Artificial Intelligence

First Order Logic
(part 2)

CS 444 – Spring 2019

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FOL – Natural Numbers

The **Peano** axioms define natural numbers and addition.

First, we can define them recursively:

$NatNum(0)$

$\forall$ n NatNum(n) $\implies$ NatNum(S(n))

S(n) is a successor function. This allows 0, S(0), S(S(0)), and so on. We need a few axions to constrain the successor function:

$\forall$ n 0 ≠ S(n)

$\forall$m, n, m ≠ n $\implies$ S(m) ≠ S(n)

Now we can define addition in terms of the successor function:

$\forall$ m   NatNum(m) $\implies$ + (0,m) = m

$\forall$ m,n NatNum(m) $\wedge$ NatNum(n) $\implies$ +(S(m),n) = S(+(m,n))

When we make S(m) = m + 1:

$\forall$ m,n NatNum(m) $\wedge$ NatNum(n) $\implies$ m + 1 + n = m + n + 1

Unification

Inference in FOL is accomplished through **unification**. We can get the inference immediately if we can find a substitution θ such that King (x) and Greedy(x) match King(John) and Greedy(y)

θ = {x/John, y/ John} works

Unify(α, β) = θ if $\alpha\theta = \beta\theta$

p	q	θ
Knowns(John, x)	Knowns(John, Jane)	{x/ Jane}
Knowns(John, x)	Knowns(y, OJ)	{x/ OJ, y/ John}
Knowns(John, x)	Knowns(y, Mother(y))	{y/John, x/Mother(John)}
Knowns(John, x)	Knowns(x, OJ)	fail

Standardizing apart eliminates overlap of variables, e.g., Knowns (z_{17}, OJ).

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Example Knowledge Base

The law says that it is a crime for an American to sell weapons to hostile nations. The country, Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is an American.

Prove that Col. West is a criminal.

Example Knowledge Base

The law says that it is a crime for an American to sell weapons to hostile nations. The country, Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is an American.

… it is a crime for an American to sell weapons to hostile nations:

$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Longrightarrow Criminal(x)$$

Nono .. Has some missiles

$$\exists x \; Owns(Nono, x) \wedge Missile\,(x):$$

$$Owns(Nono, M_1) \; and \; Missle(M_1)$$

.. All of its missiles were sold to it by Col. West:

$$\forall x \; Owns(Nono, x) \wedge Missile\,(x) \Longrightarrow Sells(West, x, Nono)$$

Missiles are weapons:

$$\forall x \; Missile\,(x) \Longrightarrow Weapon(x)$$

An enemy of America counts as hostile: $\forall \, x \; Enemy(x, America) \Longrightarrow Hostile(x)$

Example Knowledge Base

The law says that it is a crime for an American to sell weapons to hostile nations. The country, Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is an American.

… it is a crime for an American to sell weapons to hostile nations:

$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \implies Criminal(x)$$

Nono .. Has some missiles

$$\exists x \; Owns(Nono, x) \wedge Missile\,(x): \quad Owns(Nono, M_1) \text{ and } Missle(M_1)$$

.. All of its missiles were sold to it by Col. West:

