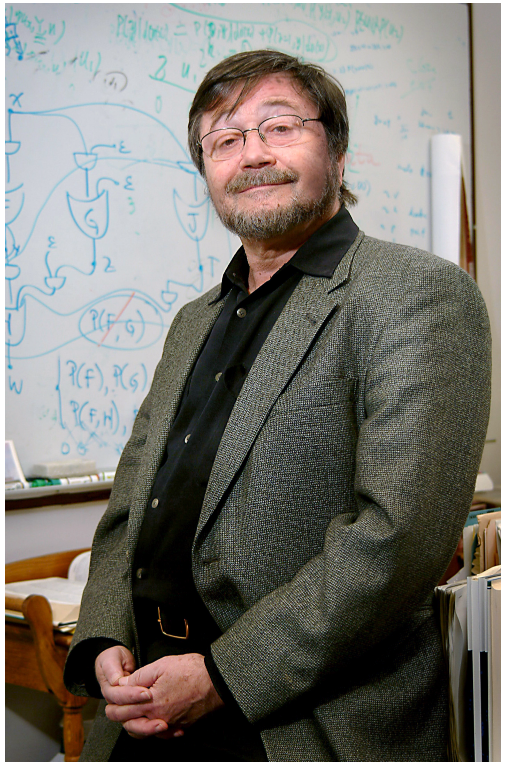
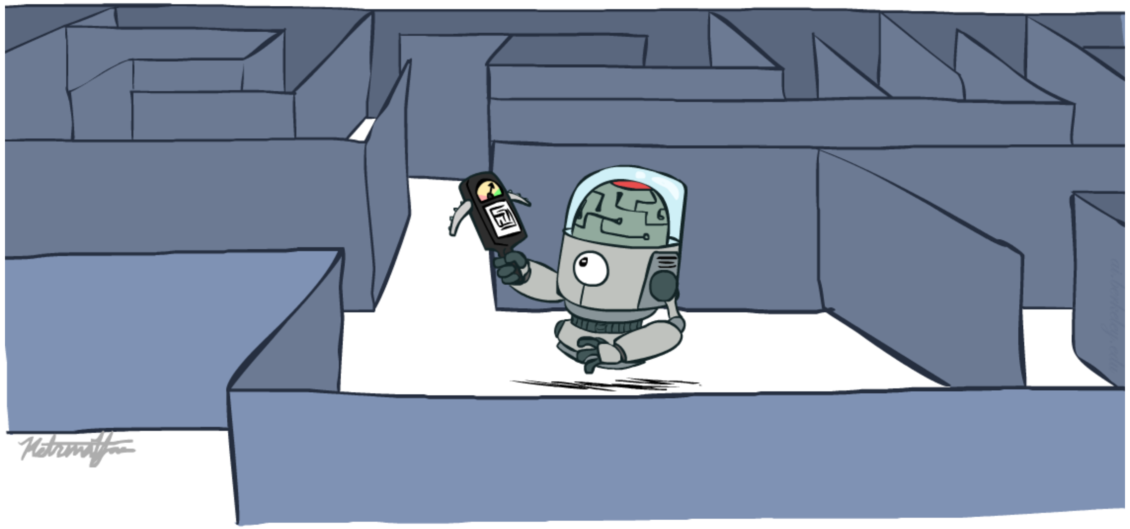


Artificial Intelligence Informed Search



CS 444 – Spring 2021

Dr. Kevin Molloy

Department of Computer Science

James Madison University

JAMES MADISON UNIVERSITY

Much of this lecture is taken from
Dan Klein and Pieter Abbeel AI class at UC Berkeley

Announcements

- HW 2 is due tomorrow night.
- PA 1 is due Feb 1st You should have started by now.
- First mastery quiz due this Friday. Topics will be posted on the website.

Learning Objectives for Today

- Informed Search
  - Heuristics
  - Greedy Search
  - A* Search

- Graph Search

Figure from Berkley AI

Search Recap

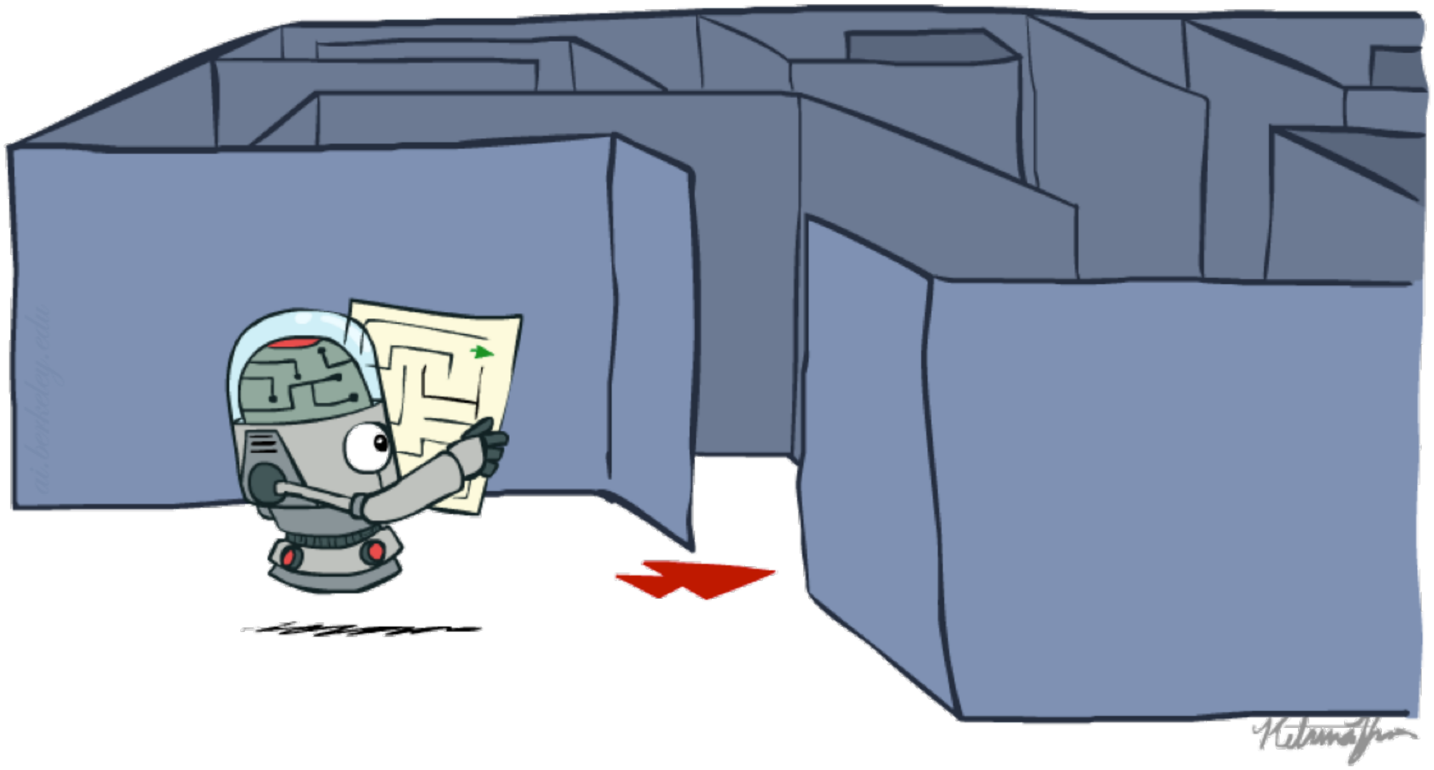


Figure from Berkley AI

Search Recap

- Search problem:
  - States (configurations of the world)
  - Actions and costs
  - Successor function (world dynamics)
  - Start state and goal test
- Search tree:
  - Nodes: represent plans for reaching states
  - Plans have costs (sum of action costs)
- Search algorithm:
  - Systematically builds a search tree
  - Chooses an ordering of the fringe (unexplored nodes)
  - Optimal: finds least-cost plans

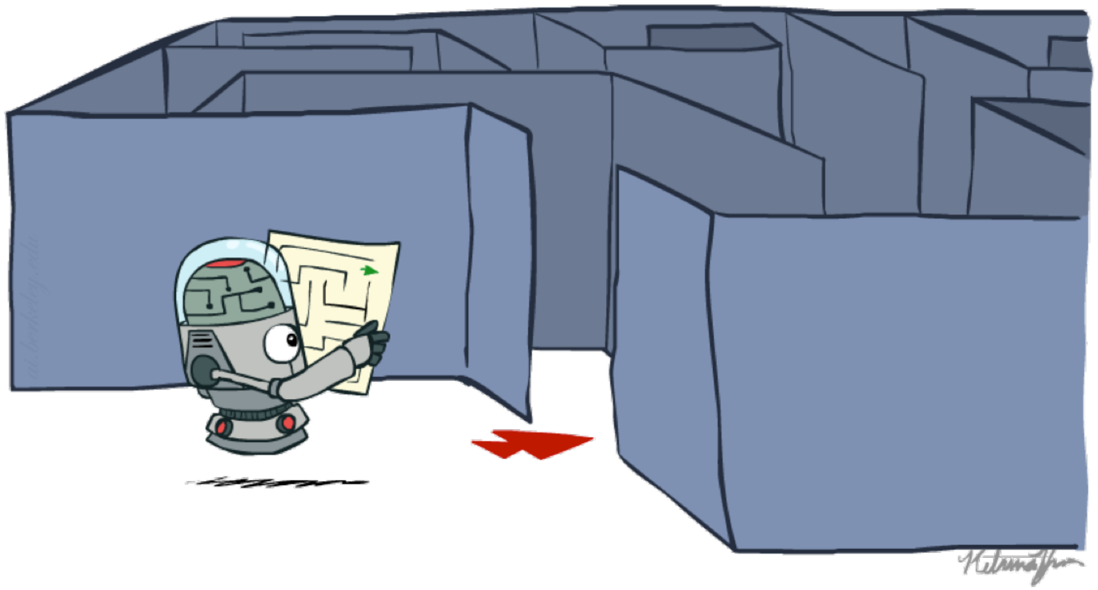


Figure from Berkley AI

Example: Pancake Problem

BOUNDS FOR SORTING BY PREFIX REVERSAL

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Received 18 January 1978
Revised 28 August 1978

For a permutation σ of the integers from 1 to n, let $f(\sigma)$ be the smallest number of prefix reversals that will transform σ to the identity permutation, and let $f(n)$ be the largest such $f(\sigma)$ for all σ in (the symmetric group) S_n. We show that $f(n) \leqslant (5n+5)/3$, and that $f(n) \geqslant 17n/16$ for n a multiple of 16. If, furthermore, each integer is required to participate in an even number of reversed prefixes, the corresponding function $g(n)$ is shown to obey $3n/2-1 \leqslant g(n) \leqslant 2n+3$.

Cost: Number of pancakes flipped

Example: Pancake Problem

State space graph with costs as weights

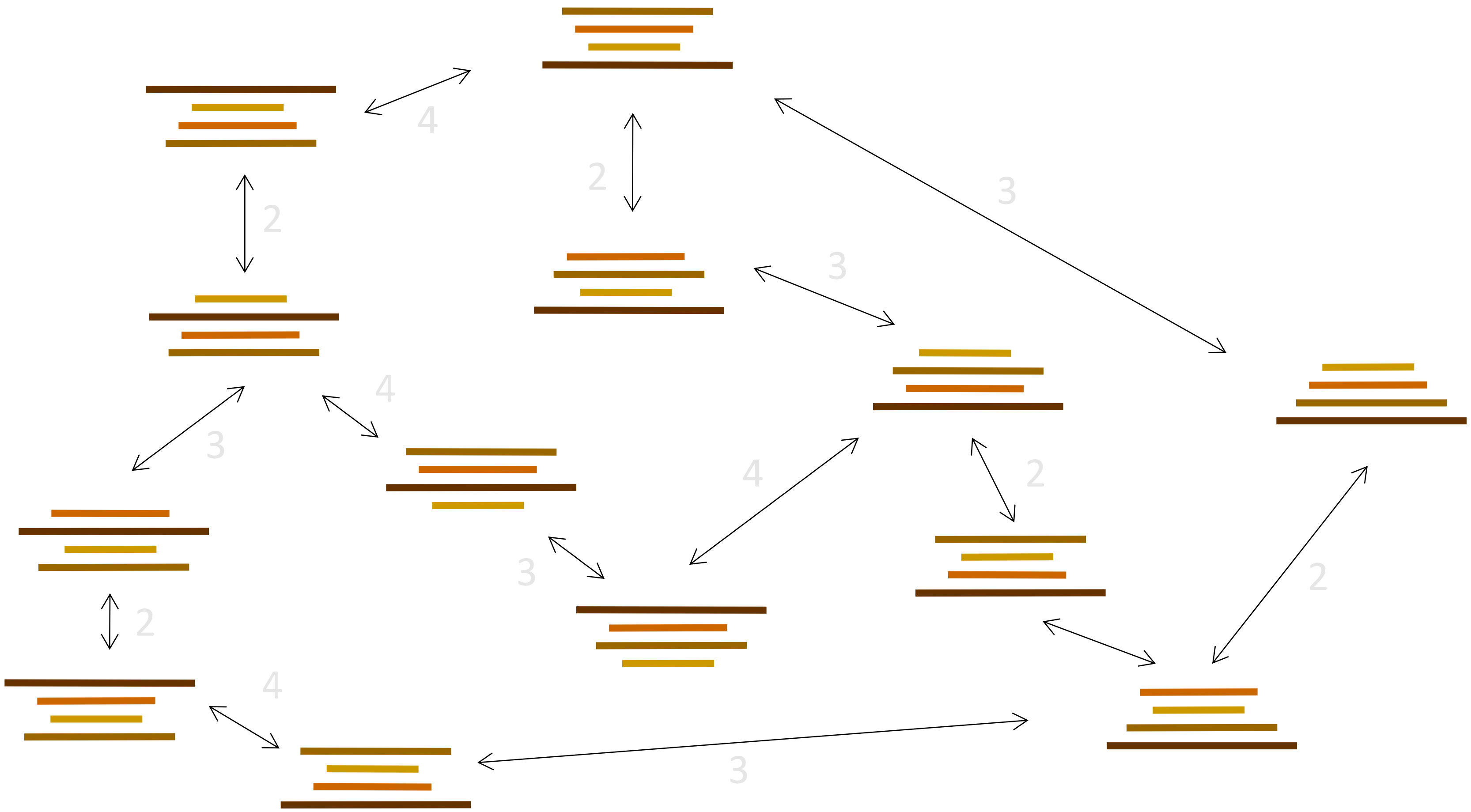


Figure from Berkley AI

General Tree Search

```
function Tree-Search( problem, strategy) returns a solution, or failure
    initialize the search tree using the initial state of problem
    loop do
        if there are no candidates for expansion then return failure
        choose a leaf node for expansion according to strategy
        if the node contains a goal state then return the corresponding solution
        else expand the node and add the resulting nodes to the search tree
    end
```

