Artificial Intelligence

Probabilistic Reasoning

CS 444 – Spring 2019
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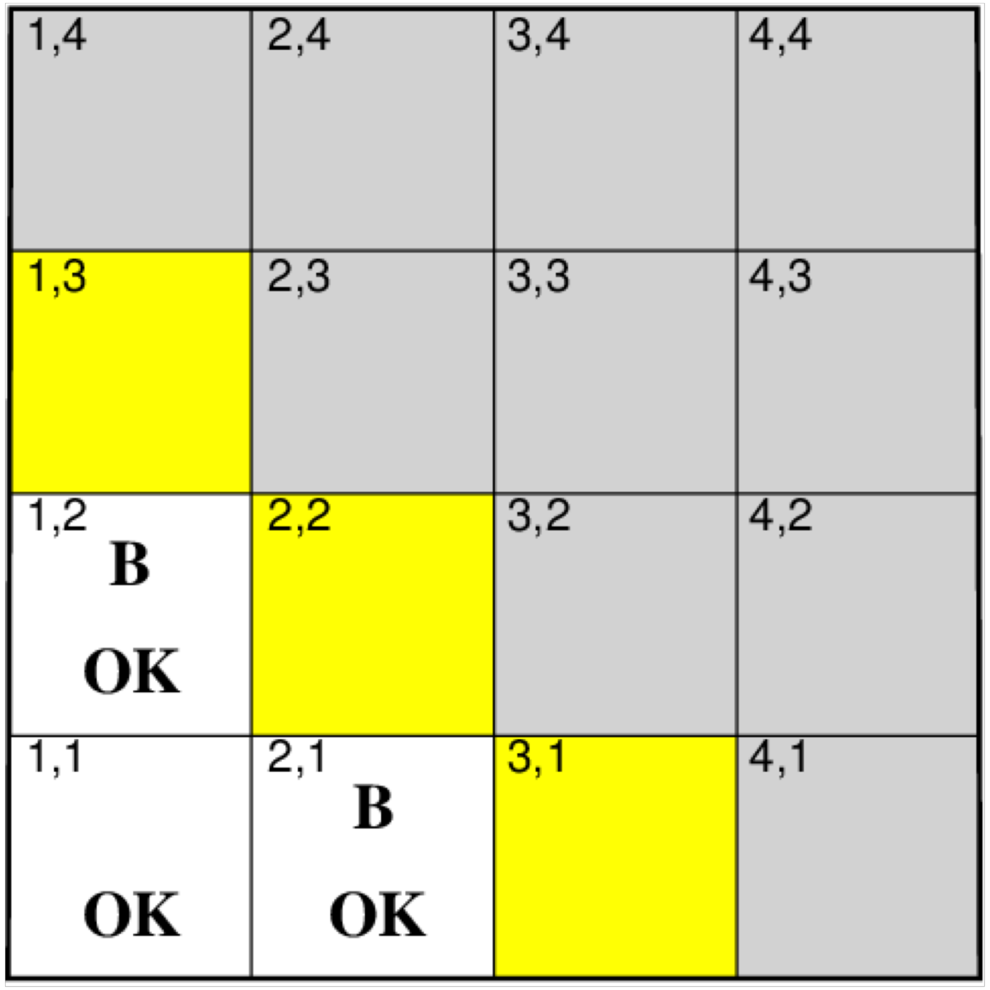
Wumpus World



Knowledge:

- A pit causes a breeze in all adjacent squares
- Each square other than [1, 1] contains a pit with probability 0.2

P_{ij} = true iff [i, j] contains a pit

B_{ij} true iff [i, j] is breezy

Include only $B_{1,1}$, $B_{1,2}$, $B_{2,1}$ in the probability model

Specifying the Probability Model

The full joint distribution is $P(P_{1,1}, \ldots, P_{4,4}, B_{1,1}, B_{1,2}, B_{2,1})$

Apply product rule: $P(B_{1,1}, B_{1,2}, B_{2,1} \mid P_{1,1}, \ldots, P_{4,4})\, P(P_{1,1}, \ldots, P_{4,4})$

(Do this to get P(Effect | Cause).)

First term: 1 if pits are adjacent to breezes, 0 otherwise.