$$\forall x \; Owns(Nono, x) \wedge Missile\,(x) \implies Sells(West, x, Nono)$$

Missiles are weapons: $\quad \forall x \; Missile\,(x) \implies Weapon(x)$

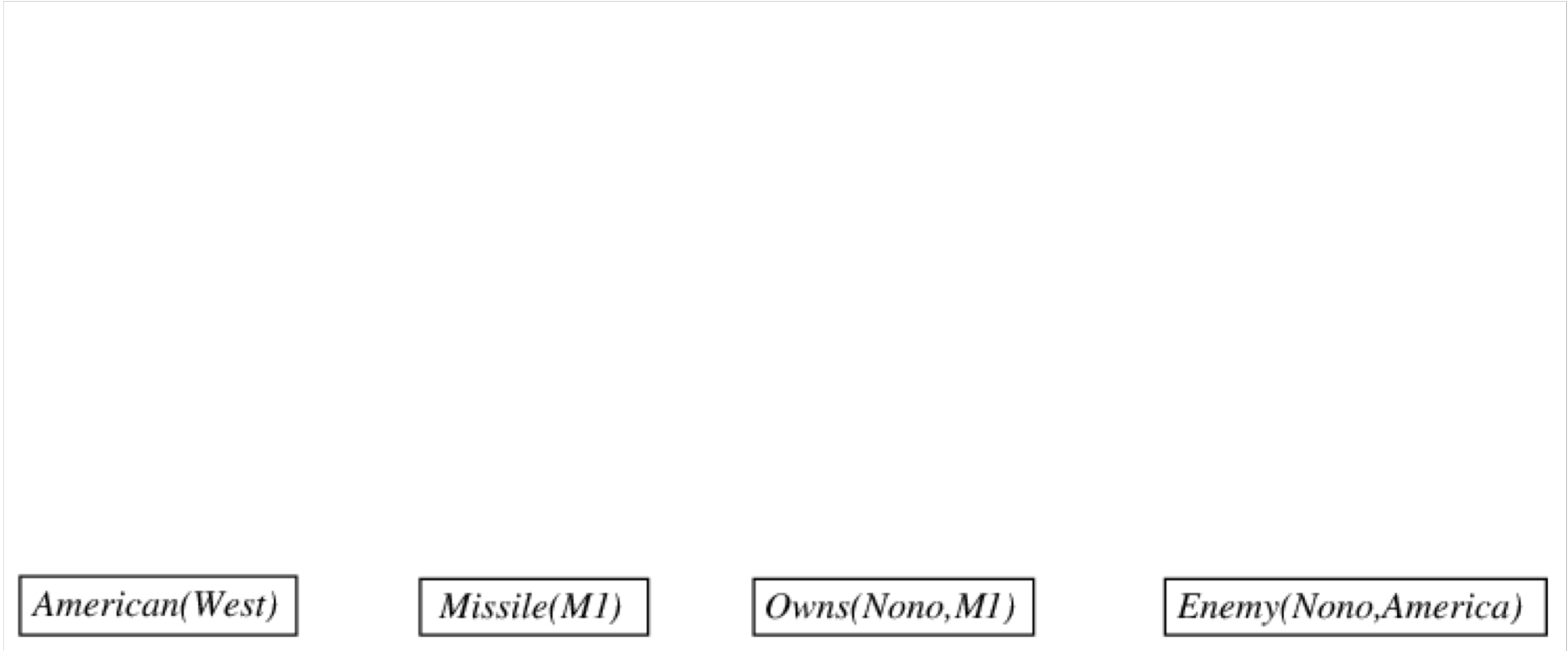An enemy of America counts as hostile: $\forall\, x \; Enemy(x, America) \implies Hostile(x)$
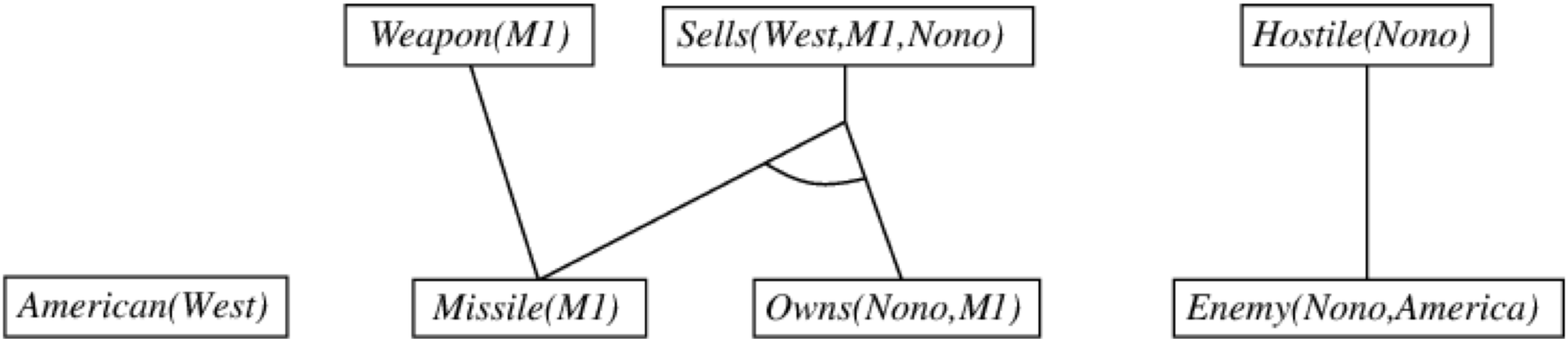
West, who is an American..

American(West)