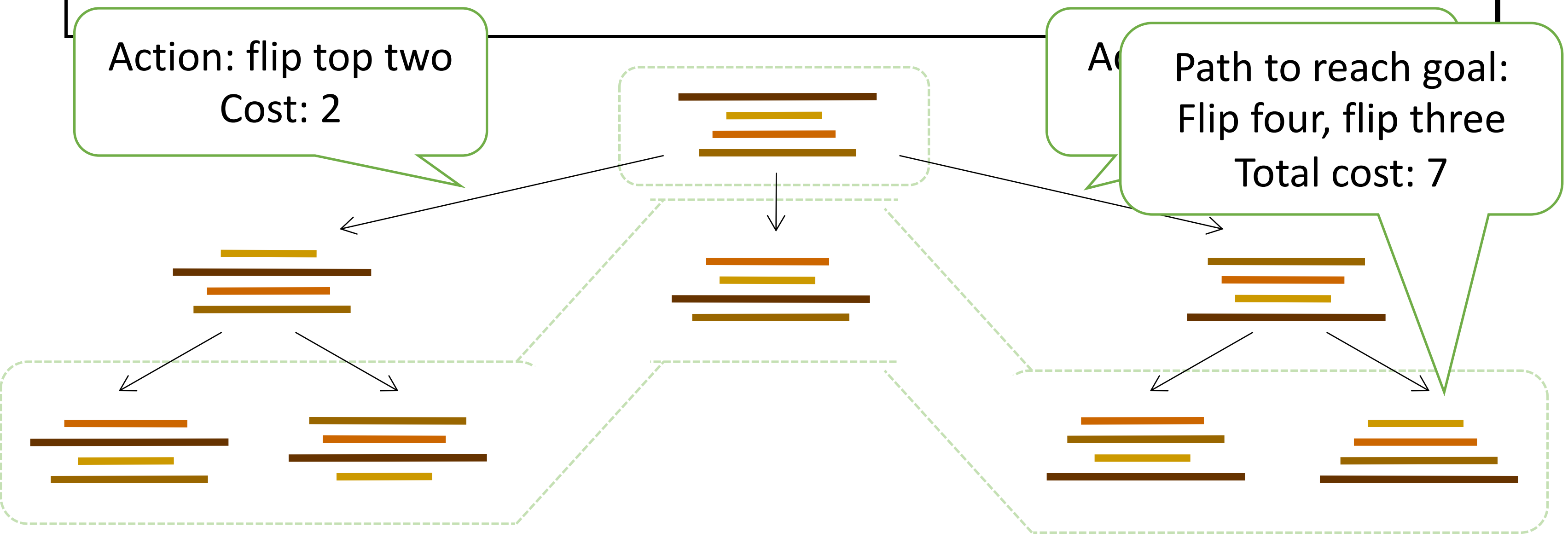


Figure from Berkley AI

Search – Only Differences are in the Queue

- All these search algorithms are the same except for fringe strategies
  - Conceptually, all fringes are priority queues (i.e. collections of nodes with attached priorities)
  - Practically, for DFS and BFS, you can avoid the log(n) overhead from an actual priority queue, by using stacks and queues
  - Can even code one implementation that takes a variable queuing object

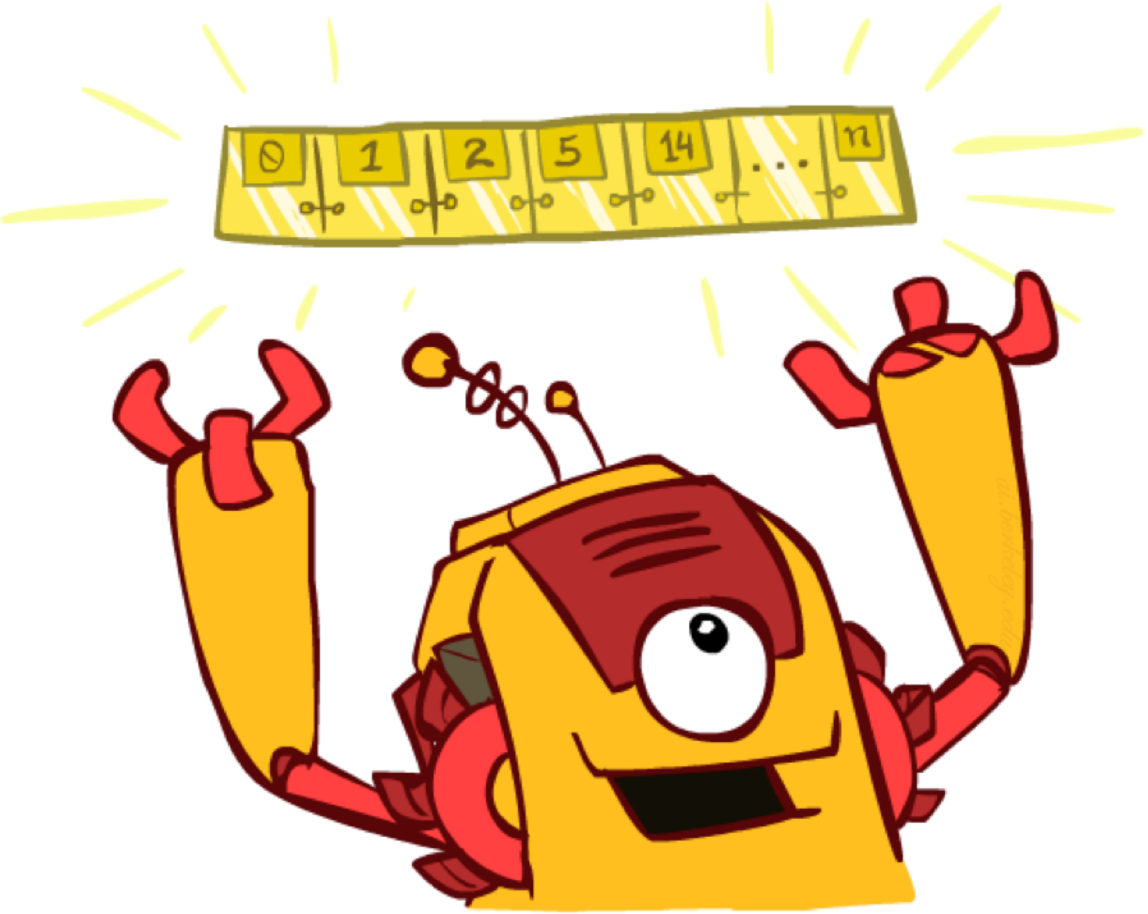


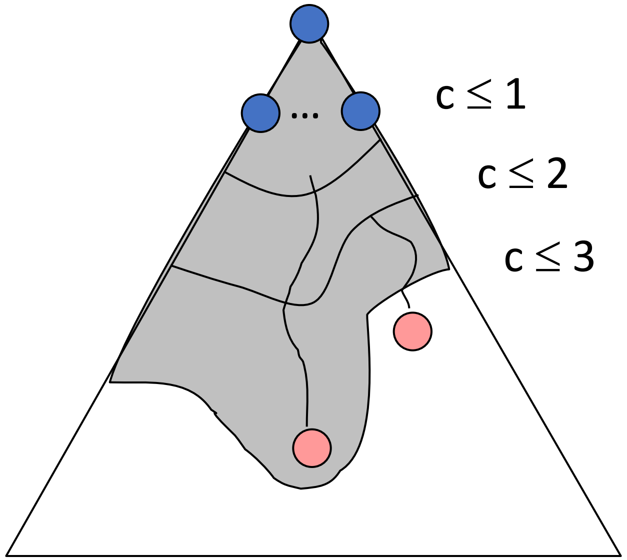
Figure from Berkley AI

Uniform Cost Search

- Strategy: expand lowest path cost

- The good: UCS is complete and optimal!

- The bad:
  - Explores options in every “direction”
  - No information about goal location



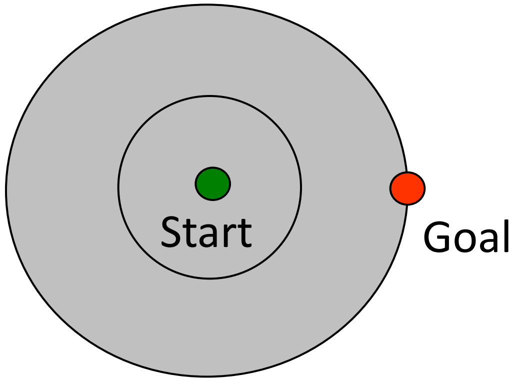


Figure from Berkley AI

Informed Search

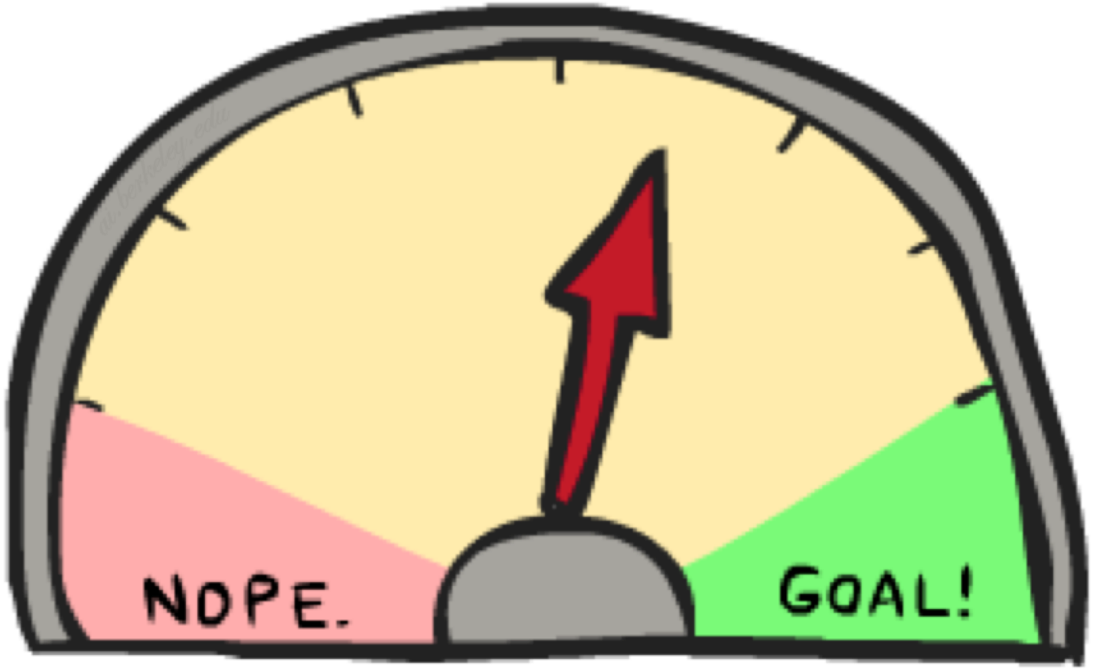
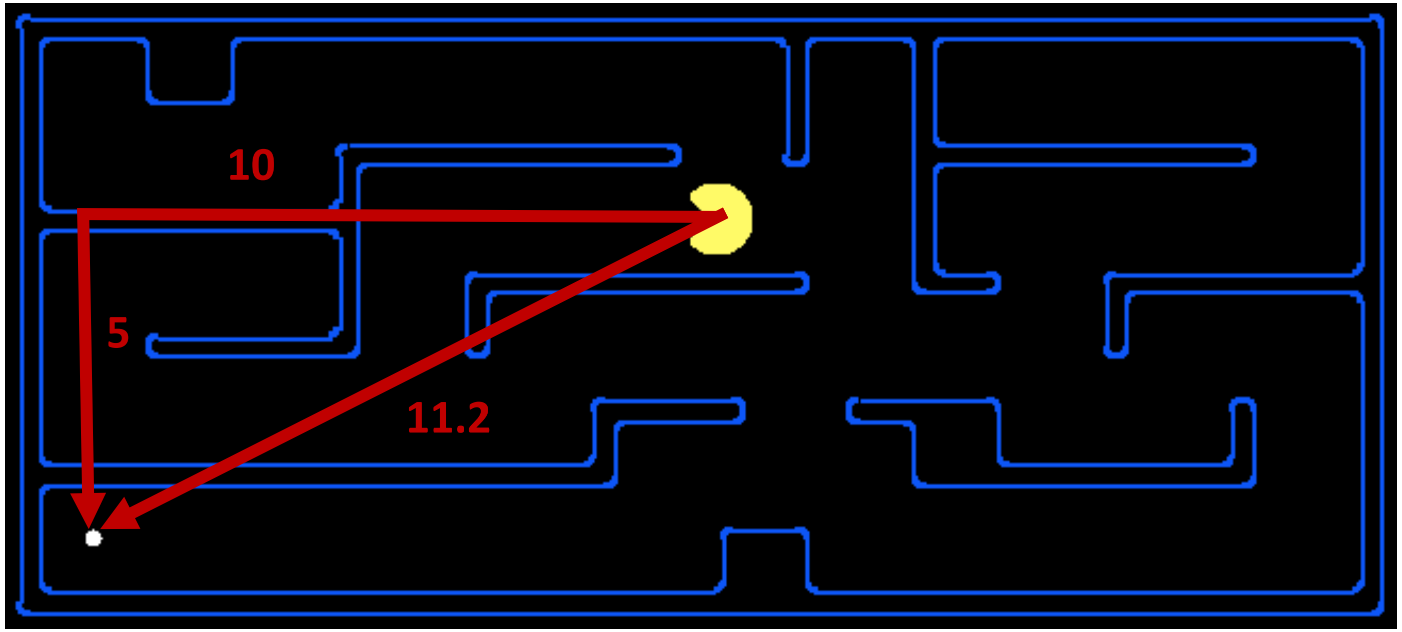


Figure from Berkley AI

Search Heuristic

- A heuristic is:
  - A function that *estimates* how close a state is to a goal
  - Designed for a particular search problem
  - Examples: Manhattan distance, Euclidean distance for pathing



