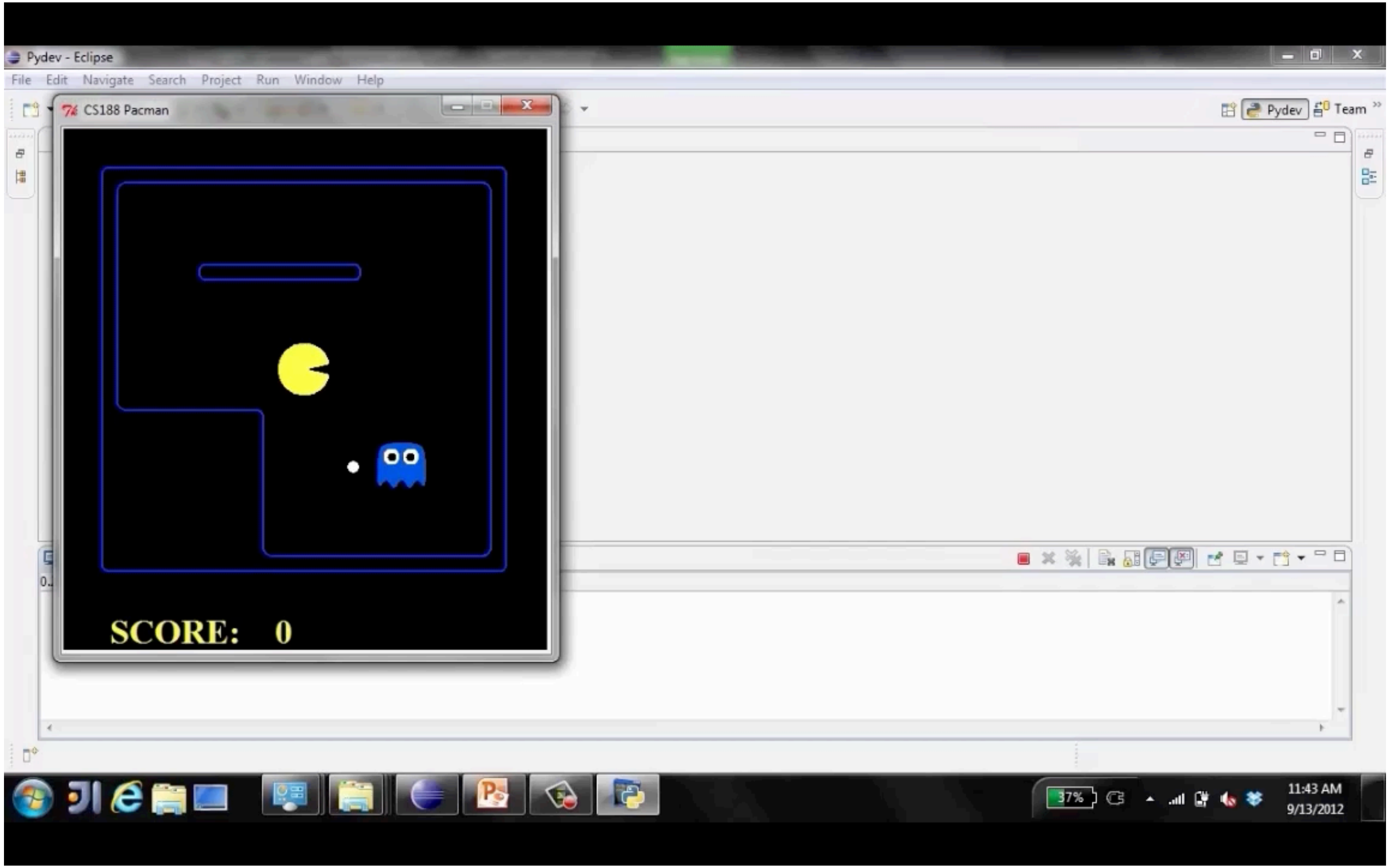


Figure from Berkley AI

World Assumption Demo – Adversarial Ghost Expectimax Pacman

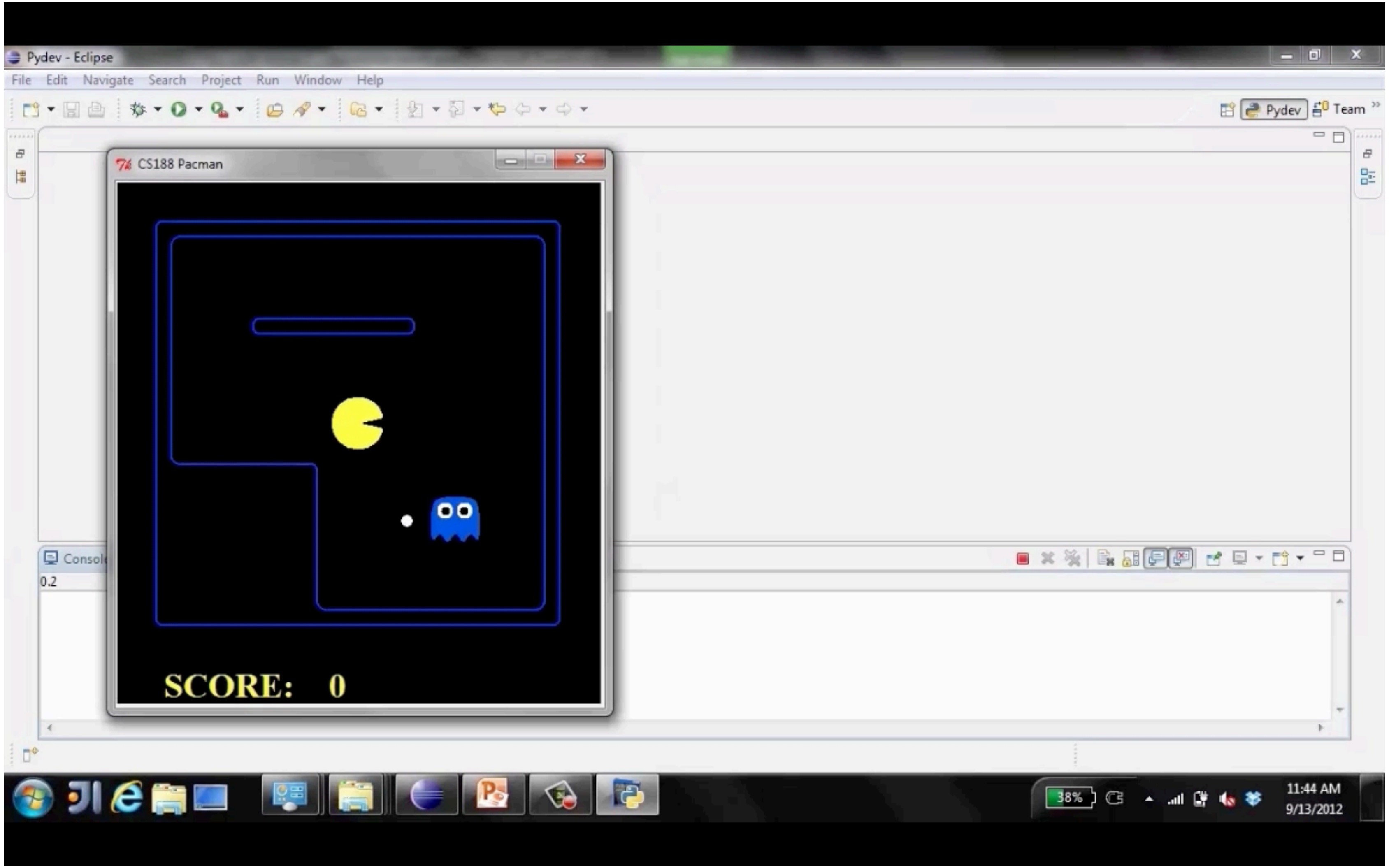


Figure from Berkley AI

World Assumption Demo – Random Ghost Minimax Pacman

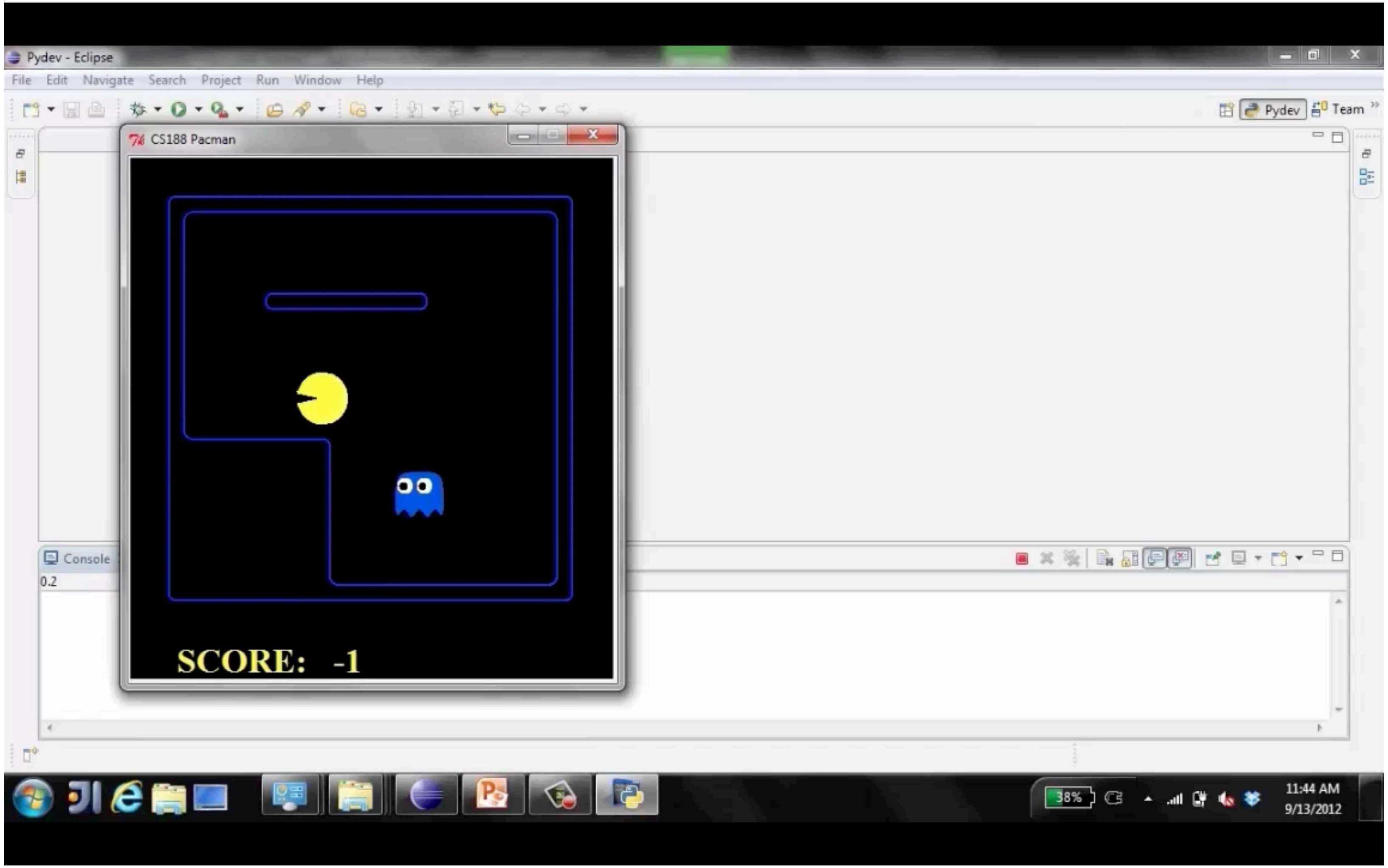
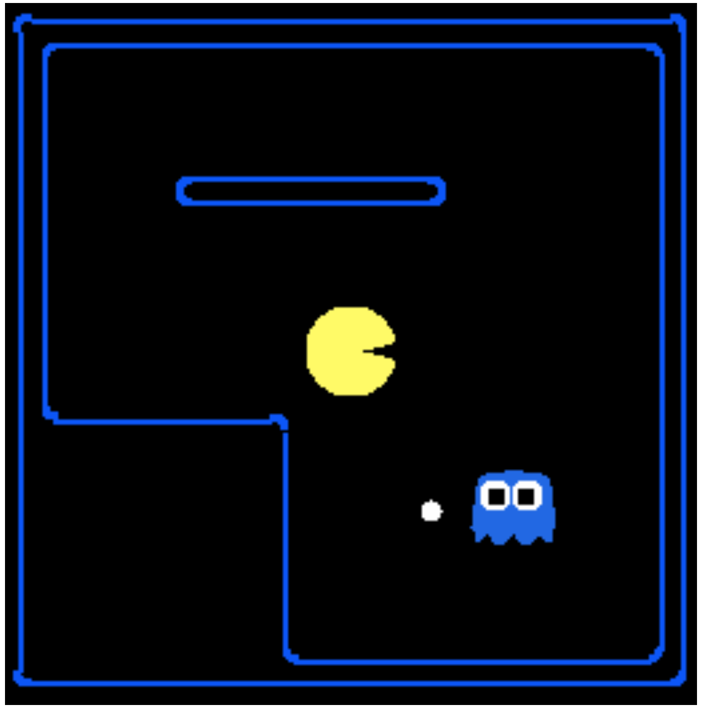


Figure from Berkley AI

Assumptions vs. Reality



	Adversarial Ghost	Random Ghost
Minimax Pacman	Won 5/5 Avg. Score: 483	Won 5/5 Avg. Score: 493
Expectimax Pacman	Won 1/5 Avg. Score: -303	Won 5/5 Avg. Score: 503

Results from playing 5 games

Pacman used depth 4 search with an eval function that avoids trouble
Ghost used depth 2 search with an eval function that seeks Pacman

Example: Backgammon

- Dice rolls increase *b*: 21 possible rolls with 2 dice
  - Backgammon ≈ 20 legal moves
  - Depth 2 = 20 x (21 x 20)^3 = 1.2 x 10^9

- As depth increases, probability of reaching a given search node shrinks
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier…

- Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play

- 1st AI world champion in any game!