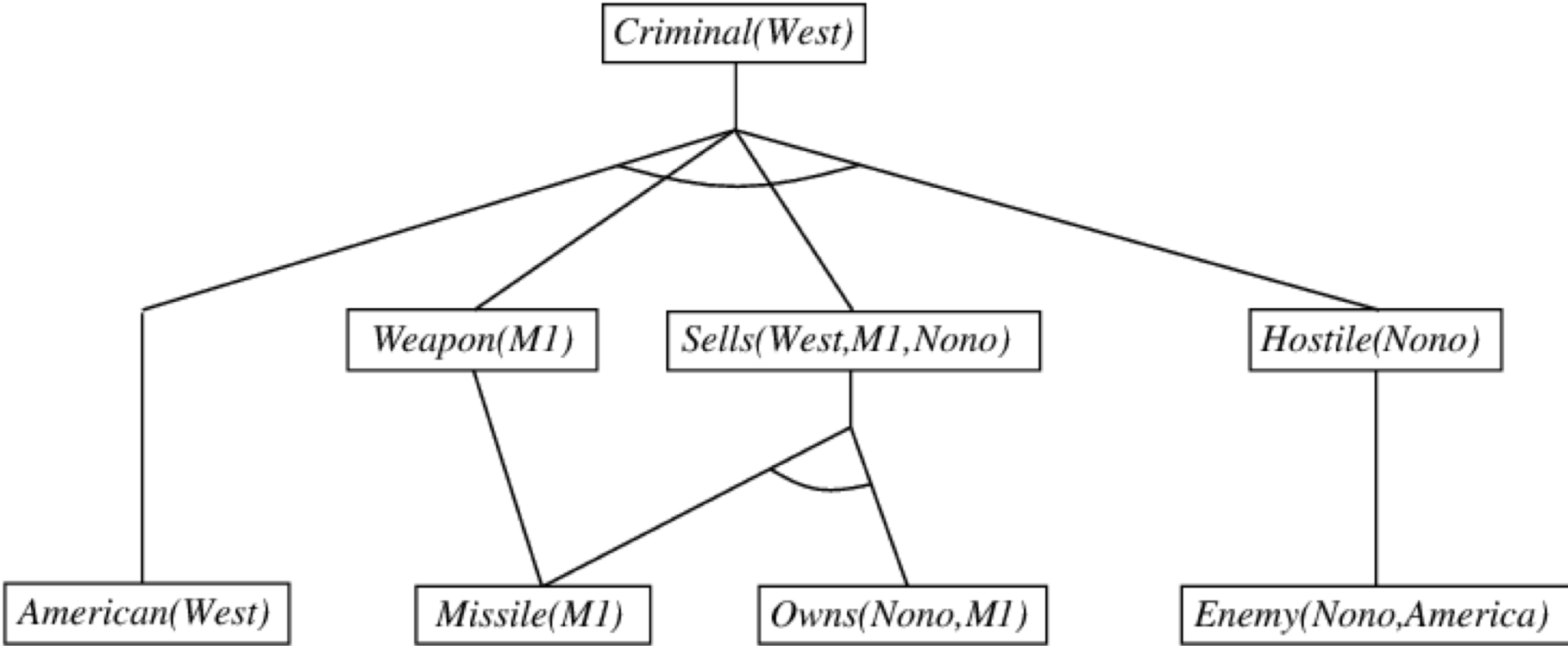The country Nono, an enemy of America

Enemy(Nono, America)

Forward Chaining Example

American(West)   Missile(M1)   Owns(Nono,M1)   Enemy(Nono,America)

Forward Chaining Example

Forward Chaining Example

Forward Chaining Algorithm

Operates the same as with propositional logic. Combines sentences

until it reaches a fixed point. When KB is written with definite clauses, and does not contain functional symbols, it is **complete**. It is bound by $O(pn^k)$ where k is the maximum arity of all predicate functions, n is the number of constant symbols, and p is the number of predicates.

Is it complete for knowledge bases with functional symbols?

The short answer is yes, but logically complete means that if the statement is true, we can proof that in a finite amount of time.