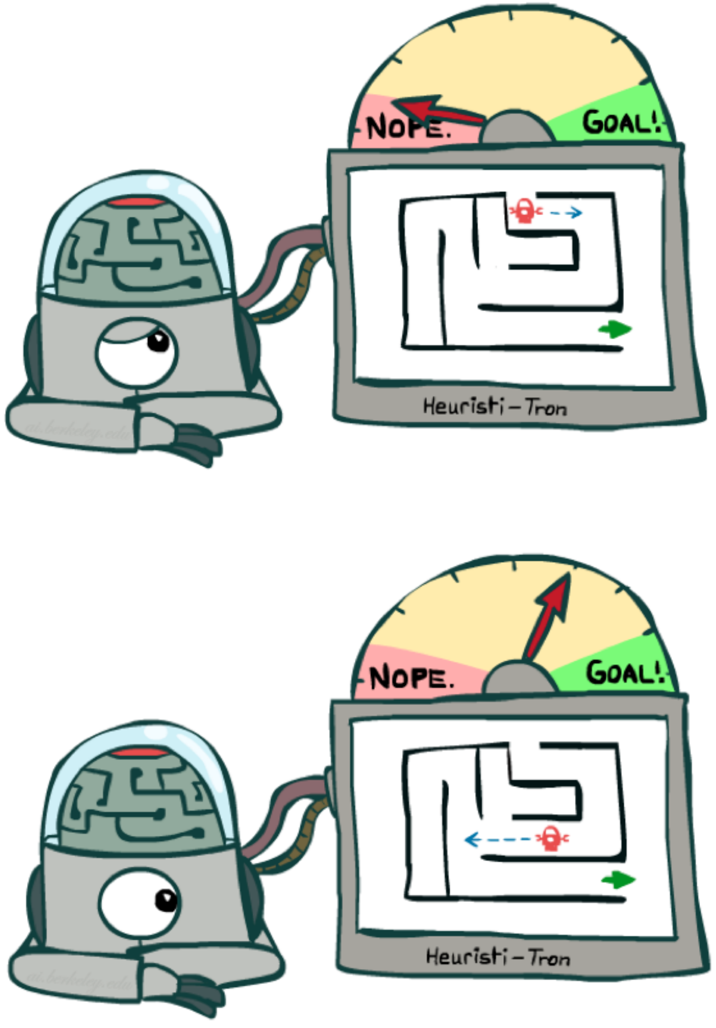
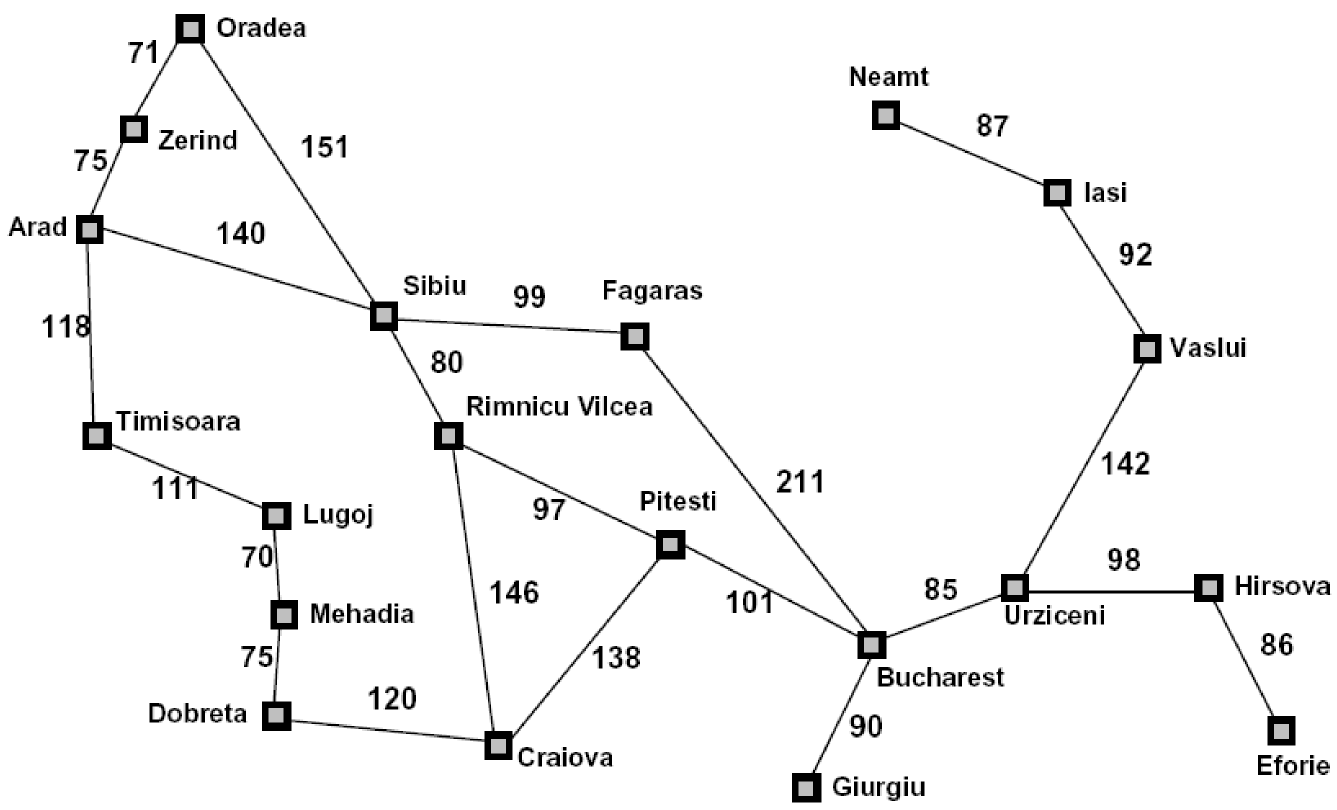


Figure from Berkley AI

Example: Heuristic Function



Straight−line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

h(x)

Figure from Berkley AI

Example: Heuristic Function

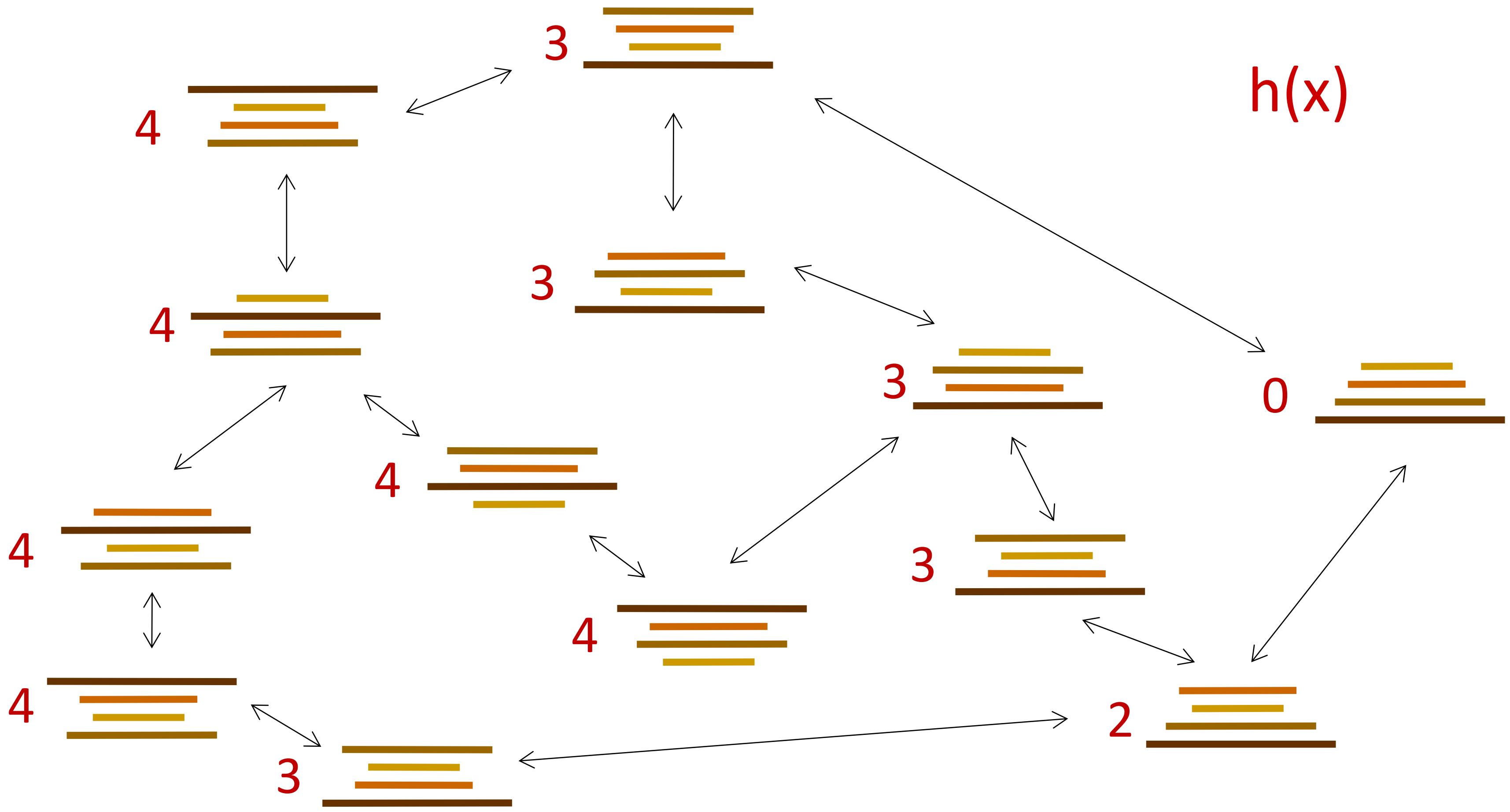
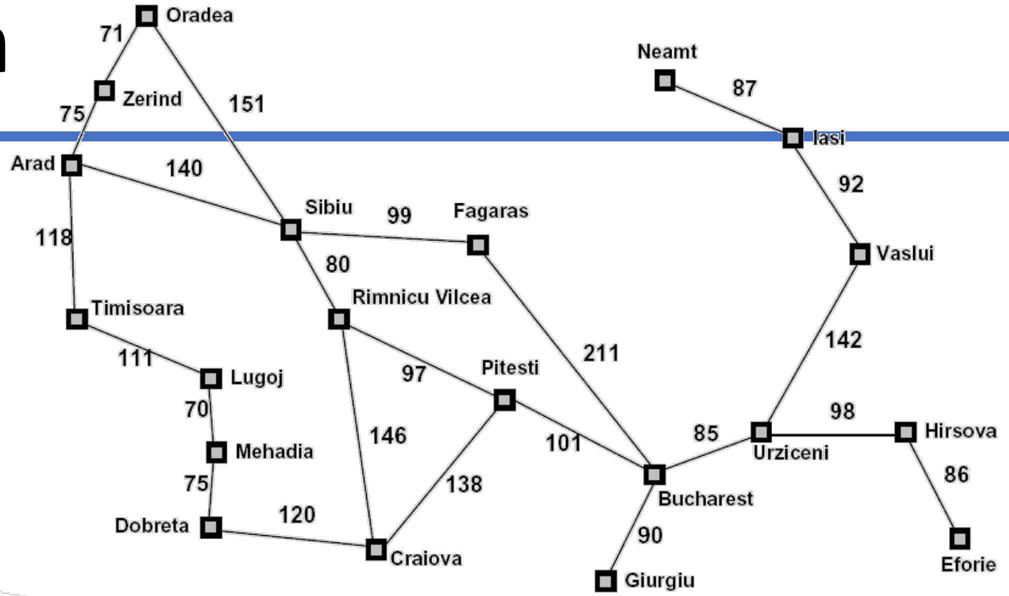


Figure from Berkley AI

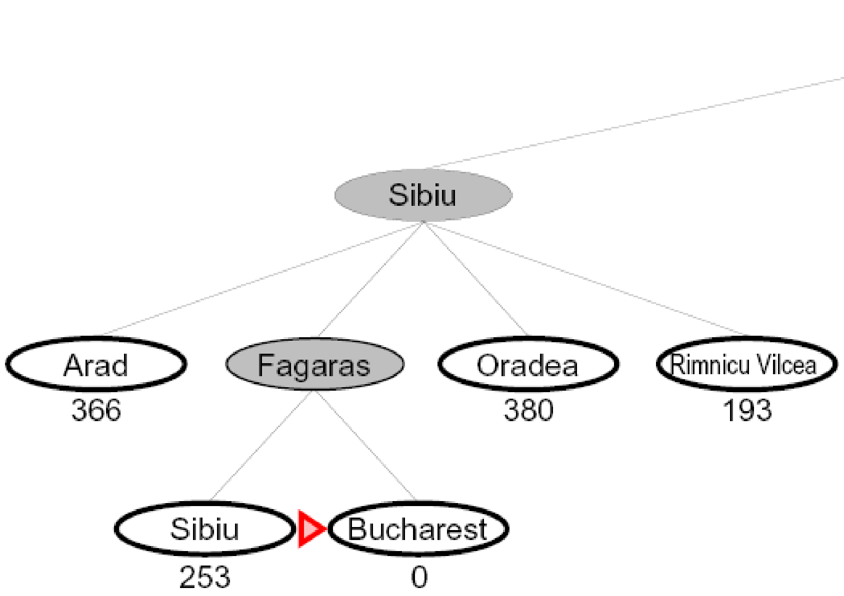
Greedy Search

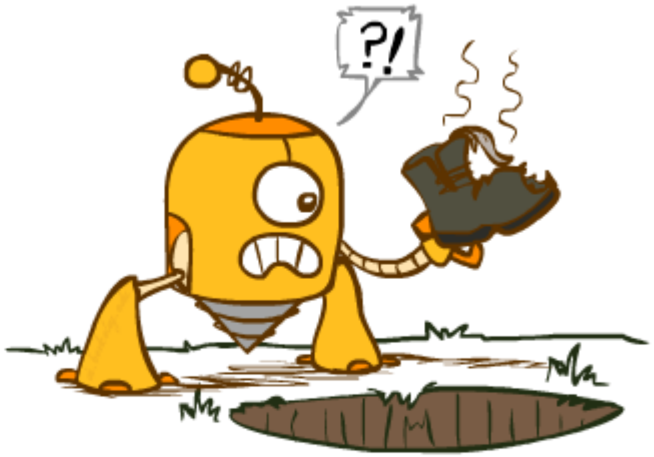
Figure from Berkley AI

Greedy Search



- Expand the node that seems closest…



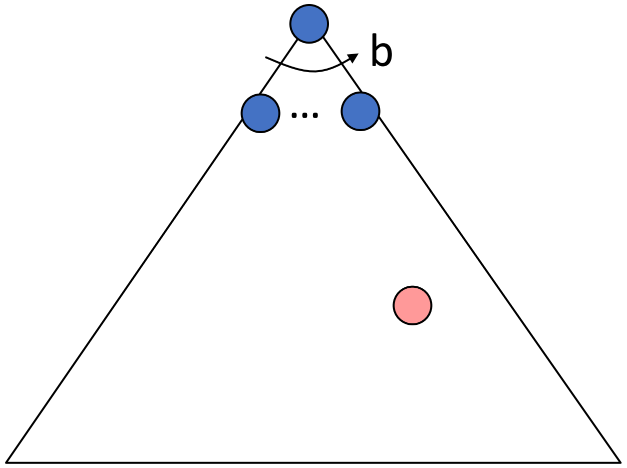


- What can go wrong?