Second term: pits are placed randomly, probability 0.2 per square:

$$P_{1,1}, \ldots, P_{4,4} = \prod_{i,j=1,1}^{4,4} \boldsymbol{P}(P_{i,j}) = 0.2^{n} \; x \; 0.8^{16-n} \; for \; n \; pits$$

Observations and Query

We know the following facts:

$b = \neg b_{1,1} \wedge b_{1,2} \wedge b_{2,1}$

$known = \neg p_{1,1} \wedge \neg p_{1,2} \wedge \neg p_{2,1}$

Query is P($P_{1,3}$ | known, b)

Define Unknown = Pij s other than P1,3 and Known

For inference by enumeration, we have:

$$\mathbf{P}(P_{1,3} \mid known, b) = \alpha \sum_{unknown} \mathbf{P}(P_{1,3}, unknown, known, b)$$

Yikes !!! This grows exponentially with the number of squares.

Employing Conditional Independence

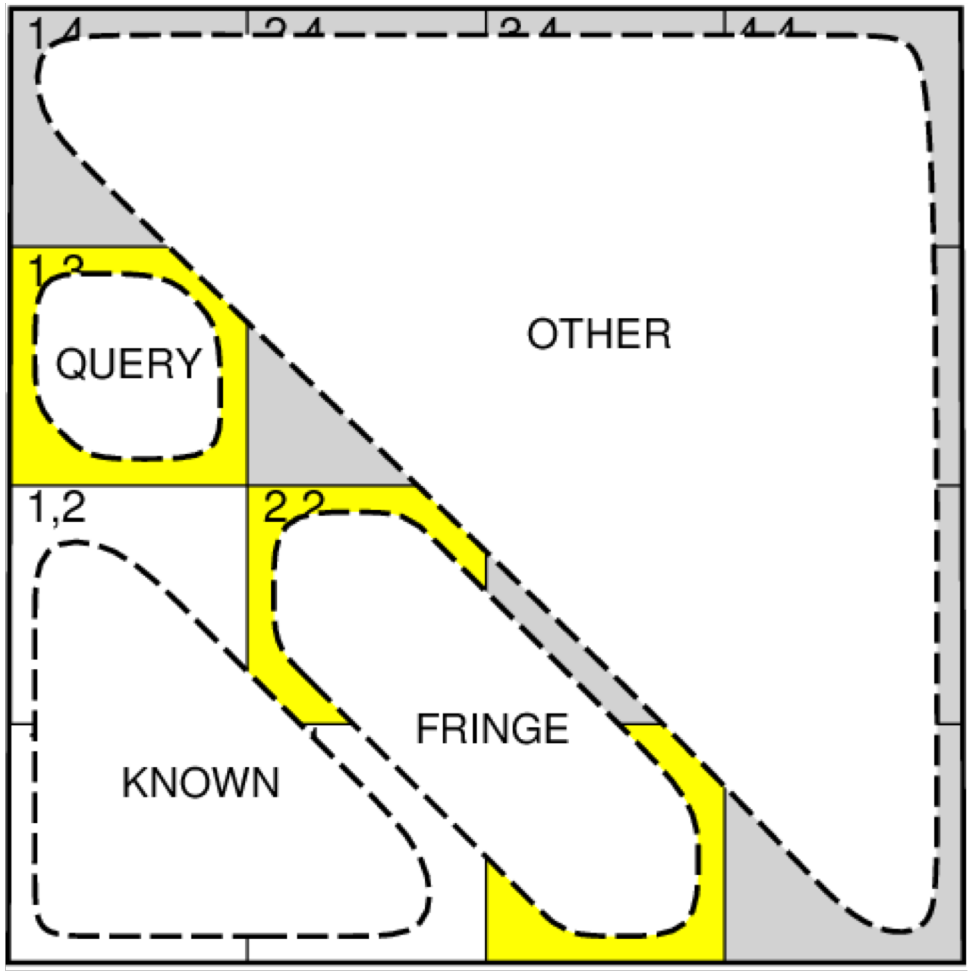
Basic insight: observations are conditionally independent of other hidden squares given neighboring hidden squares.

Define Unknown = Fringe ∪ Other

$P(b \mid P_{1,3}, \text{Known}, \text{Unknown}) = P(b \mid P_{1,3}, \text{Known}, \text{Fringe})$

Manipulate query into a form where we can use this.

$$= \alpha P(P_{1,3}) \sum_{fringe} P(b \mid known, P_{1,3}, fringe) P(fringe)$$



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