Because functions can be applied recursively, (recall S(S(S(n))) for example. So, it is not possible to decide whether the knowledge base entails some fact (when it doesn't, we would loop forever). When it does, you can imagine a routine like IDS (iterative deepening search), where the level of recursion is controlled. We call this **semi-decidable**.
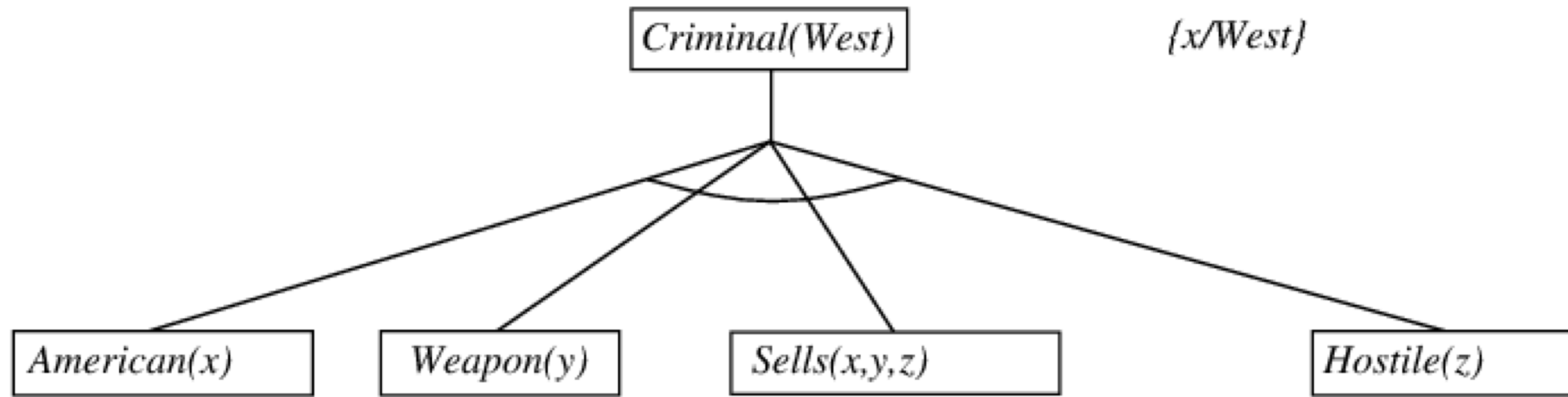
Similar Problems
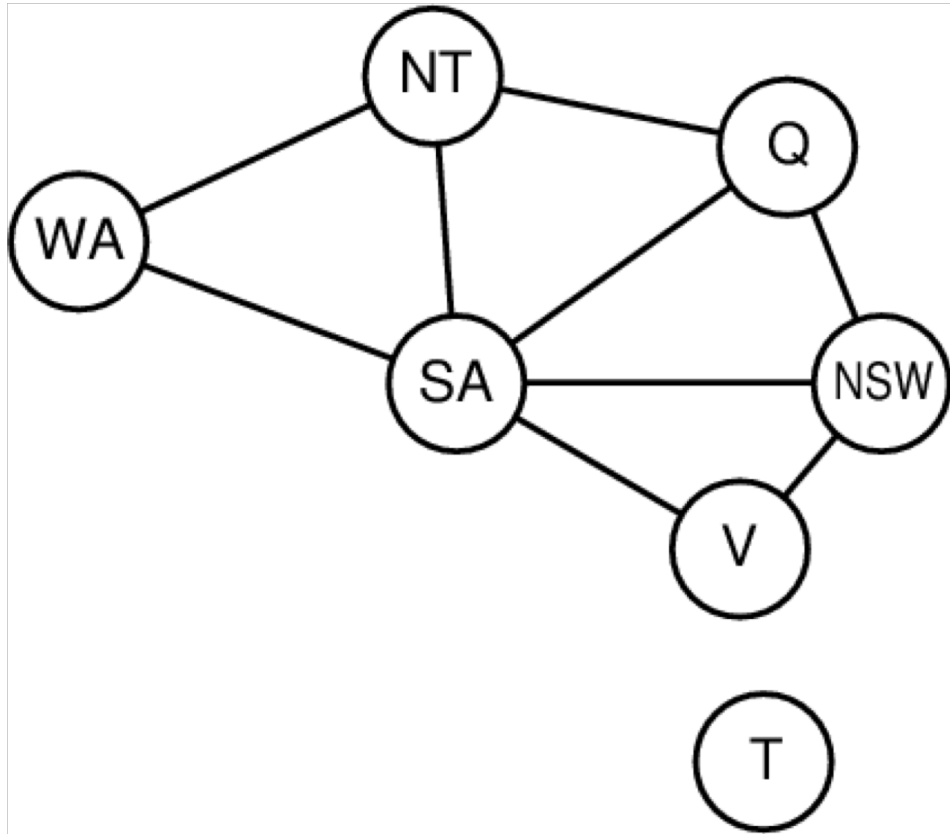
Diff(wa, nt) ∧ Diff(wa, sa) ∧ Diff(nt, q) ∧ Diff(nt, sa) ∧ Diff(q, nsw) ∧ Diff(q, sa) ∧ Diff(nsw, v) ∧ Diff(nsw, sa) ∧ Diff(v, sa)

$$\Longrightarrow Colorable()$$

Colorable() is inferred iff the CSP has a solution. CSPs include 3SAT as a special case, hence, matching is NP-Hard.