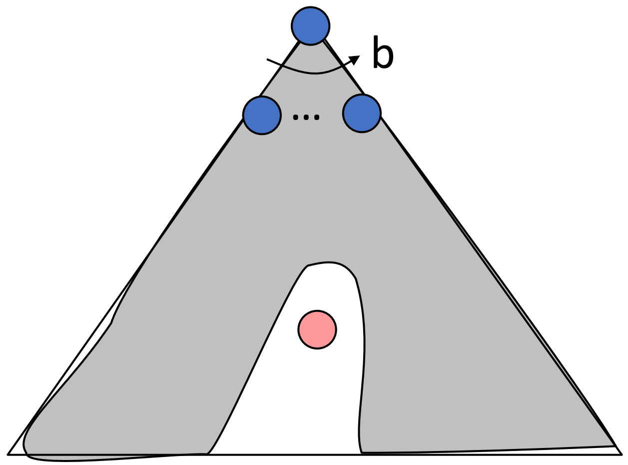
Figure from Berkley AI

Greedy Search

- Strategy: expand a node that you think is closest to a goal state
  - Heuristic: estimate of distance to nearest goal for each state



- A common case:
  - Best-first takes you straight to the (wrong) goal



- Worst-case: like a badly-guided DFS

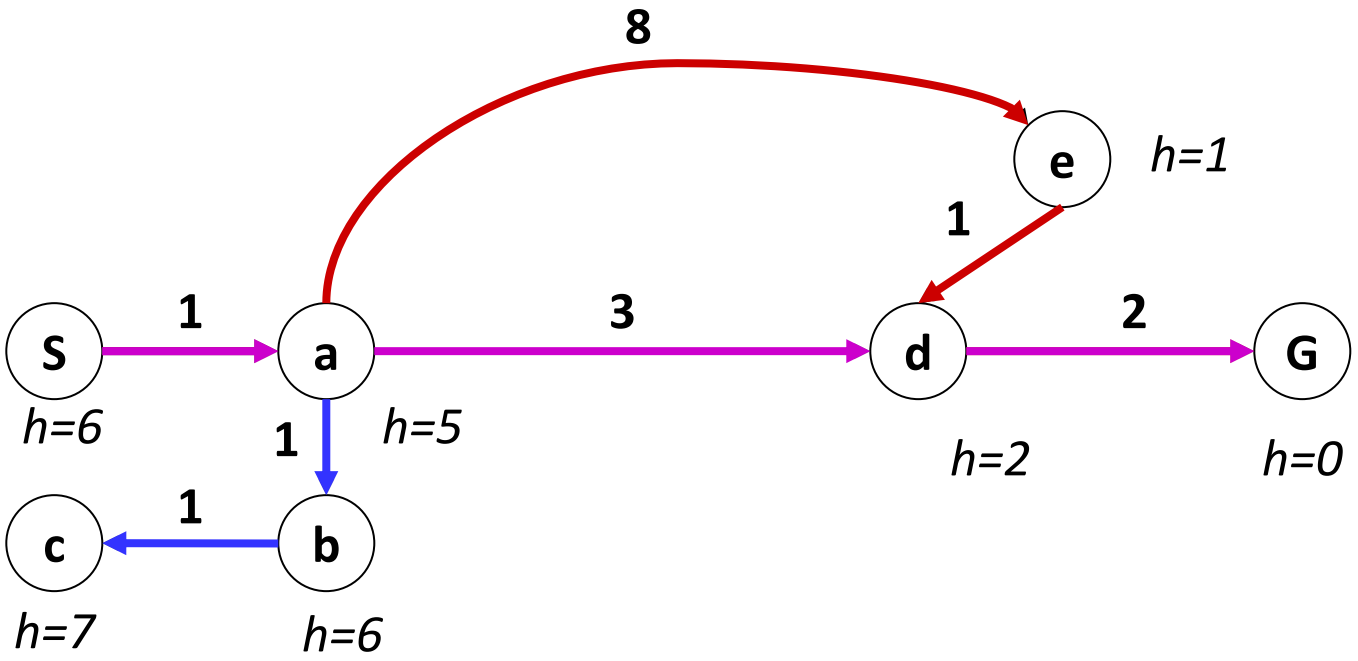
Figure from Berkley AI

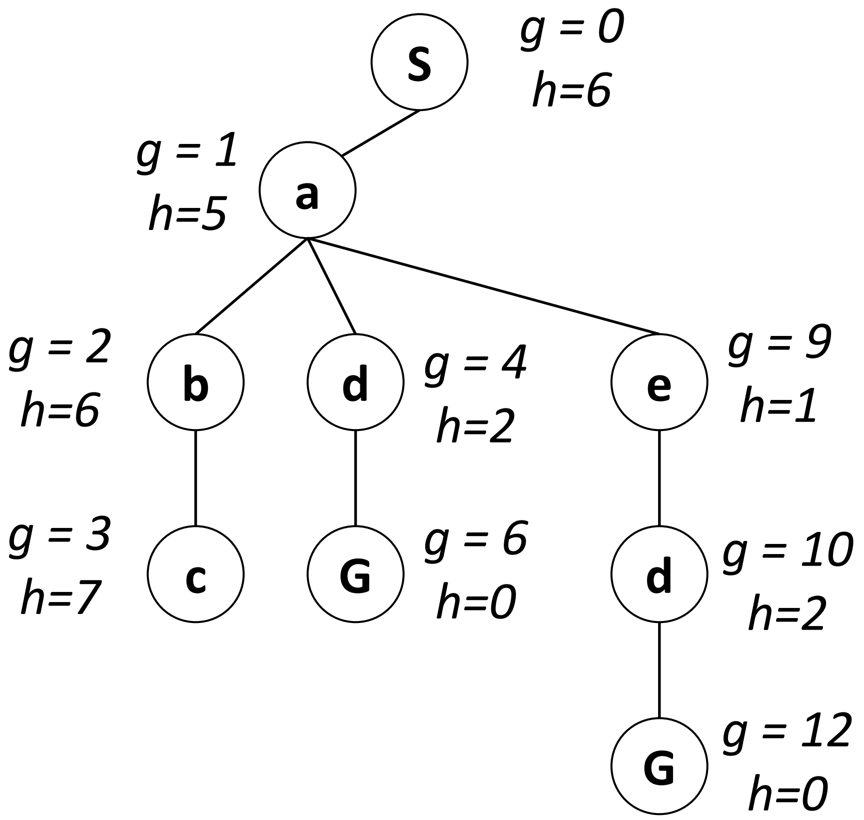
A* Search

Figure from Berkley AI

Combining UCS and Greedy

- Uniform-cost orders by path cost, or *backward cost* g(n)
- Greedy orders by goal proximity, or *forward cost* h(n)

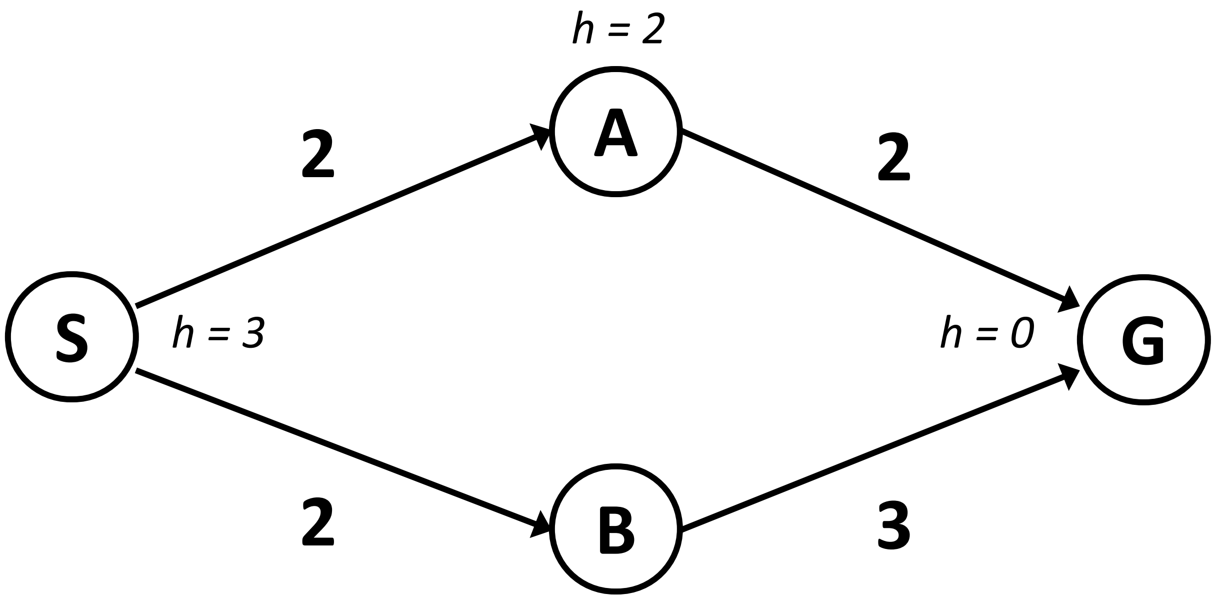




- A* Search orders by the sum: f(n) = g(n) + h(n)

When should A* terminate?

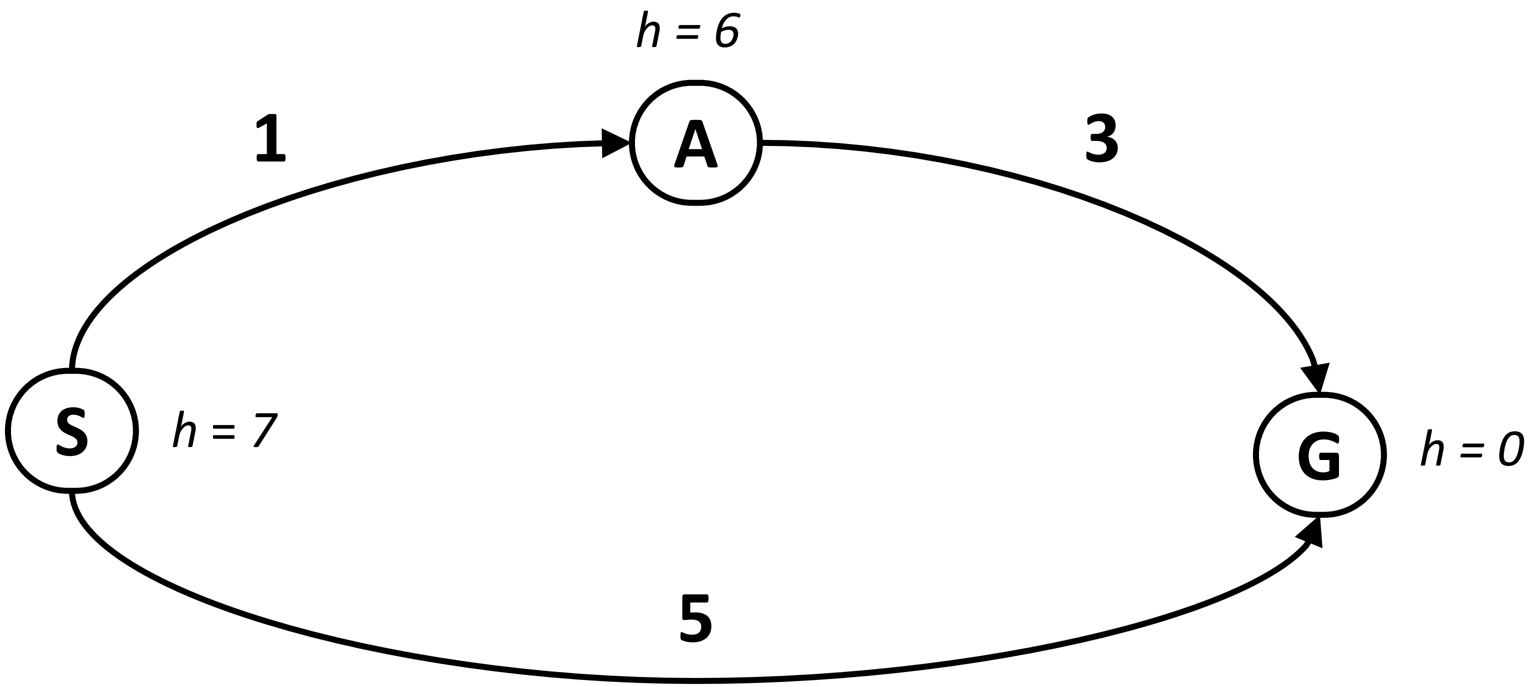
- Should we stop when we enqueue a goal?



- No: only stop when we dequeue a goal

Figure from Berkley AI

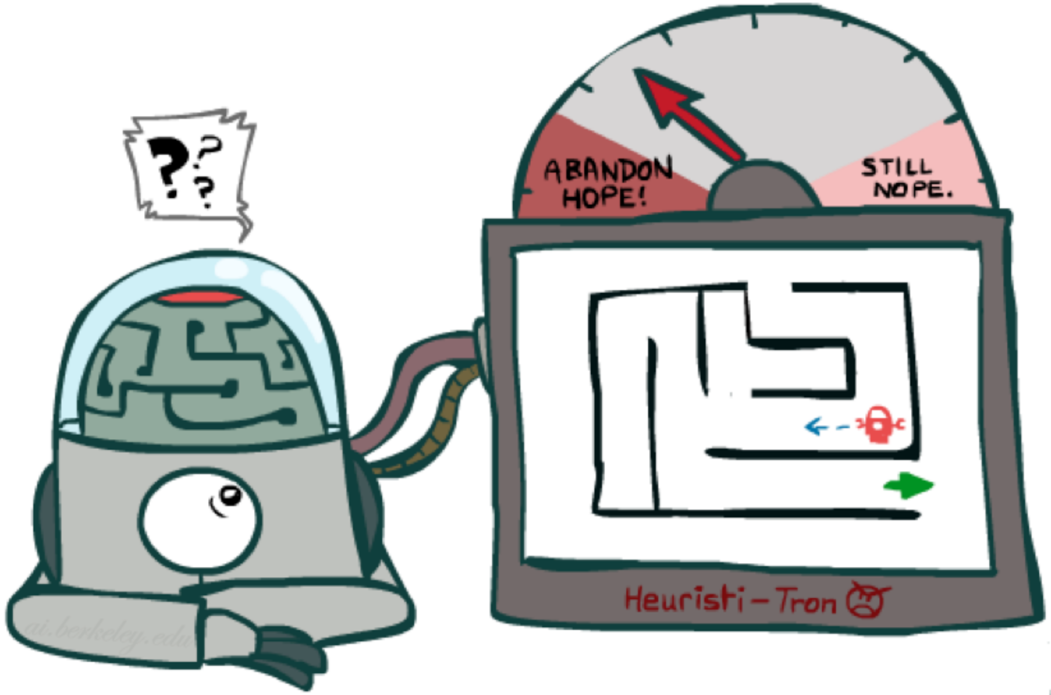
Is A* Optimal?