Image: Wikipedia

Figure from Berkley AI

Other Game Types

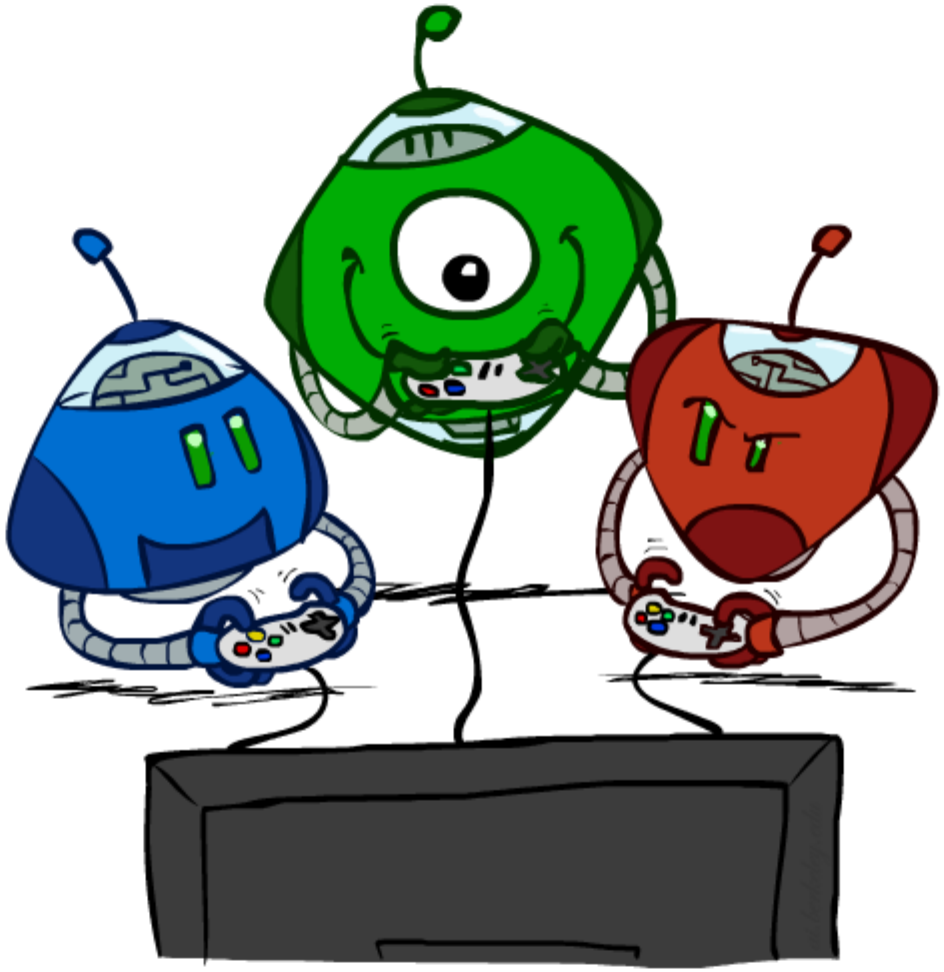
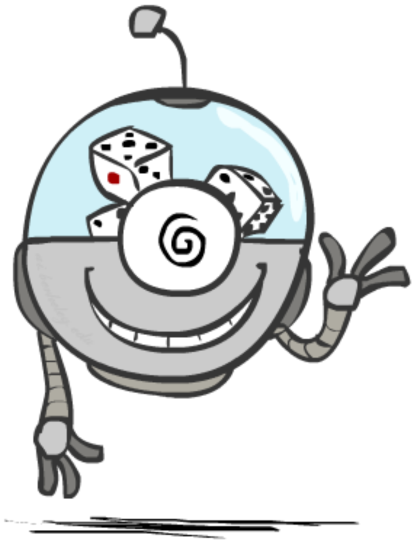


Figure from Berkley AI

Mixed Layer Types

- E.g. Backgammon
- Expectiminimax
  - Environment is an extra “random agent” player that moves after each min/max agent
  - Each node computes the appropriate combination of its children



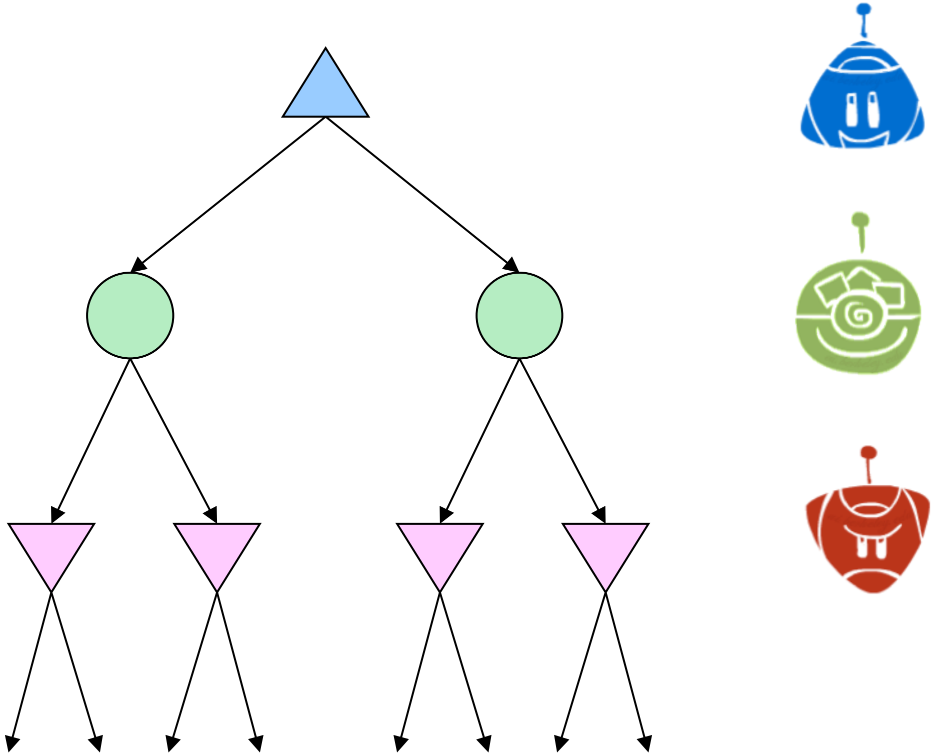
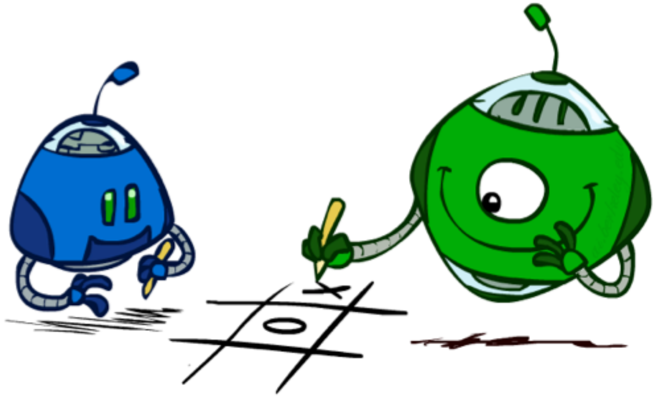
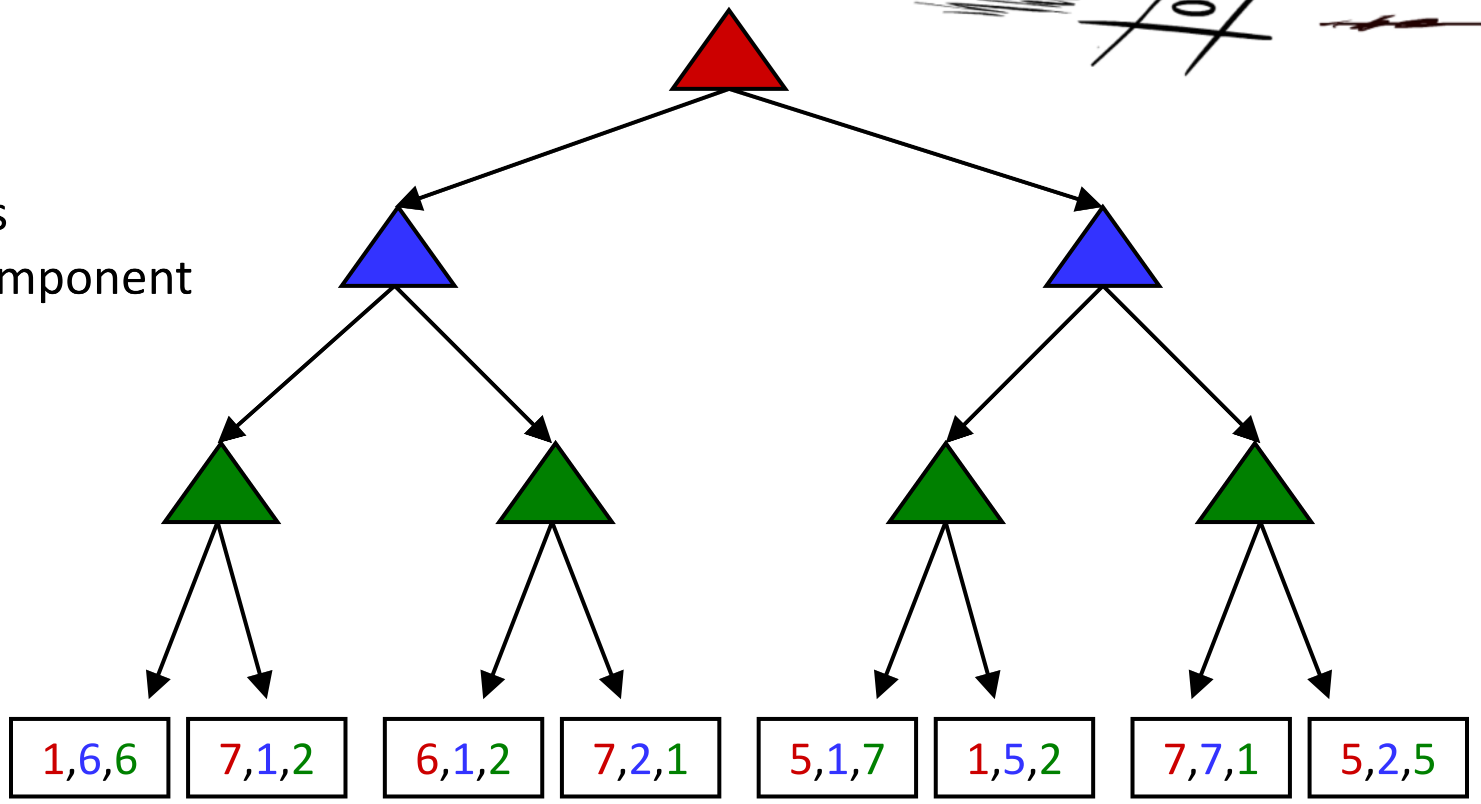