Backward Chaining Examples
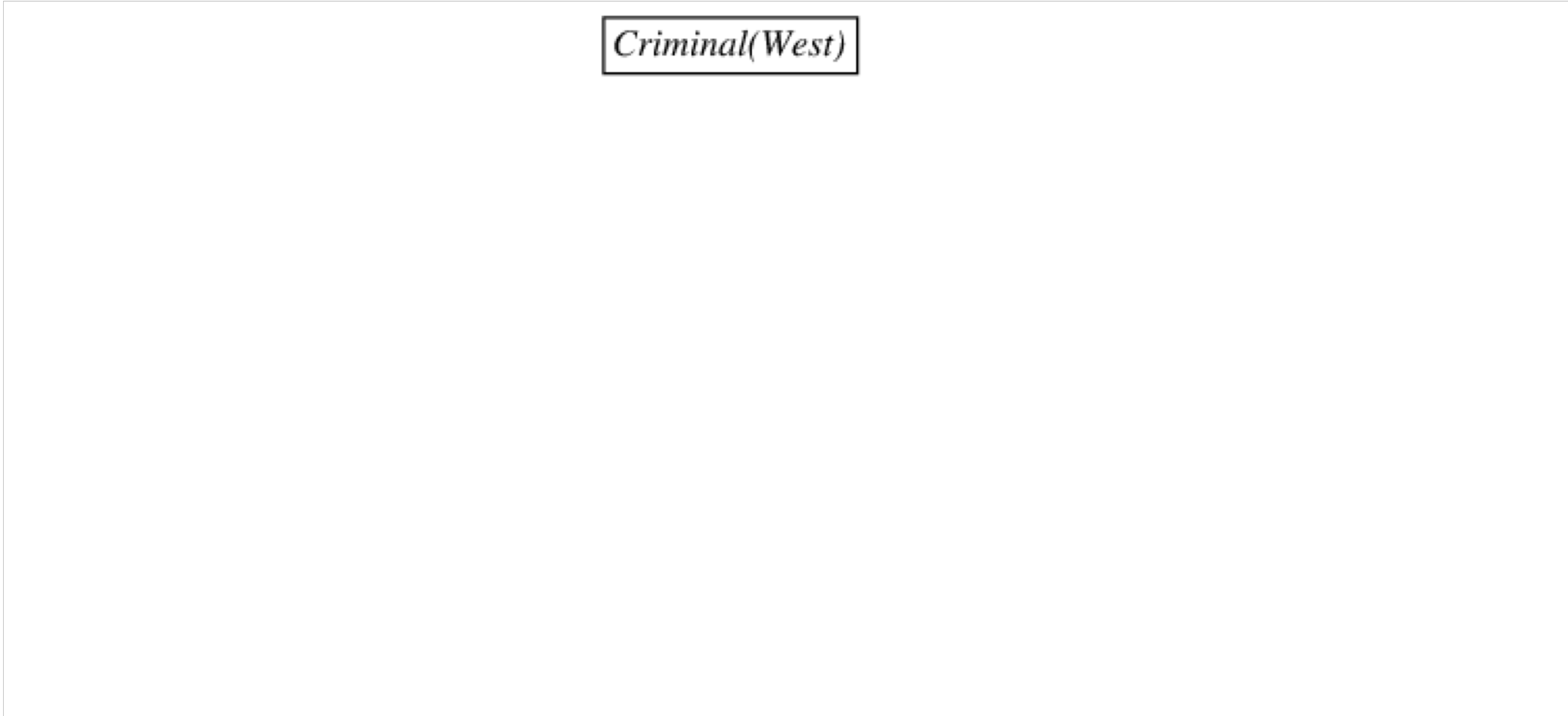