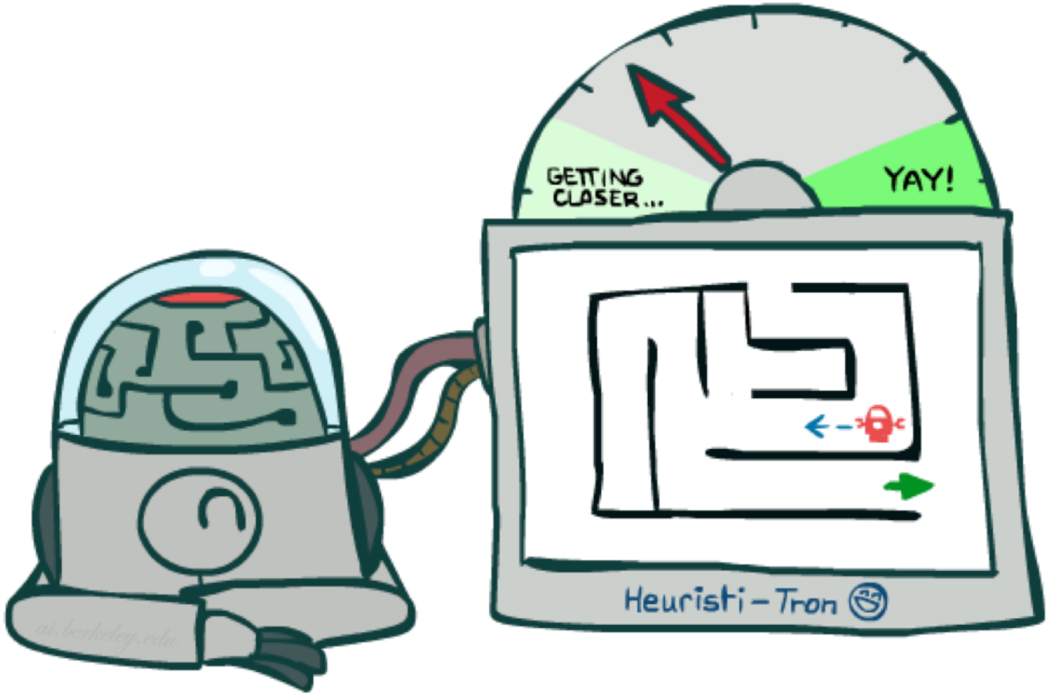


- What went wrong?
- Actual bad goal cost < estimated good goal cost
- We need estimates to be less than actual costs!

Figure from Berkley AI

Idea: Admissibility





Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the fringe

Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

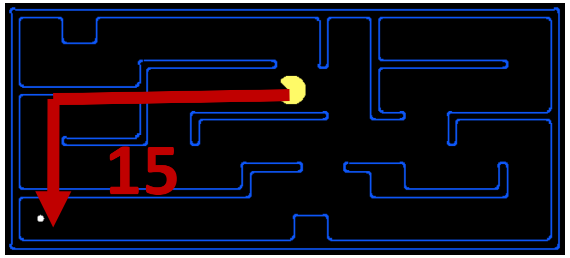
Admissible Heuristics

- A heuristic h is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

- Examples:



- Coming up with admissible heuristics is most of what's involved in using A* in practice.

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Figure from Berkley AI

Optimality of A* Tree Search

Assume:

- A is an optimal goal node
- B is a suboptimal goal node
- h is admissible

Claim:

- A will exit the fringe before B

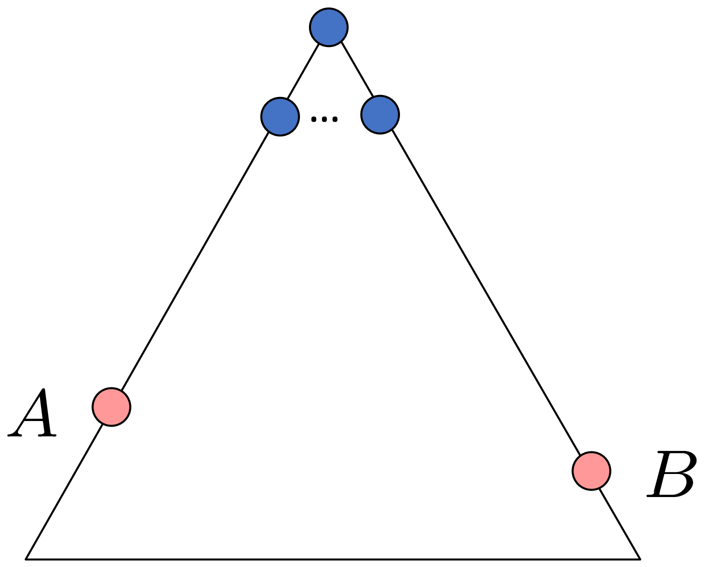
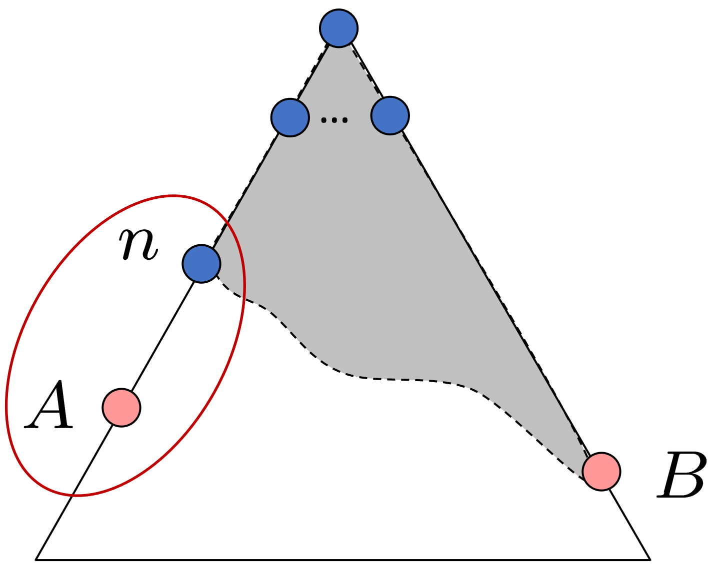


Figure from Berkley AI

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
    1. f(n) is less or equal to f(A)



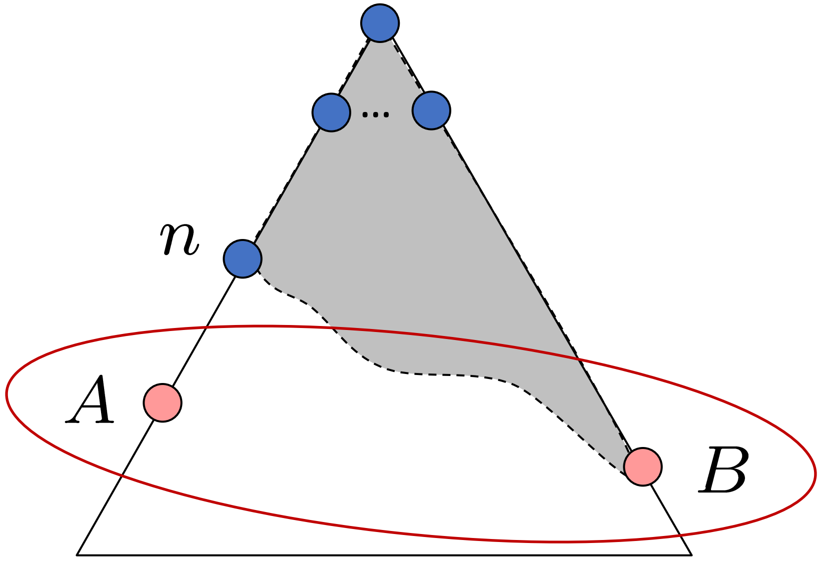
$f(n) = g(n) + h(n)$	Definition of f-cost
$f(n) \leq g(A)$	Admissibility of h
$g(A) = f(A)$	h = 0 at a goal

Figure from Berkley AI

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
  1. f(n) is less or equal to f(A)
  2. f(A) is less than f(B)



$$g(A) < g(B)$$ B is suboptimal

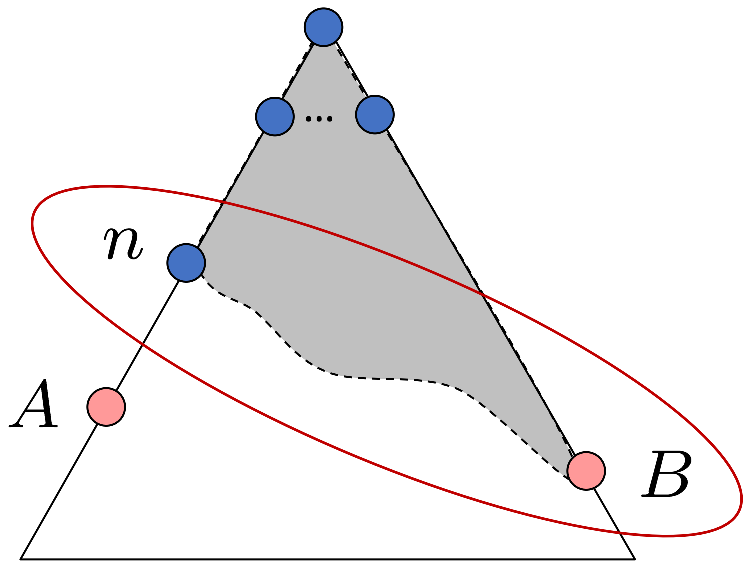
$$f(A) < f(B)$$ h = 0 at a goal

Figure from Berkley AI

Optimality of A* Tree Search: Blocking

Proof:

- Imagine B is on the fringe
- Some ancestor *n* of A is on the fringe, too (maybe A!)
- Claim: *n* will be expanded before B
    1. f(n) is less or equal to f(A)
    2. f(A) is less than f(B)
    3. *n* expands before B
- All ancestors of A expand before B
- A expands before B
- A* search is optimal

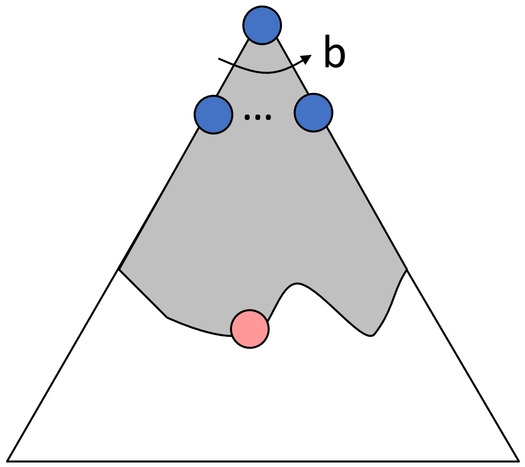


$$f(n) \leq f(A) < f(B)$$

Figure from Berkley AI

Properties of A*

Uniform-Cost



A*

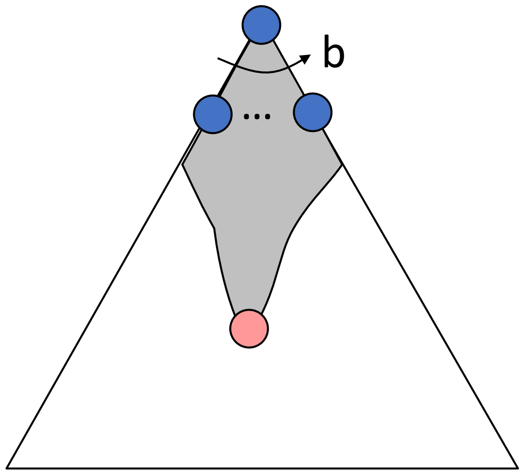
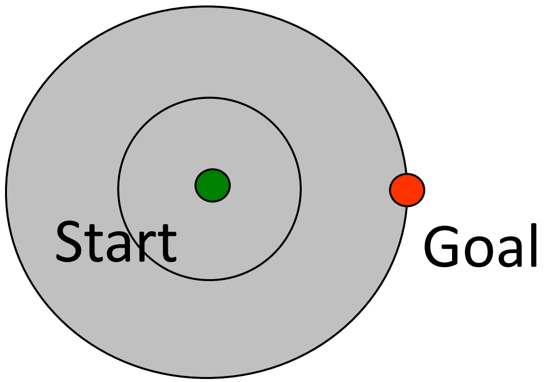


Figure from Berkley AI

Uniform-Cost versus A* Contours

- Uniform-cost expands equally in all "directions"



- A* expands mainly toward the goal, but does hedge its bets to ensure optimality

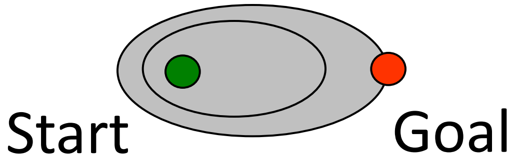
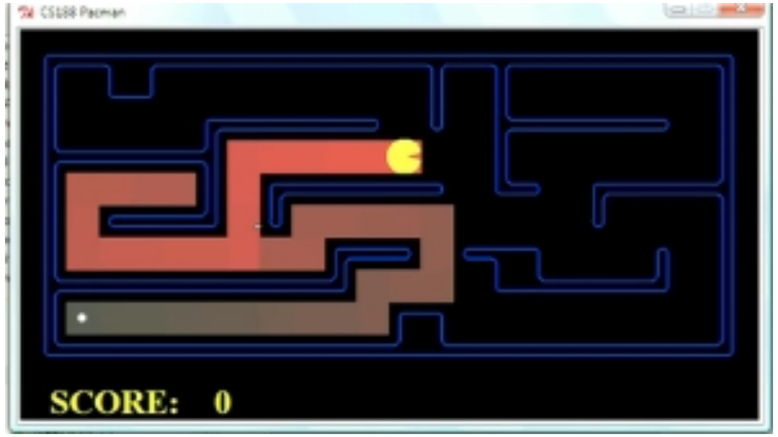


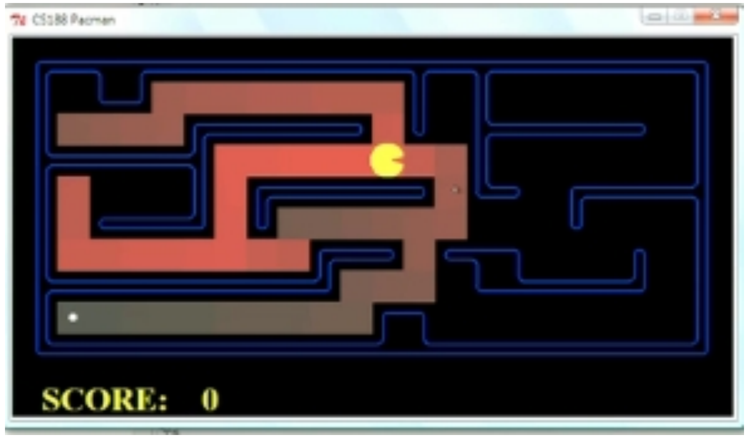
Figure from Berkley AI

Pacman Comparison



Greedy

Uniform Cost



A*

Figure from Berkley AI

A* Applications

- Video games
- Pathing / routing problems
- Resource planning problems
- Robot motion planning
- Language analysis
- Machine translation
- Speech recognition
- …