Figure from Berkley AI

Multi-Agent Utilities



- What if the game is not zero-sum, or has multiple players?
- Generalization of minimax:
  - Terminals have utility tuples
  - Node values are also utility tuples
  - Each player maximizes its own component
  - Can give rise to cooperation and competition dynamically…



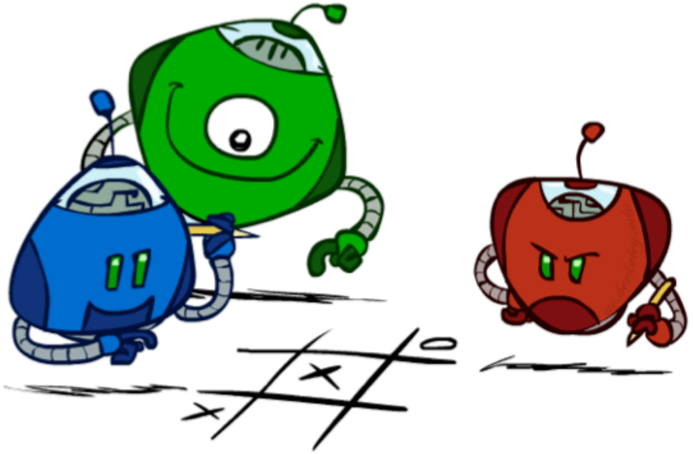


Figure from Berkley AI

Utilities

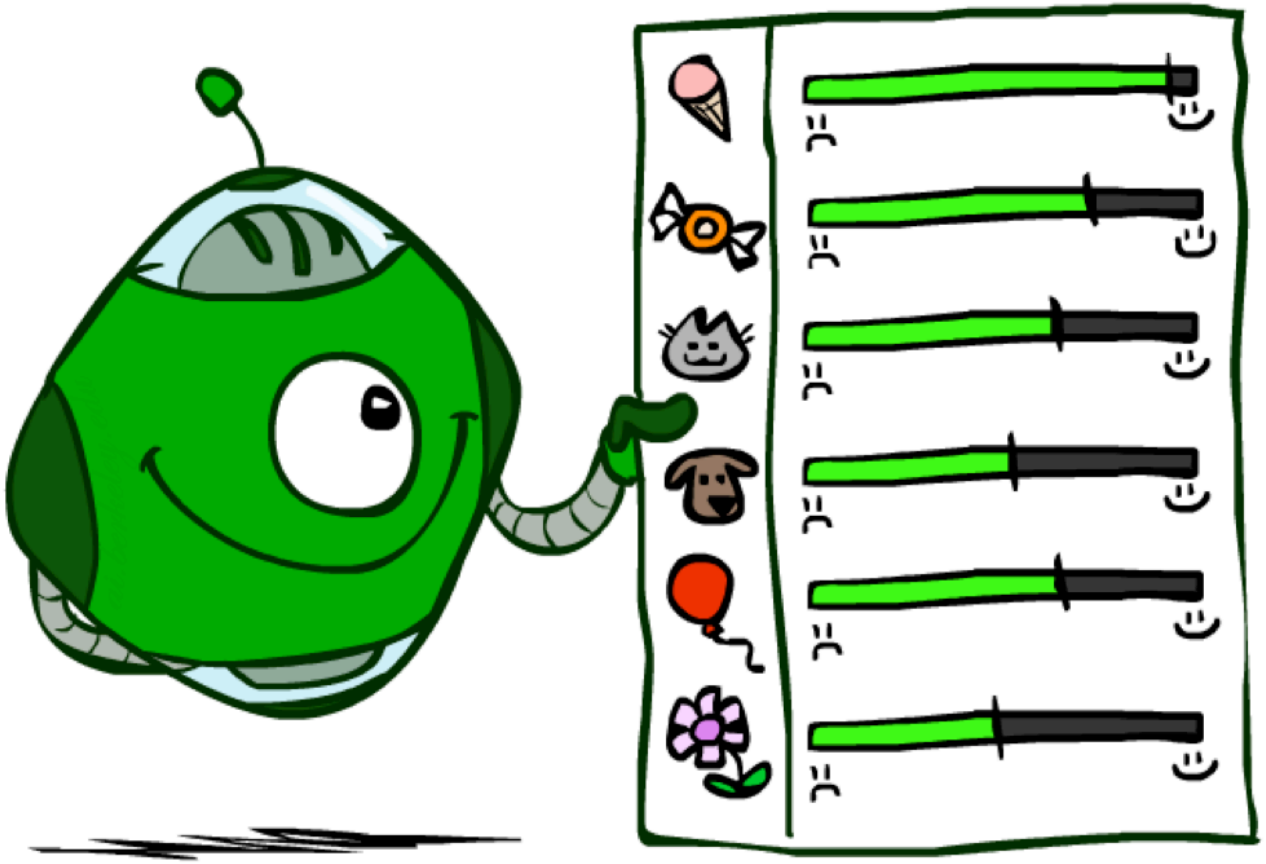


Figure from Berkley AI

Maximum Expected Utility

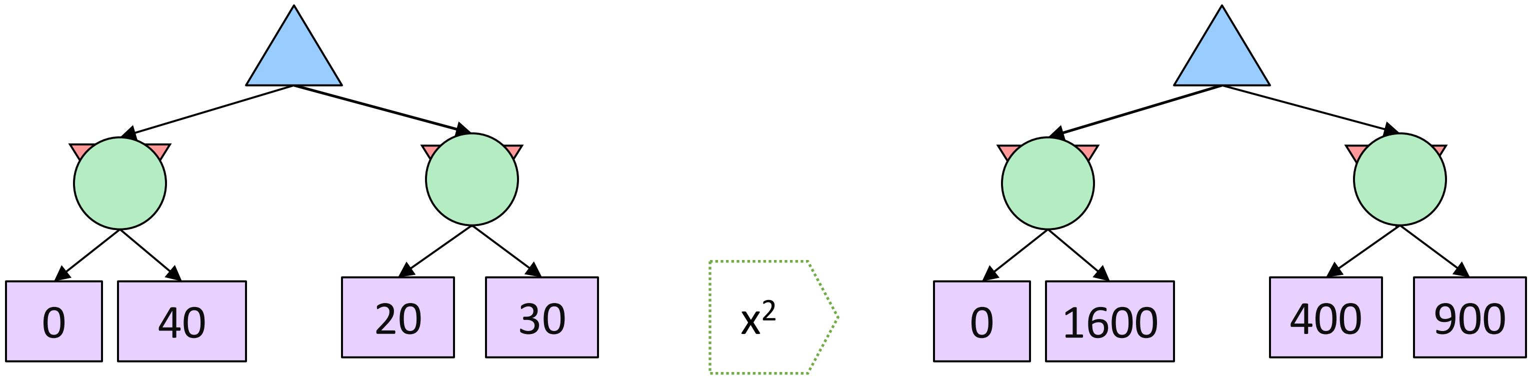
- Why should we average utilities? Why not minimax?

- Principle of maximum expected utility:
  - A rational agent should chose the action that maximizes its expected utility, given its knowledge

- Questions:
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - How do we know that averaging even makes sense?