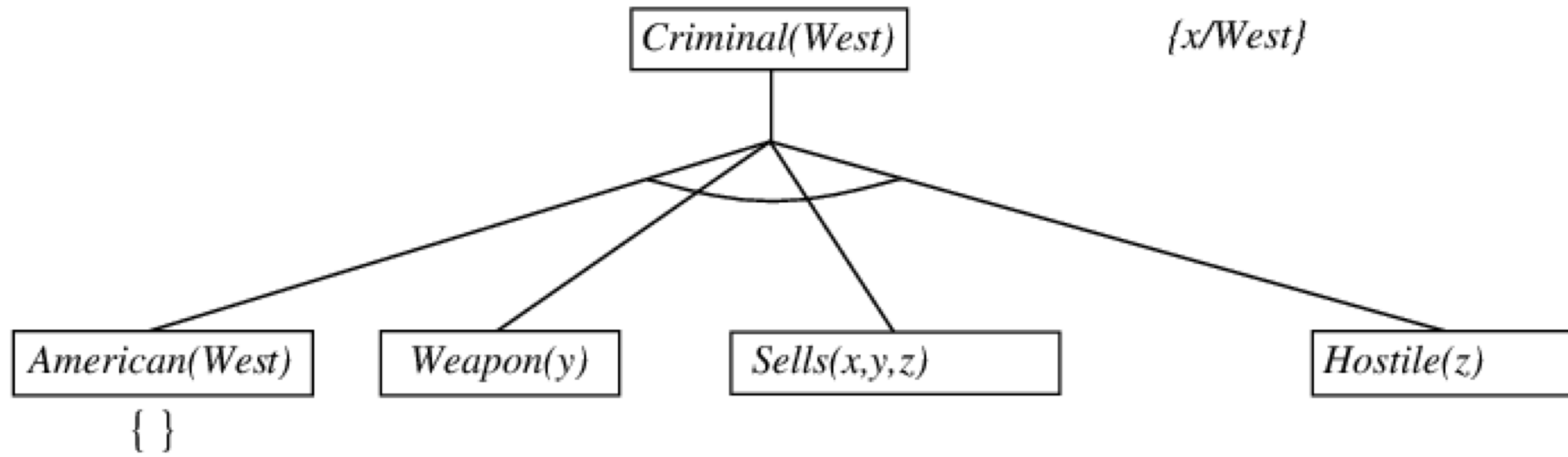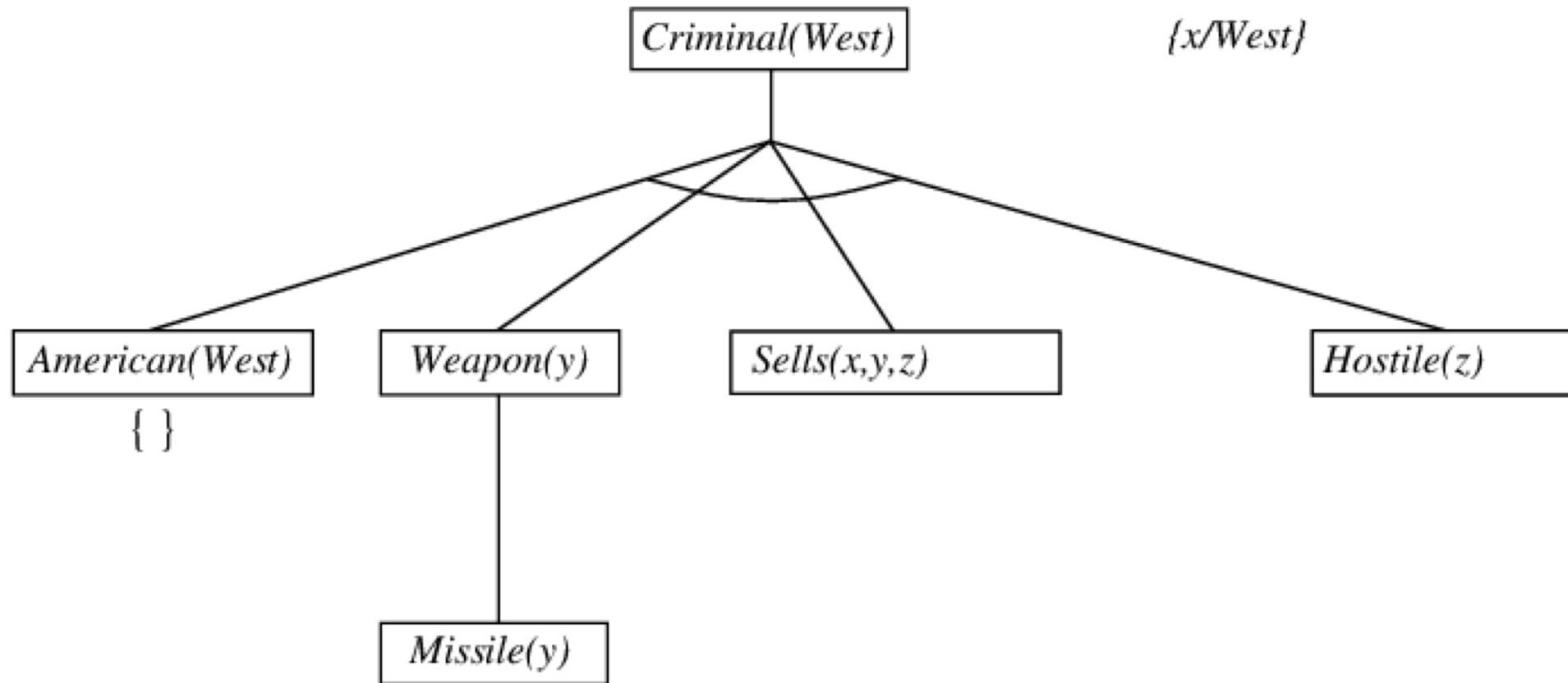