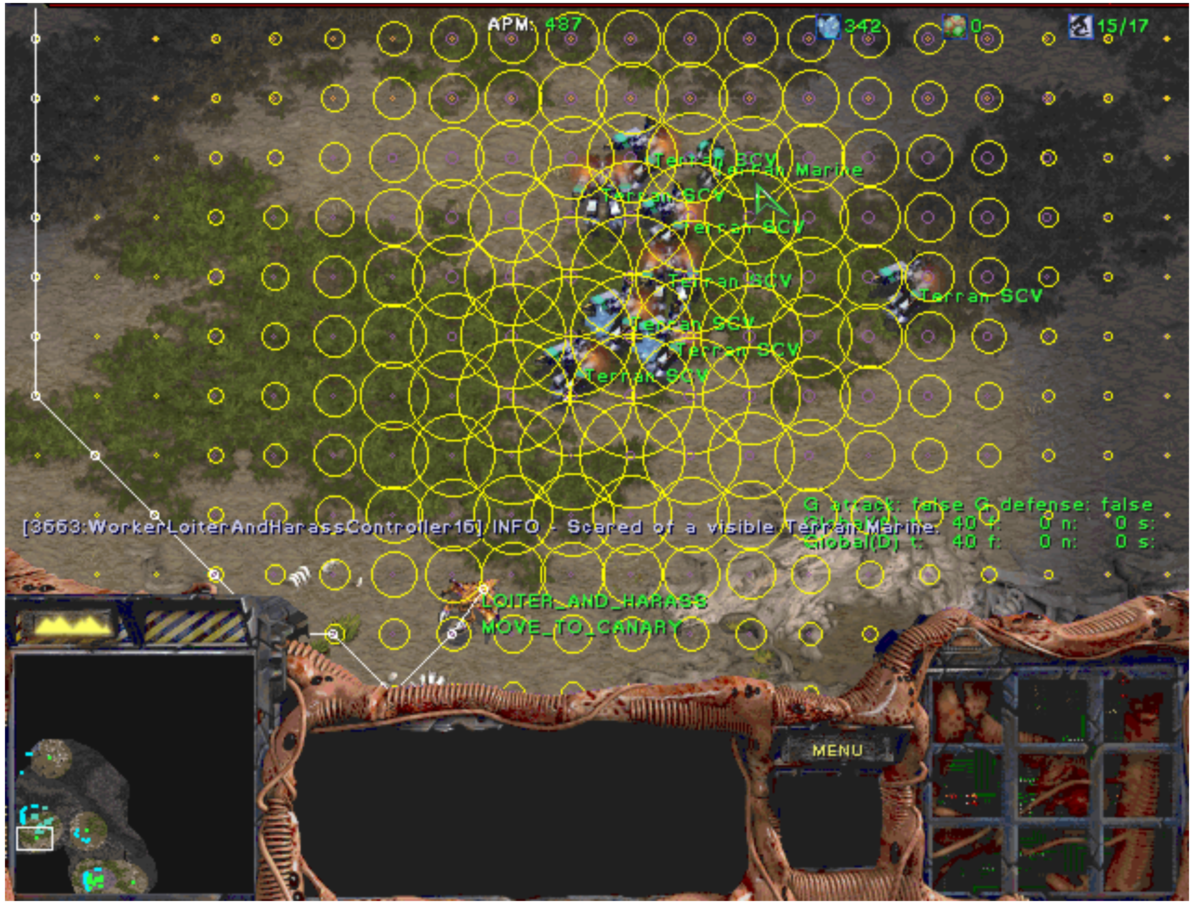
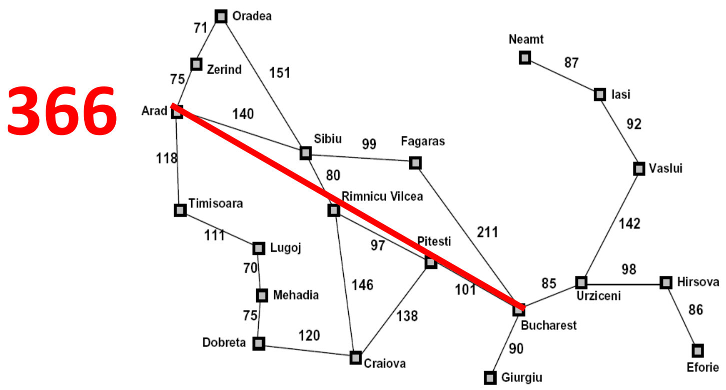


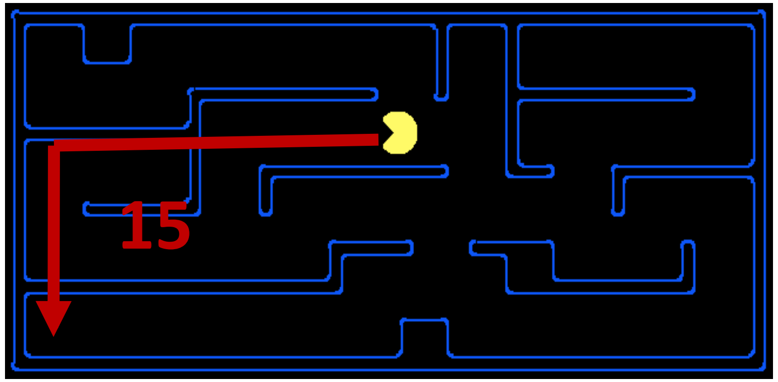
Figure from Berkley AI

A*: Creating Admissible Heuristics

- Most of the work in solving hard search problems optimally is in coming up with admissible heuristics

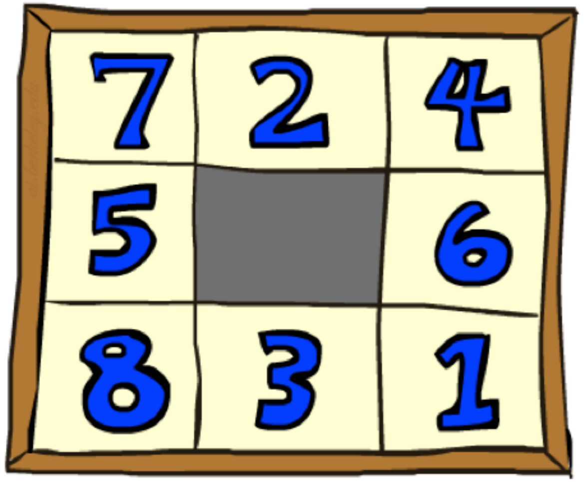
- Often, admissible heuristics are solutions to *relaxed problems,* where new actions are available



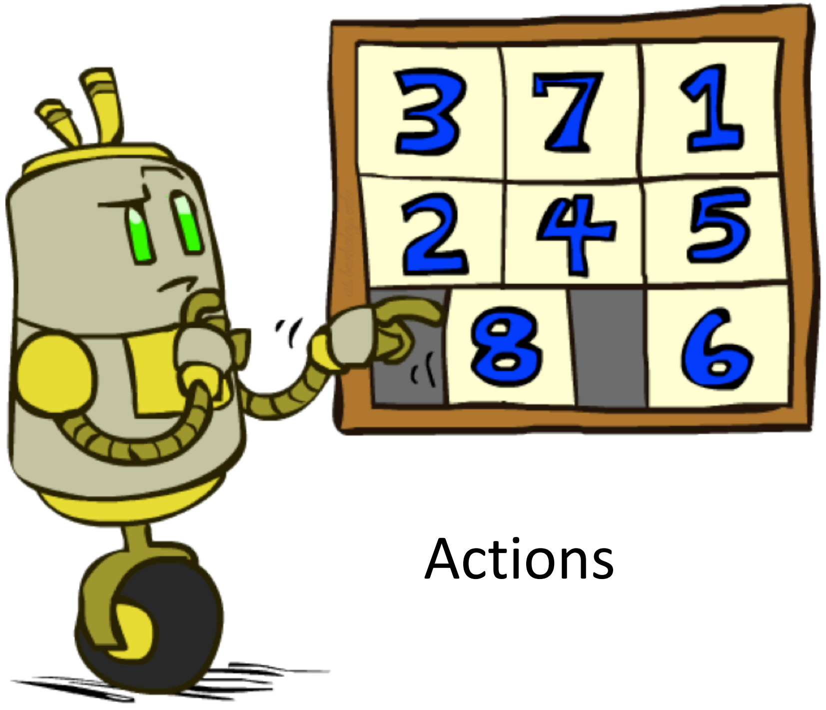


- Inadmissible heuristics are often useful too

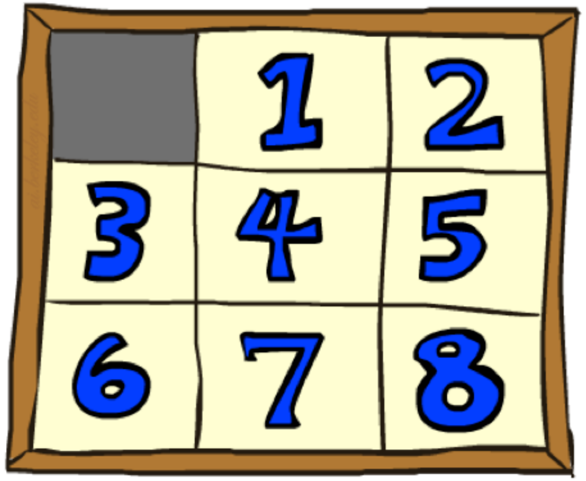
Example: 8 Puzzle



Start State



Actions



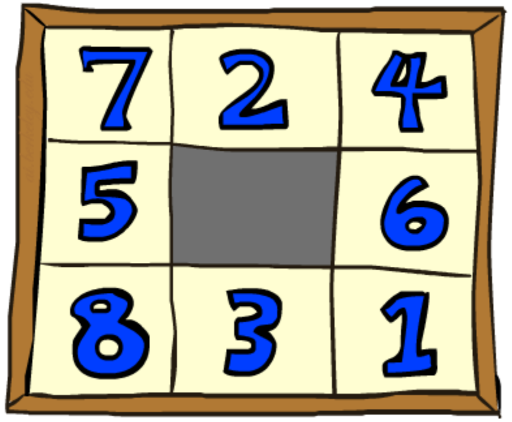
Goal State

- What are the states?
- How many states?
- What are the actions?
- How many successors from the start state?
- What should the costs be?

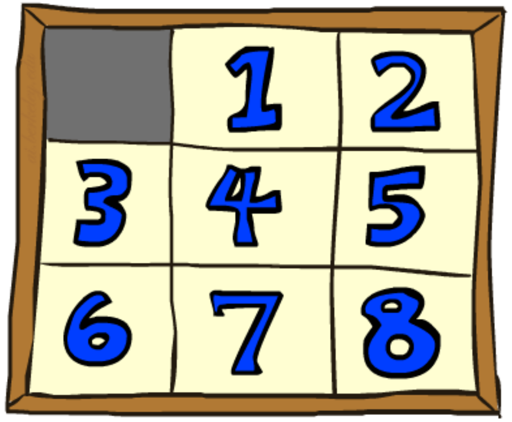
Figure from Berkley AI

8 Puzzle: Heuristic I

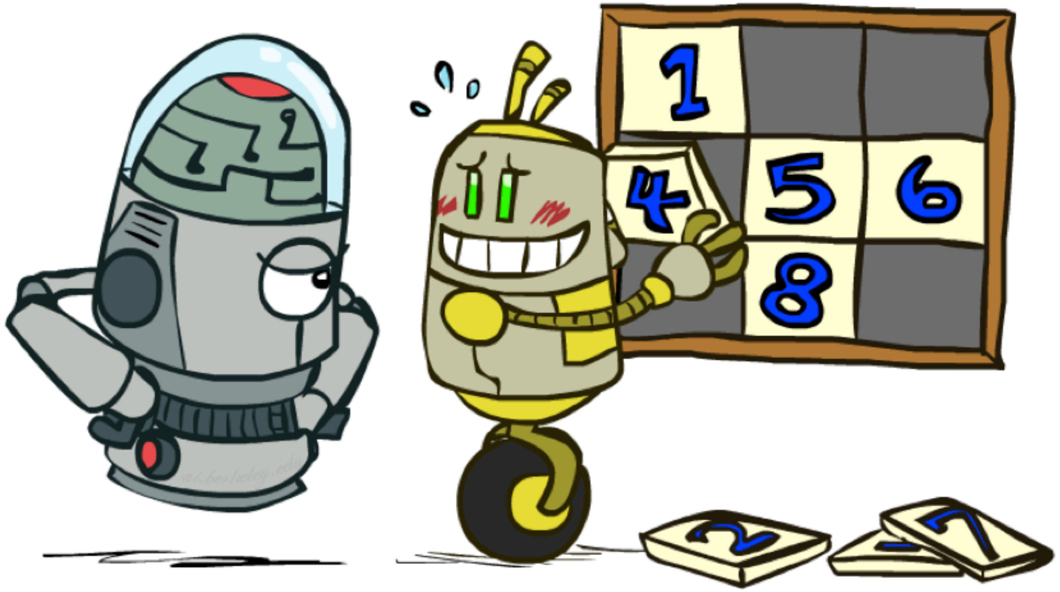
- Heuristic: Number of tiles misplaced
- Why is it admissible?
- h(start) = 8
- This is a *relaxed-problem* heuristic



Start State



Goal State

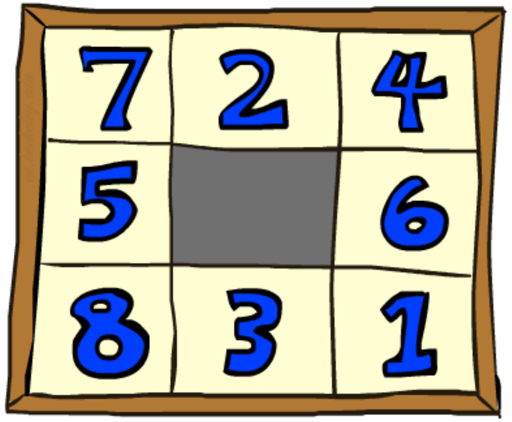


	Average nodes expanded when the optimal path has...		
	…4 steps	…8 steps	…12 steps
UCS	112	6,300	3.6×10^6
TILES	13	39	227