  - What if our behavior (preferences) can't be described by utilities?

Figure from Berkley AI

What Utilities to Use?



- For worst-case minimax reasoning, terminal function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- For average-case expectimax reasoning, we need *magnitudes* to be meaningful

Utilities

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences

- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function

- We hard-wire utilities and let behaviors emerge
  - Why don't we let agents pick utilities?
  - Why don't we prescribe behaviors?

Figure from Berkley AI

Utilities: Uncertain Outcomes

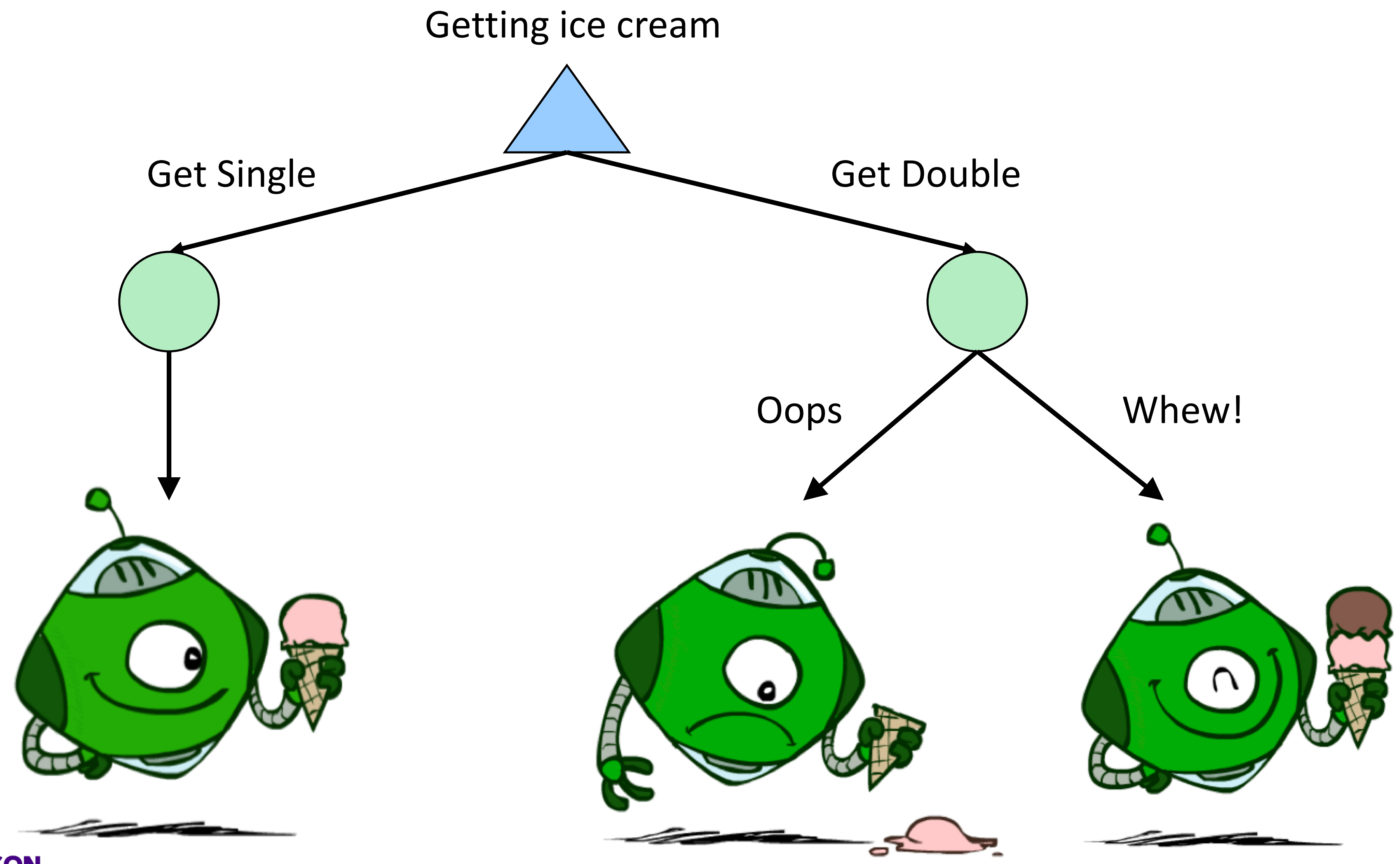


Figure from Berkley AI

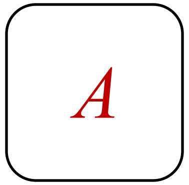
Preferences

- An agent must have preferences among:
  - Prizes: A, B, etc.
  - Lotteries: situations with uncertain prizes