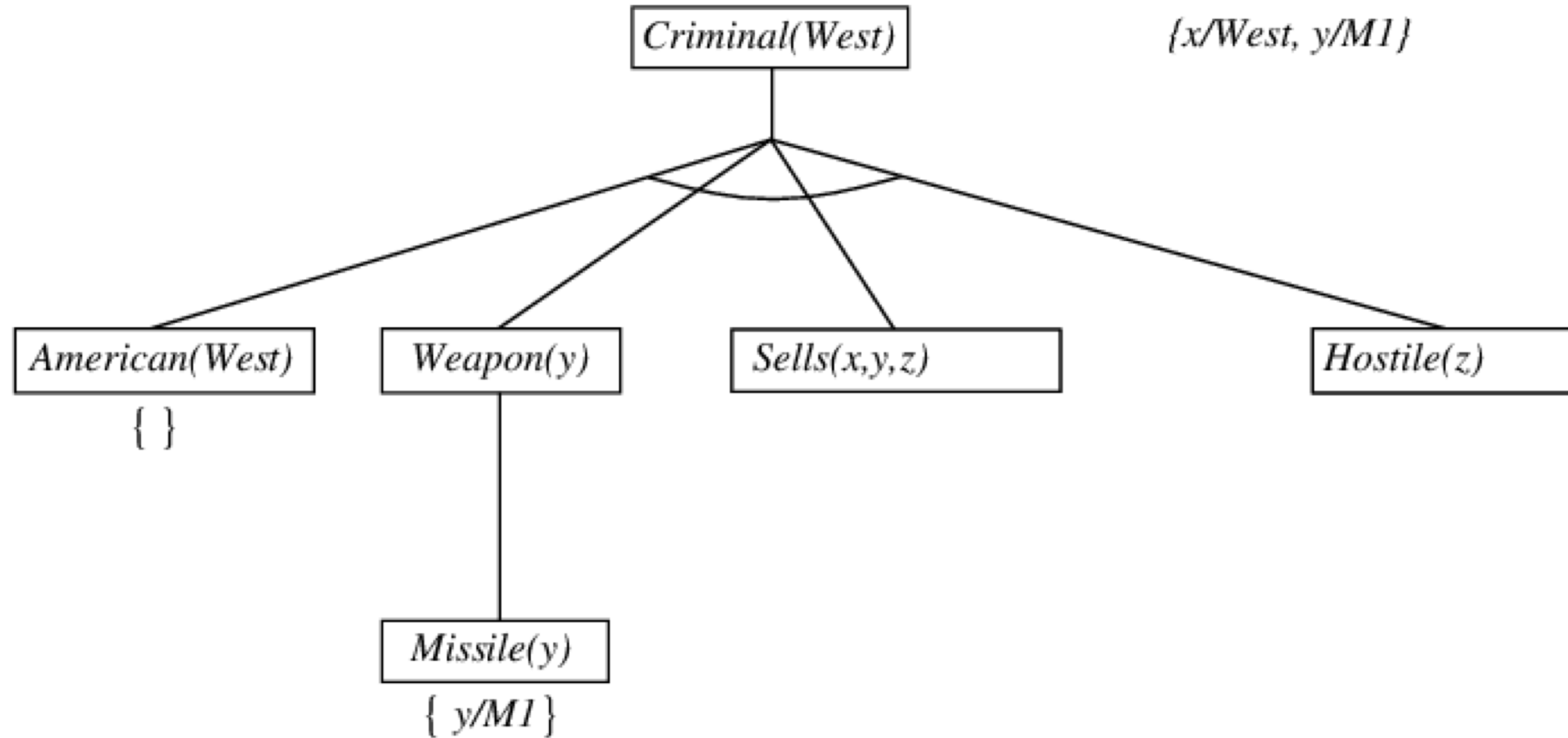
Criminal(West)