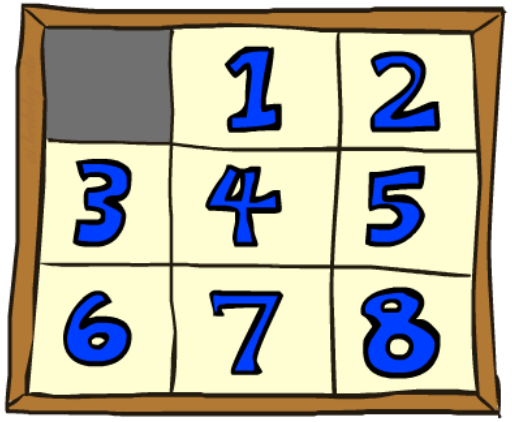
Figure from Berkley AI; Statistics from Andrew Moore

8 Puzzle: Heuristic II

- What if we had an easier 8-puzzle where any tile could slide any direction at any time, ignoring other tiles?
- Total *Manhattan* distance
- Why is it admissible?
- h(start) = 3 + 1 + 2 + … = 18



Start State



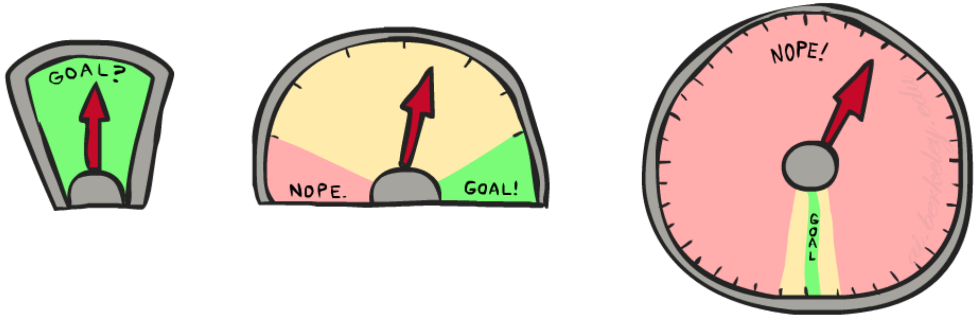
Goal State

	Average nodes expanded when the optimal path has…		
	…4 steps	…8 steps	…12 steps
TILES	13	39	227
MANHATTAN	12	25	73

Figure from Berkley AI

8 Puzzle: Heuristic III

- How about using the *actual cost* as a heuristic?
  - Would it be admissible?
  - Would we save on nodes expanded?
  - What's wrong with it?



- With A*: a trade-off between quality of estimate and work per node
  - As heuristics get closer to the true cost, you will expand fewer nodes but usually do more work per node to compute the heuristic itself

Figure from Berkley AI

Trivial Heuristics and Dominance

- Dominance: $h_a \geq h_c$ if

$$\forall n : h_a(n) \geq h_c(n)$$

- Heuristics form a semi-lattice:
  - Max of admissible heuristics is admissible

$$h(n) = max(h_a(n), h_b(n))$$

- Trivial heuristics
  - Bottom of lattice is the zero heuristic (what does this give us?)
  - Top of lattice is the exact heuristic

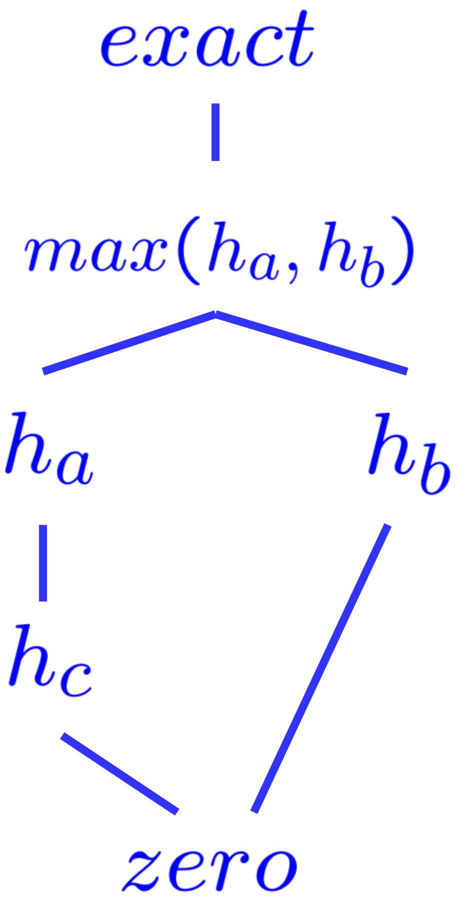


Figure from Berkley AI

Graph Search

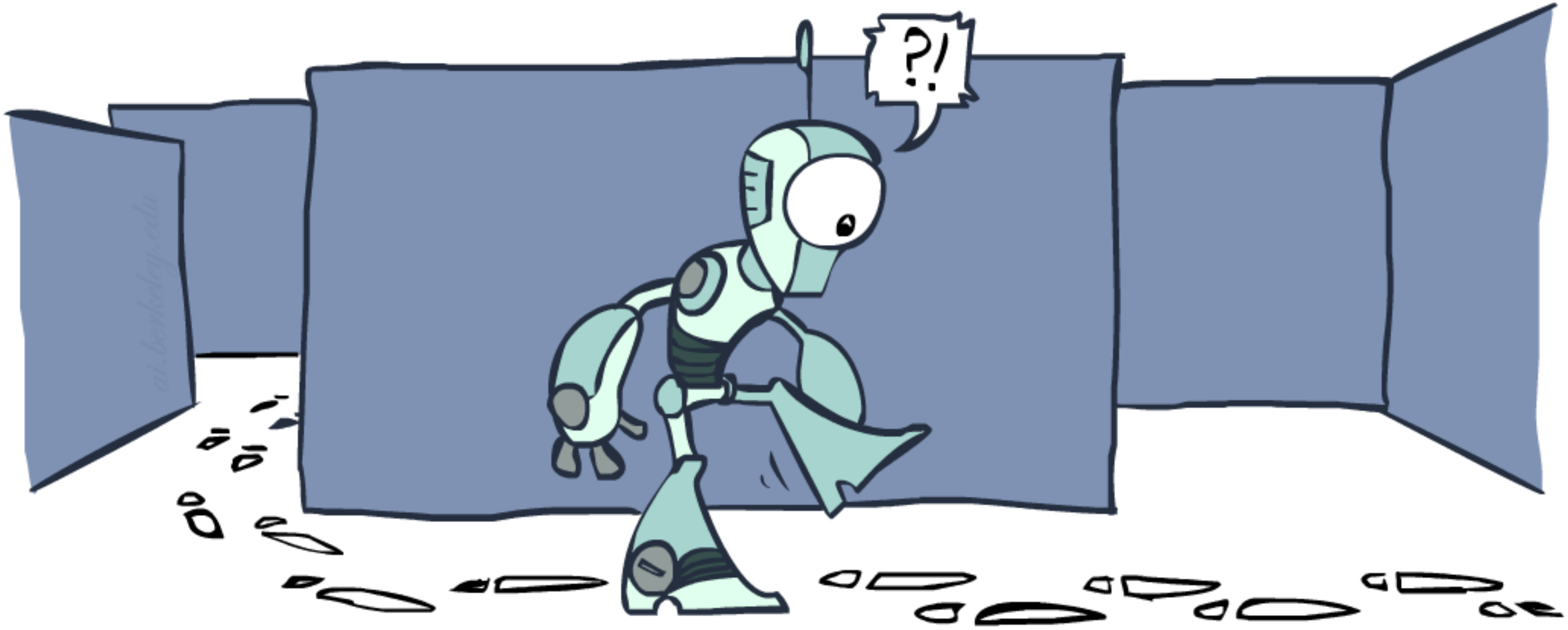
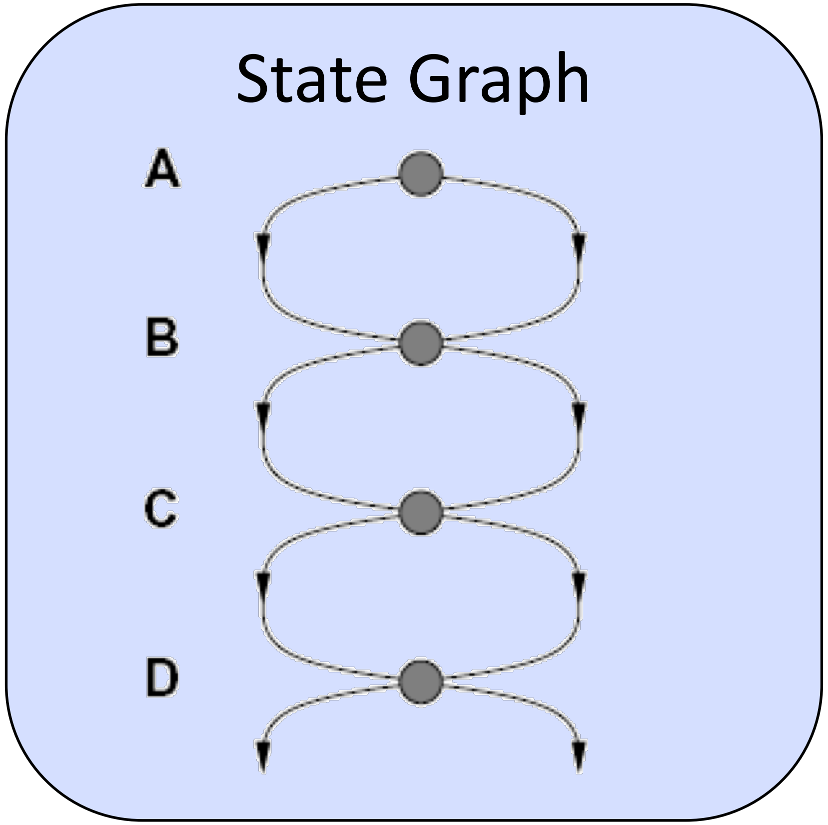


Figure from Berkley AI

Tree Search: Extra Work!

- Failure to detect repeated states can cause exponentially more work.



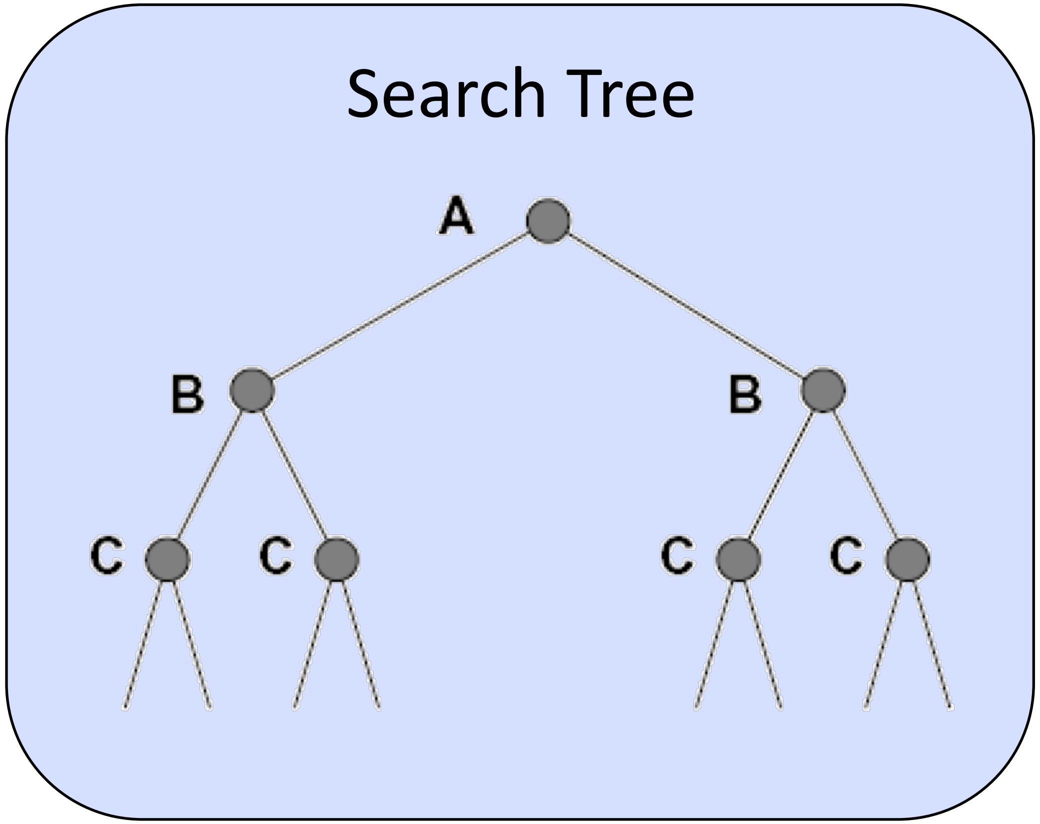


Figure from Berkley AI

Graph Search

- In BFS, for example, we shouldn't bother expanding the circled nodes (why?)