$$L = [p, A;\ (1-p), B]$$

- Notation:
  - Preference: $A \succ B$
  - Indifference: $A \sim B$

A Prize



A Lottery

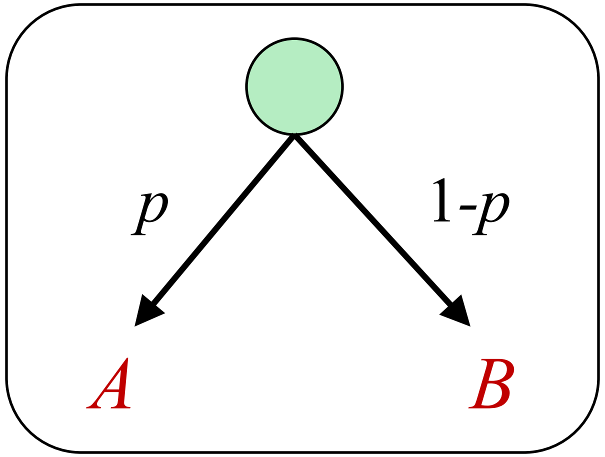


Figure from Berkley AI

Rationality

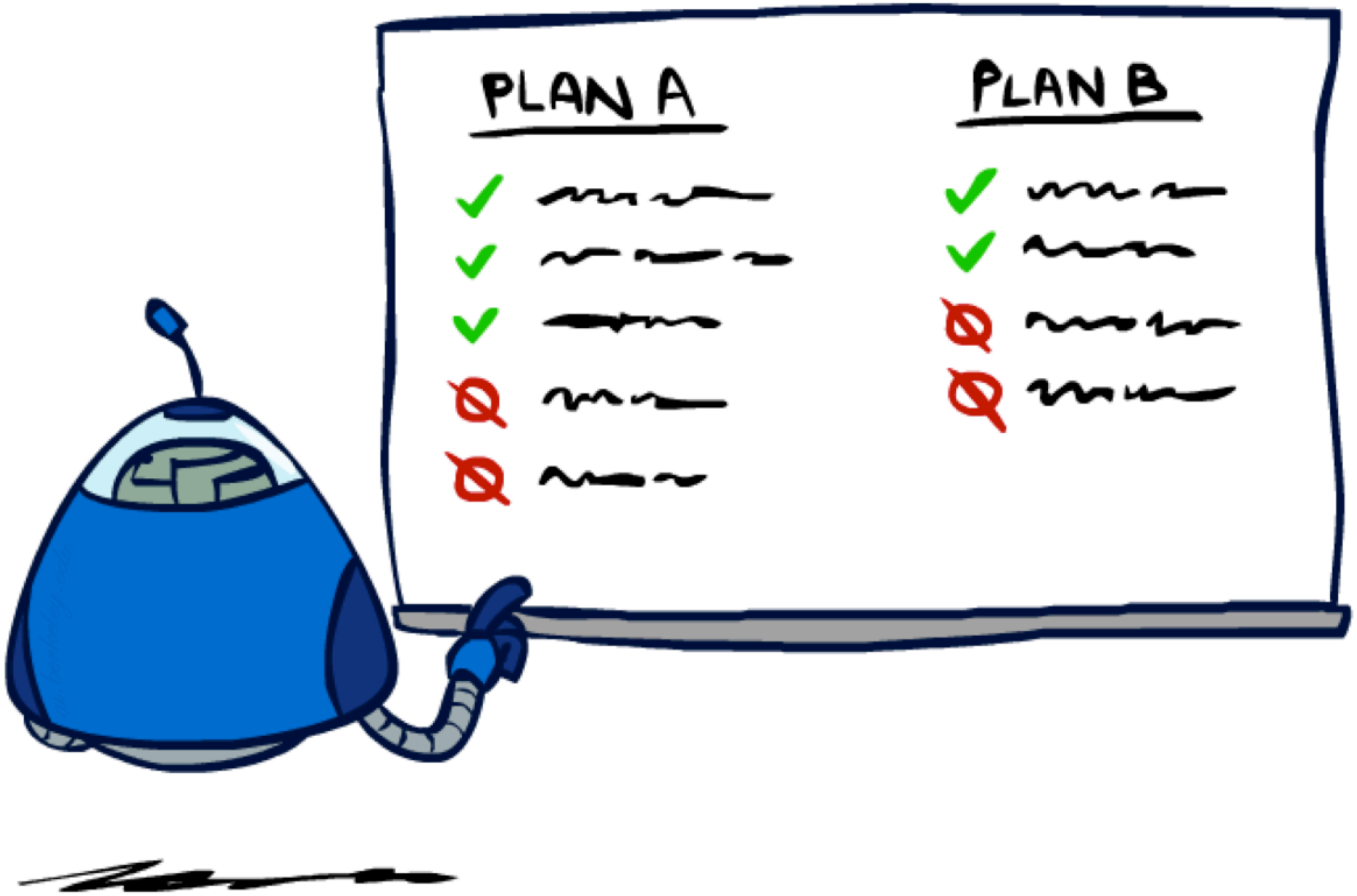


Figure from Berkley AI

Rational Preferences

- We want some constraints on preferences before we call them rational, such as:

Axiom of Transitivity: $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

- For example: an agent with intransitive preferences can be induced to give away all of its money
  - If B > C, then an agent with C would pay (say) 1 cent to get B
  - If A > B, then an agent with B would pay (say) 1 cent to get A
  - If C > A, then an agent with A would pay (say) 1 cent to get C

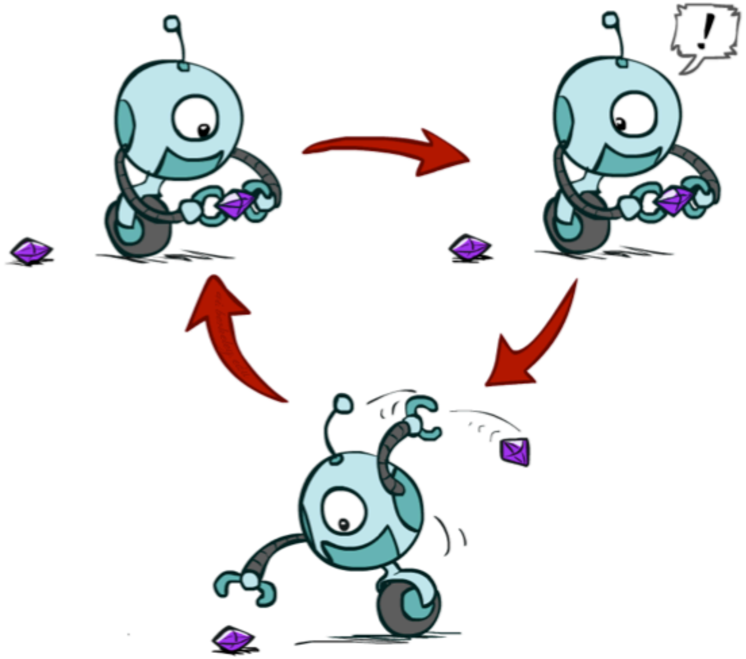


Figure from Berkley AI

Rational Preferences

The Axioms of Rationality

<u>Orderability</u>

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

<u>Transitivity</u>

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

<u>Continuity</u>

$$A \succ B \succ C \Rightarrow \exists p \ [p, A; \ 1-p, C] \sim B$$

<u>Substitutability</u>

$$A \sim B \Rightarrow [p, A; \ 1-p, C] \sim [p, B; 1-p, C]$$

<u>Monotonicity</u>

$$A \succ B \Rightarrow$$
$$(p \geq q \Leftrightarrow [p, A; \ 1-p, B] \succeq [q, A; \ 1-q, B])$$

Theorem: Rational preferences imply behavior describable as maximization of expected utility

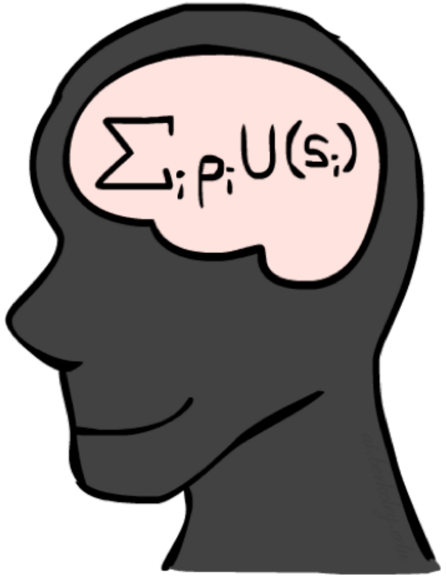
MEU Principle

- Theorem [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:

$$U(A) \geq U(B) \;\Leftrightarrow\; A \succeq B$$

$$U([p_1, S_1;\; \ldots\; ;\; p_n, S_n]) = \textstyle\sum_i p_i U(S_i)$$

  - I.e. values assigned by U preserve preferences of both prizes and lotteries!



- Maximum expected utility (MEU) principle:
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tic-tac-toe, a reflex vacuum cleaner

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Figure from Berkley AI

Human Utilities

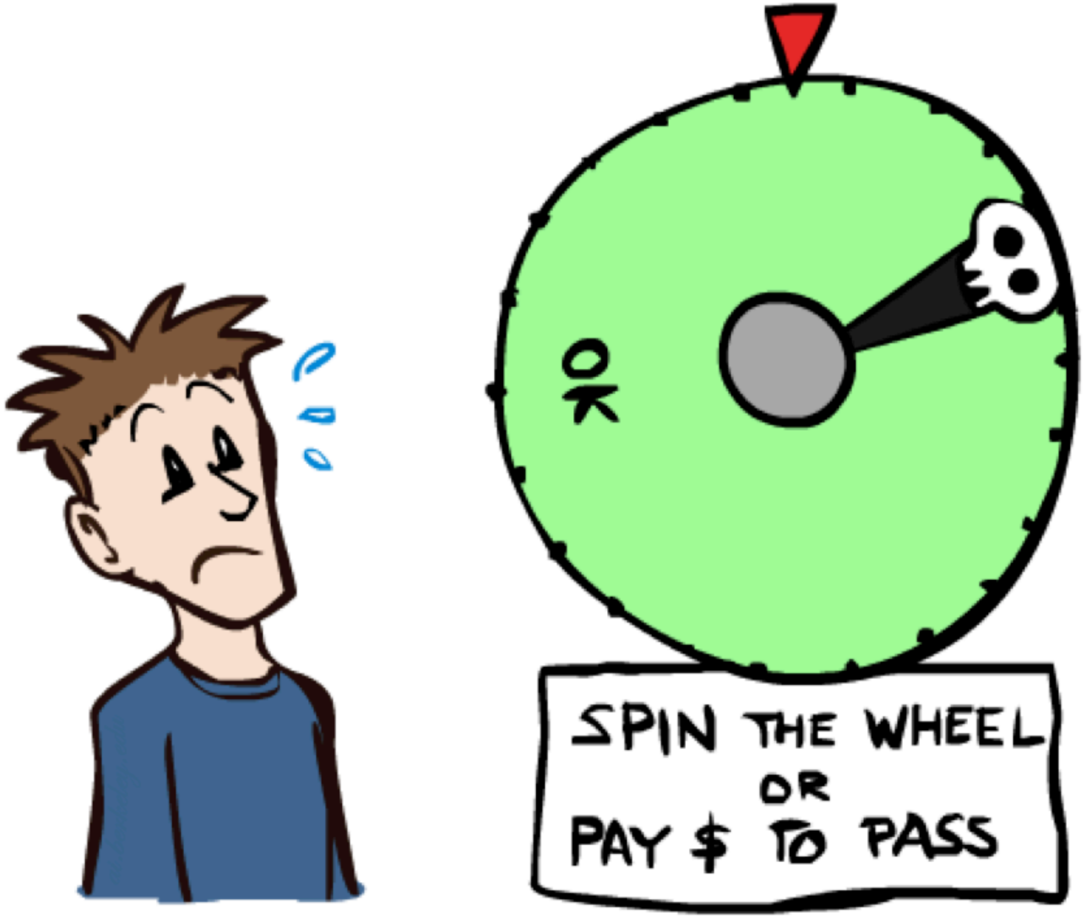


Figure from Berkley AI

Utility Scales

- Normalized utilities: $u_+ = 1.0$, $u_- = 0.0$
- Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.
- QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk
- Note: behavior is invariant under positive linear transformation

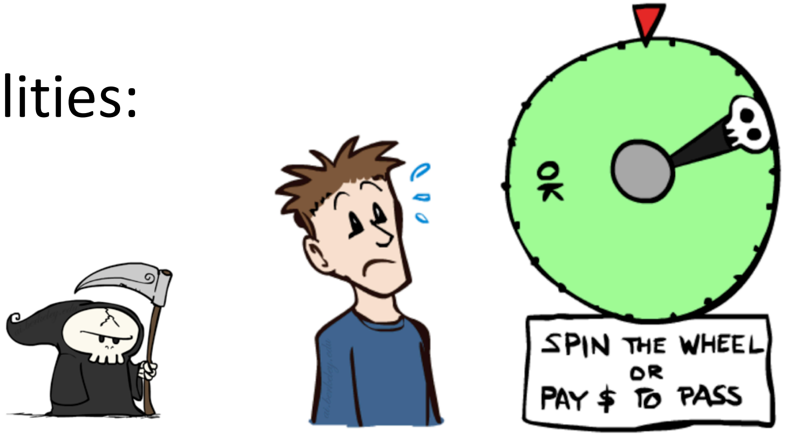
$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