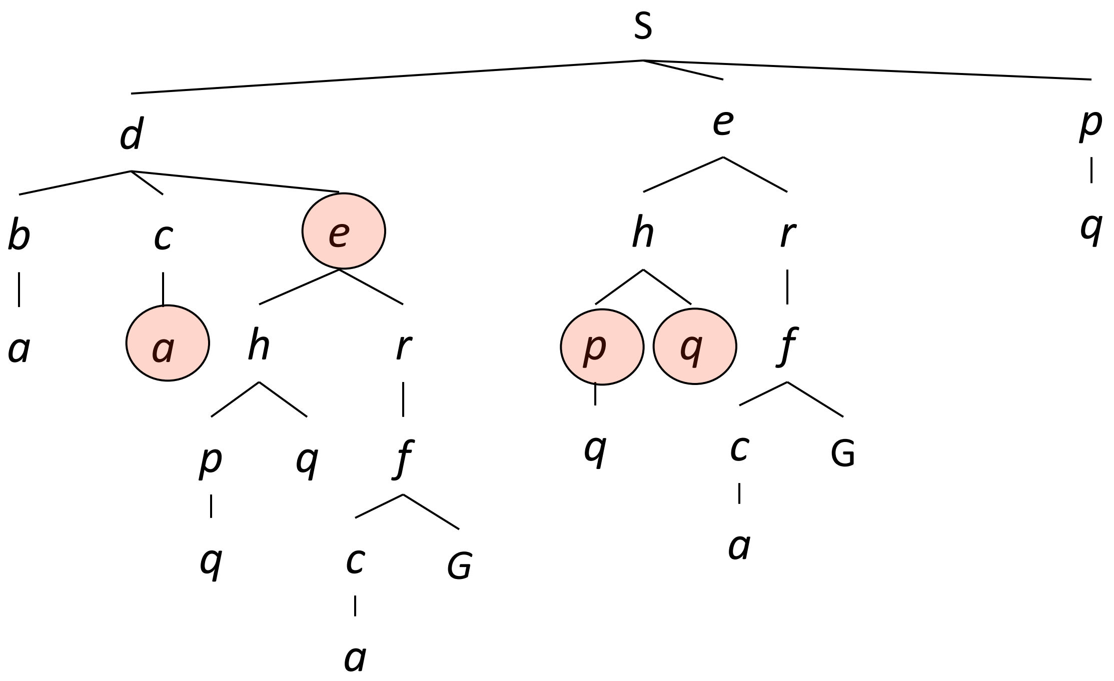


Figure from Berkley AI

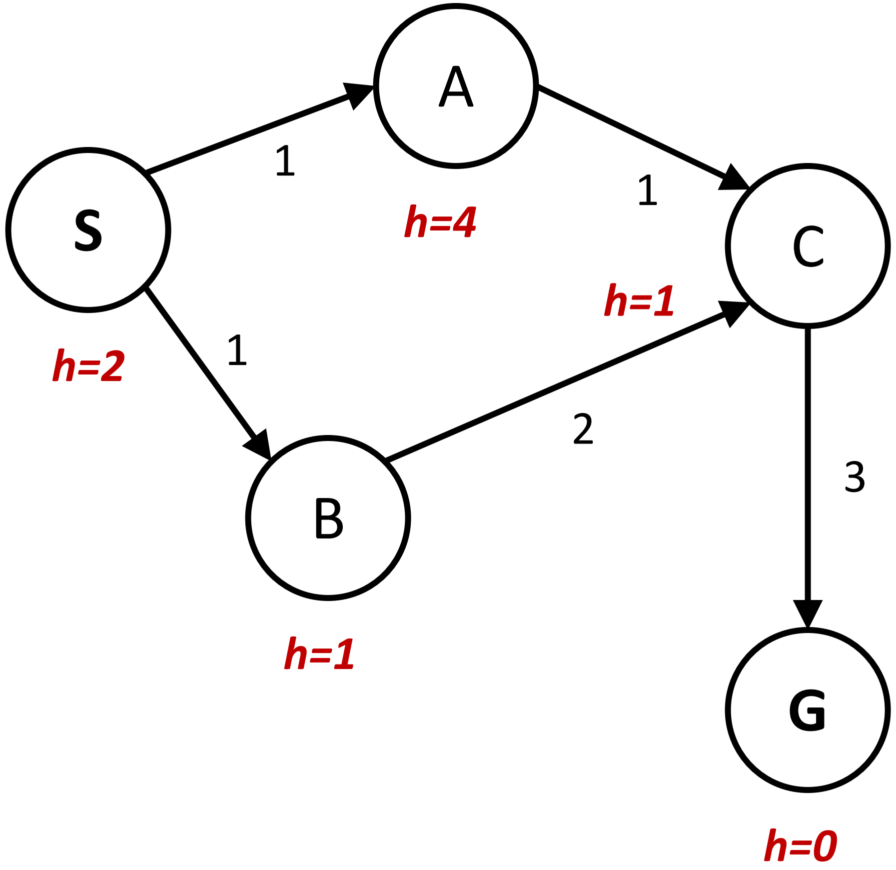
Graph Search Motivation

Idea: never expand a state twice

- How to implement:
  - Tree search + set of expanded states ("closed set")
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: store the closed set as a set, not a list
- Can graph search wreck completeness? Why/why not?
- How about optimality?

A* Graph Search Gone Wrong!

State space graph



Search tree

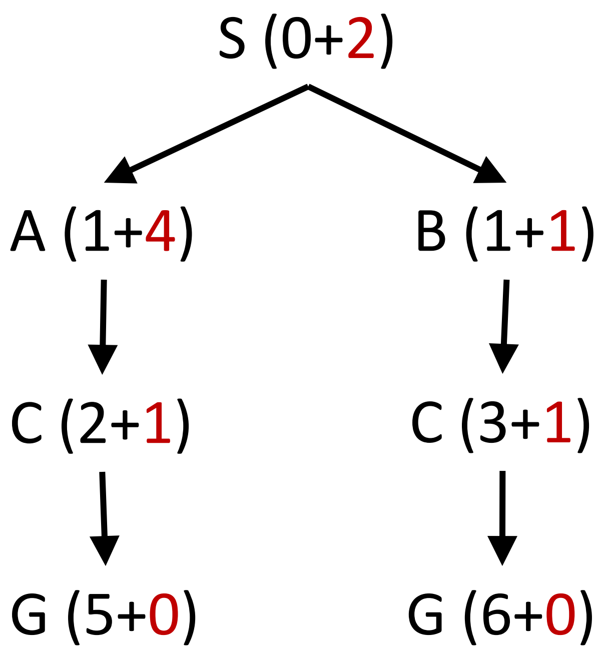
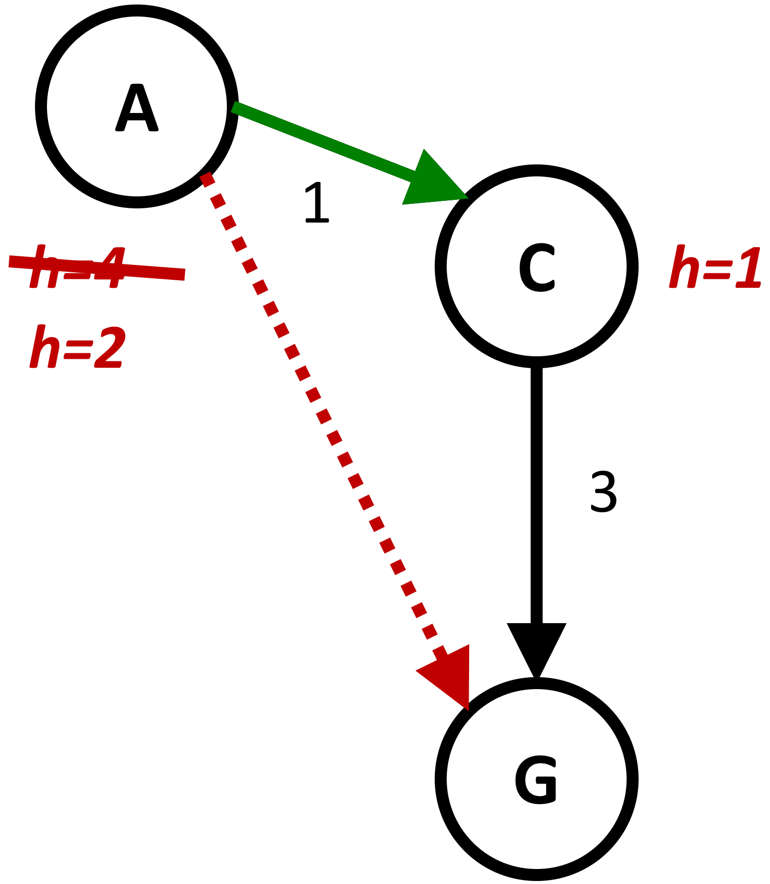


Figure from Berkley AI

Consistency of Heuristics



- Main idea: estimated heuristic costs ≤ actual costs
  - Admissibility: heuristic cost ≤ actual cost to goal

    h(A) ≤ actual cost from A to G
  - Consistency: heuristic "arc" cost ≤ actual cost for each arc

    h(A) – h(C) ≤ cost(A to C)

- Consequences of consistency:
  - The f value along a path never decreases

    h(A) ≤ cost(A to C) + h(C)
  - A* graph search is optimal

Figure from Berkley AI

Optimality of A* Graph Search

- Sketch: consider what A* does with a consistent heuristic:
  - Fact 1: In tree search, A* expands nodes in increasing total f value (f-contours)
  - Fact 2: For every state s, nodes that reach s optimally are expanded before nodes that reach s suboptimally
  - Result: A* graph search is optimal

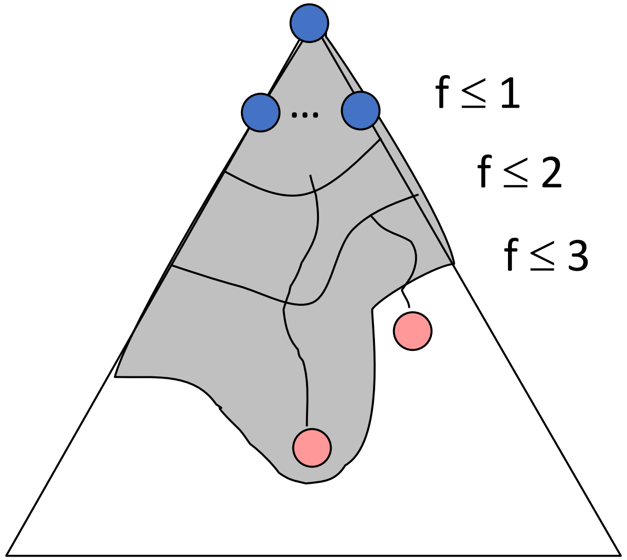


Figure from Berkley AI

Optimality

- Tree search:
  - A* is optimal if heuristic is admissible
  - UCS is a special case (h = 0)
- Graph search:
  - A* optimal if heuristic is consistent
  - UCS optimal (h = 0 is consistent)
- Consistency implies admissibility
- In general, most natural admissible heuristics tend to be consistent, especially if from relaxed problems

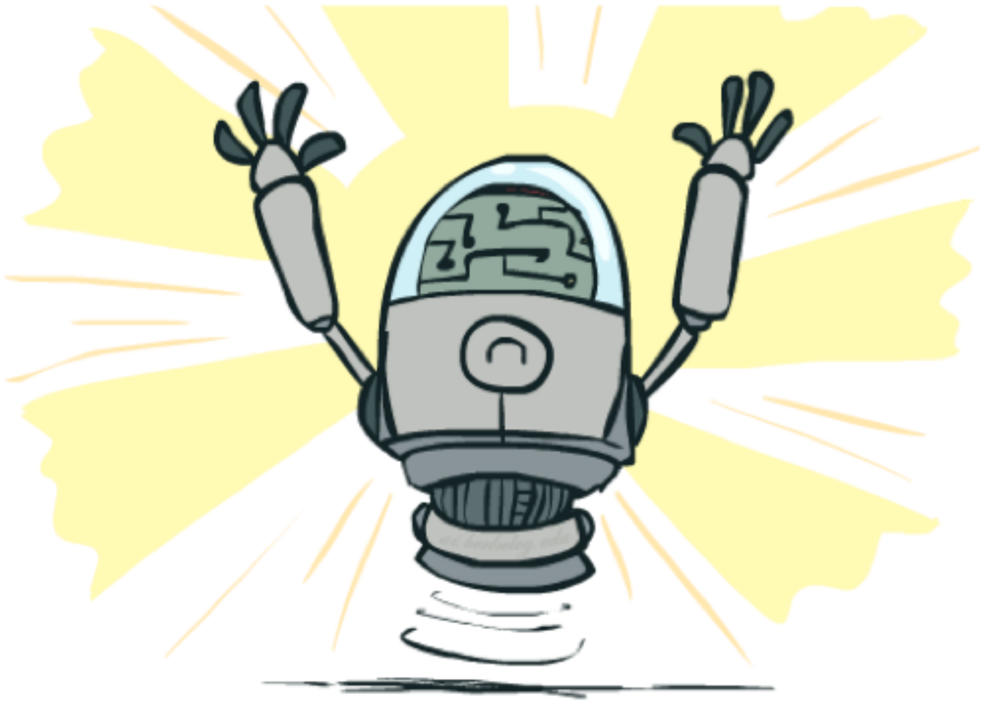


Figure from Berkley AI

A* Summary

- A* uses both backward costs and (estimates of) forward costs
- A* is optimal with admissible / consistent heuristics
- Heuristic design is key: often use relaxed problems

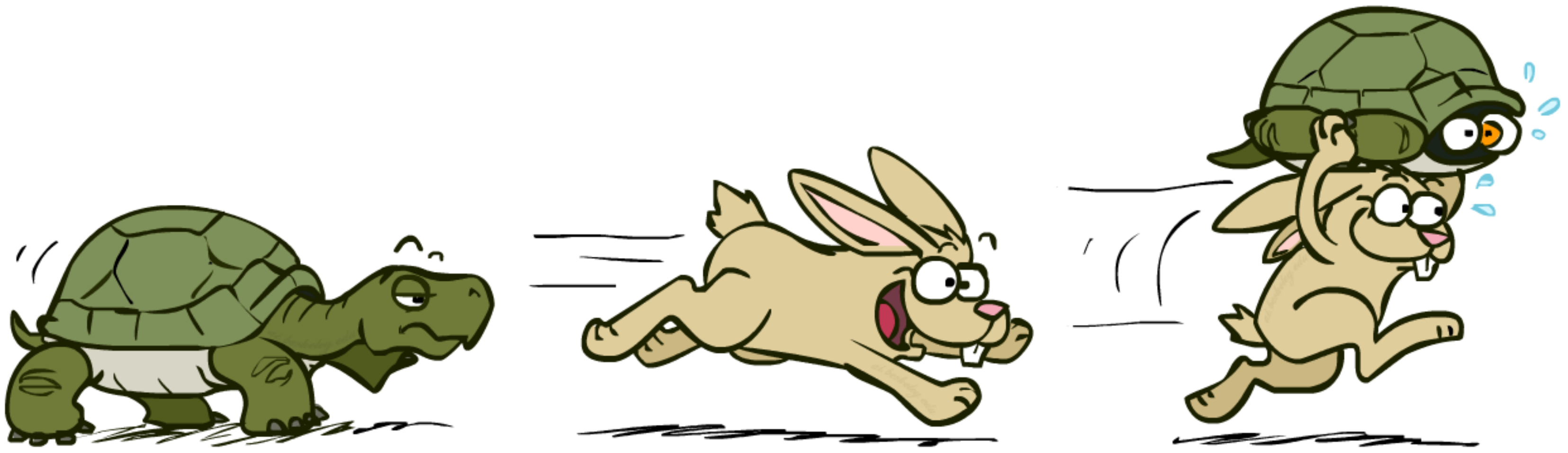


Figure from Berkley AI

Tree Search Pseudo-Code

```
function TREE-SEARCH(problem, fringe) return a solution, or failure
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        for child-node in EXPAND(STATE[node], problem) do
            fringe ← INSERT(child-node, fringe)
        end
    end
```


Figure from Berkley AI

Graph Search Pseudo-Code

```
function GRAPH-SEARCH(problem, fringe) return a solution, or failure
    closed ← an empty set
    fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
    loop do
        if fringe is empty then return failure
        node ← REMOVE-FRONT(fringe)
        if GOAL-TEST(problem, STATE[node]) then return node
        if STATE[node] is not in closed then
            add STATE[node] to closed
            for child-node in EXPAND(STATE[node], problem) do
                fringe ← INSERT(child-node, fringe)
            end
    end
```