- With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes

Figure from Berkley AI

Human Utilities

- Utilities map states to real numbers. Which numbers?
- Standard approach to assessment (elicitation) of human utilities:
  - Compare a prize A to a standard lottery L_p between
    - "best possible prize" u_+ with probability p
    - "worst possible catastrophe" u_- with probability 1-p
  - Adjust lottery probability p until indifference: A ~ L_p
  - Resulting p is a utility in [0,1]



Pay $30 ~

0.999999 → *No change*

0.000001 → *Instant death*

Figure from Berkley AI

Money

- Money does not behave as a utility function, but we can talk about the utility of having money (or being in debt)
- Given a lottery L = [p, $X; (1-p), $Y]
  - The expected monetary value EMV(L) is p*X + (1-p)*Y
  - U(L) = p*U($X) + (1-p)*U($Y)
  - Typically, U(L) < U(EMV(L))
  - In this sense, people are risk-averse
  - When deep in debt, people are risk-prone

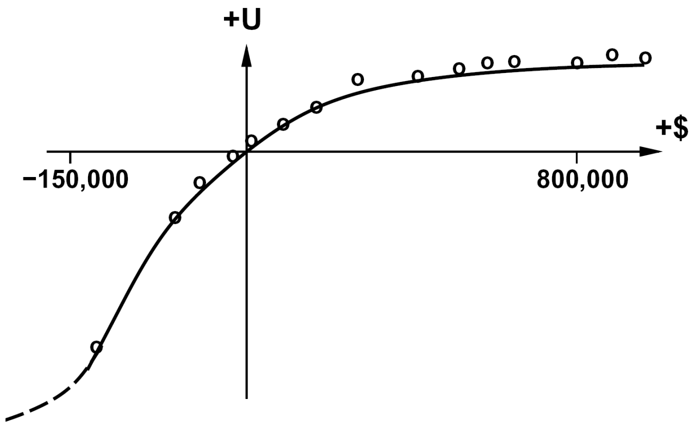


Figure from Berkley AI

Example: Insurance

- Consider the lottery [0.5, $1000; 0.5, $0]
  - What is its expected monetary value? ($500)
  - What is its certainty equivalent?
    - Monetary value acceptable in lieu of lottery
    - $400 for most people
  - Difference of $100 is the insurance premium
    - There's an insurance industry because people will pay to reduce their risk
    - If everyone were risk-neutral, no insurance needed!
  - It's win-win: you'd rather have the $400 and the insurance company would rather have the lottery (their utility curve is flat and they have many lotteries)

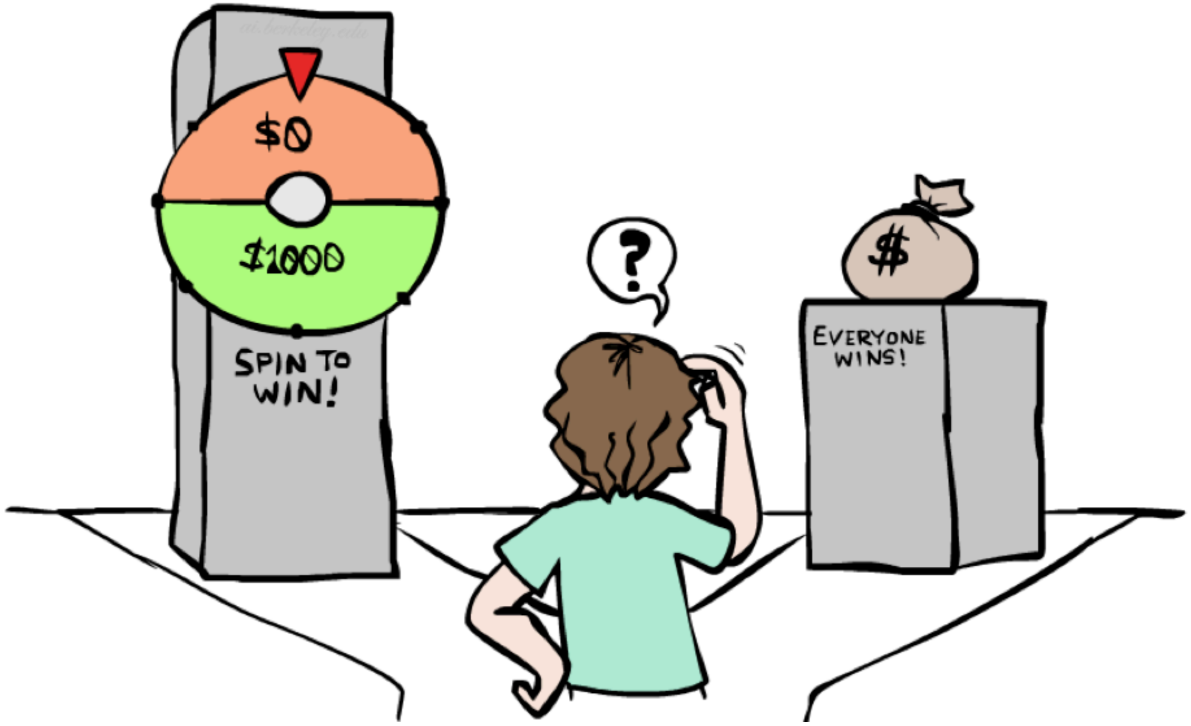


Figure from Berkley AI

Example: Human Rationality?

- Famous example of Allais (1953)
  - A: [0.8, \$4k;   0.2, \$0]
  - B: [1.0, \$3k;   0.0, \$0]
  - C: [0.2, \$4k;   0.8, \$0]
  - D: [0.25, \$3k;   0.75, \$0]
- Most people prefer B > A, C > D
- But if U(\$0) = 0, then
  - $B > A \Rightarrow U(\$3k) > 0.8\, U(\$4k)$
  - $C > D \Rightarrow 0.8\, U(\$4k) > U(\$3k)$

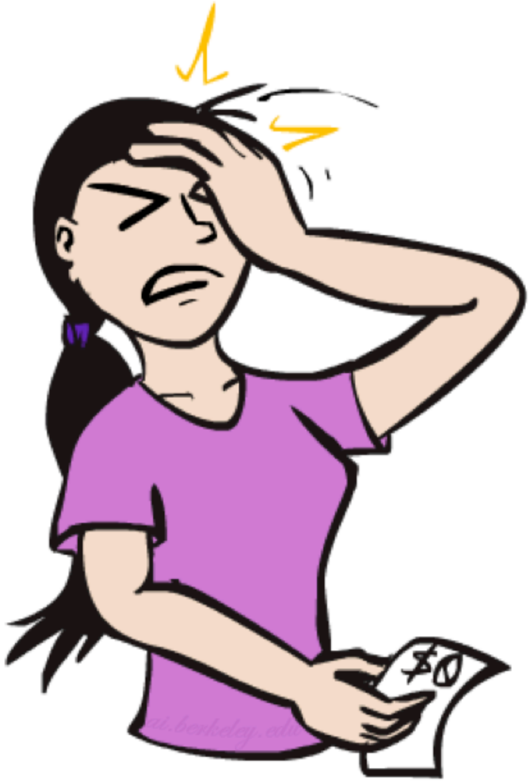


Figure from Berkley AI

Next Time

- Quiz 1b and Quiz 2 Release tomorrow. Due Monday
- PA 2 Due next Friday
- Next time we will start discussion Markov Decision Processes (MDPs)