Backward Chaining Examples

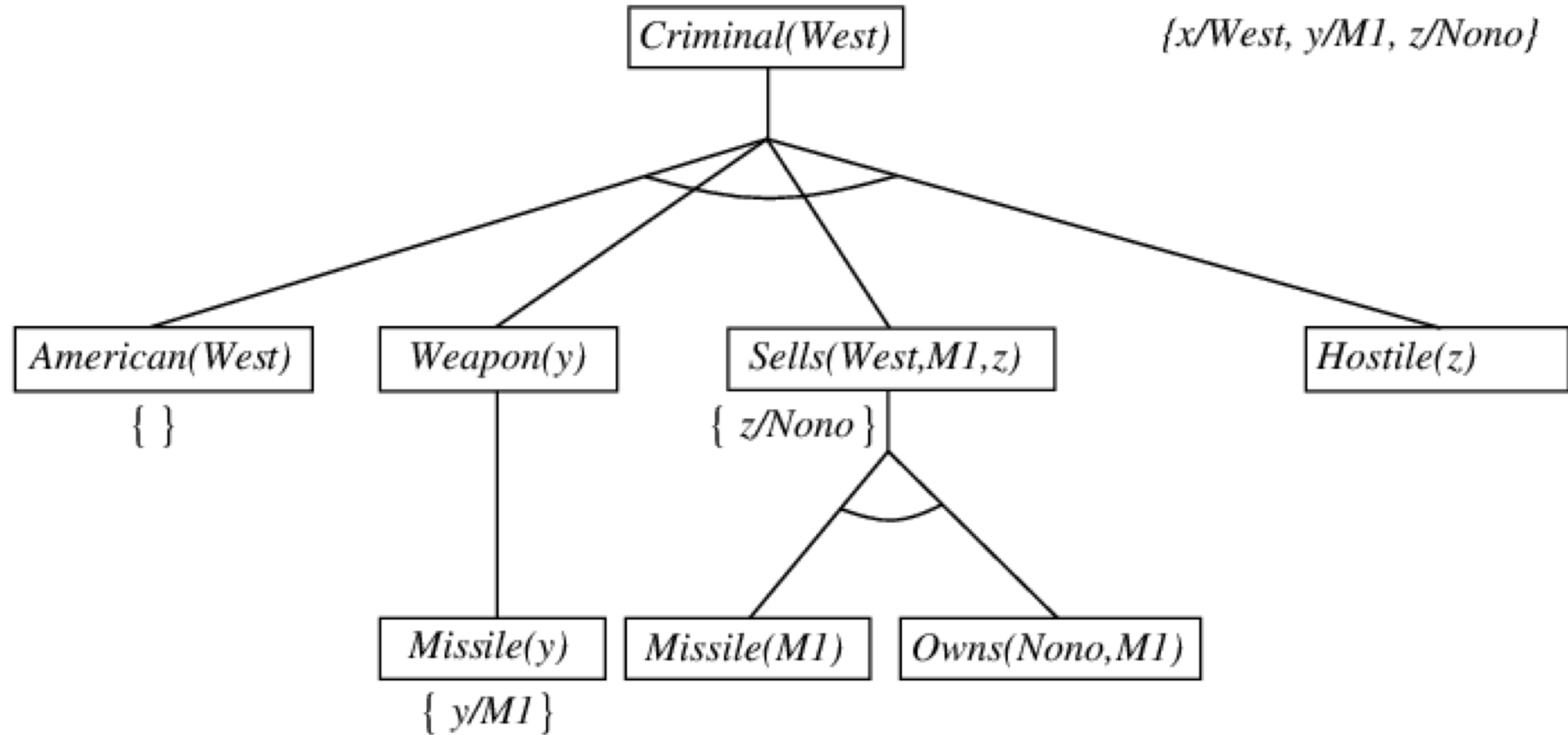# Backward Chaining Examples
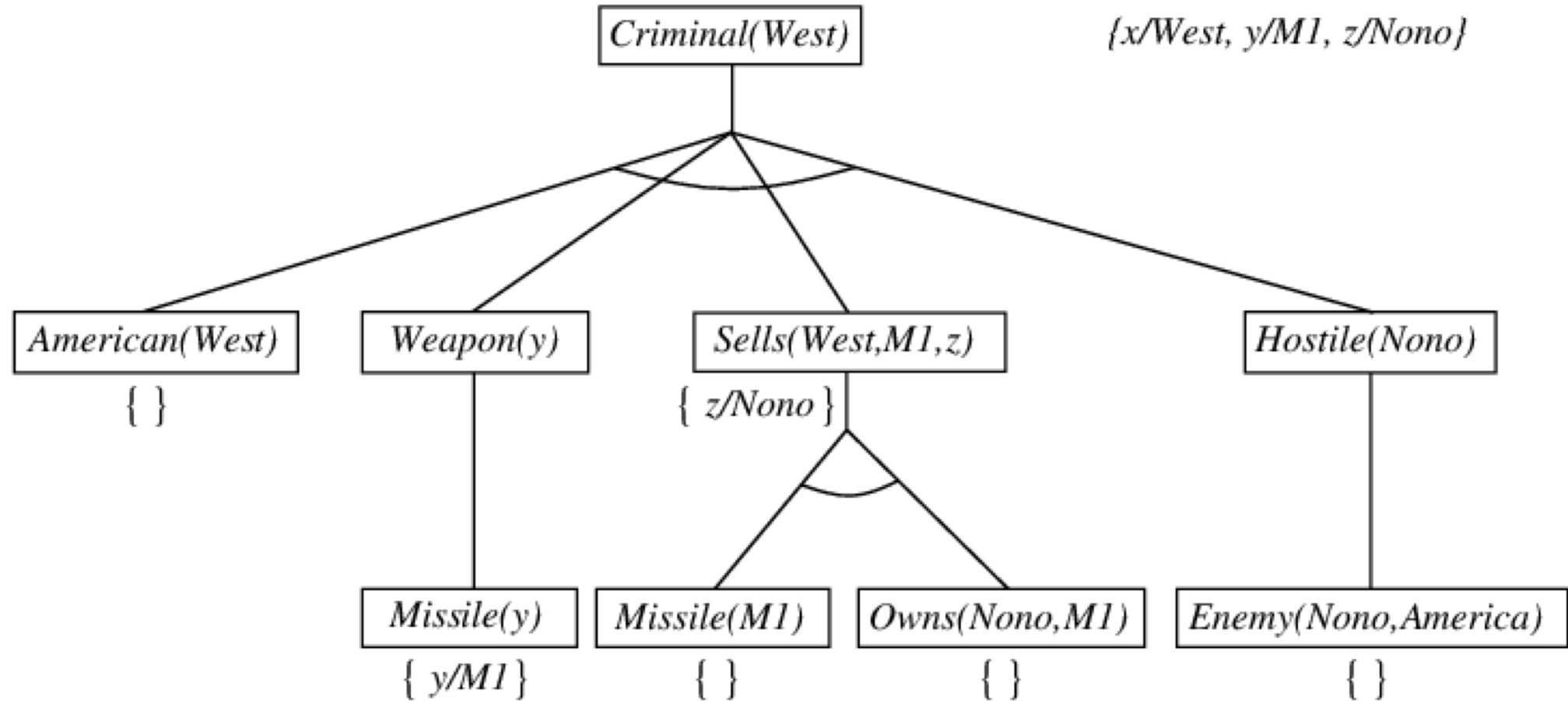
Backward Chaining Examples

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Backward-Chaining

Depth-first recursive proof search: space is linear in size of proof (good)

But, incomplete because of infinite loops (can be fixed) as we will see on Thursday.

Exam 2

Covers CSP, Propositional Logic, and FOL.

Review questions will be posted on Thursday.

Question 9.3

Suppose a knowledge base contains just one sentence:

$\exists$x AsHighAs(x, Everest).

Which of the following are legitimate results of applying Existential Instantiation?

a) AsHighAs(Everest, Everest)

b) AsHighAs(Kilimanjaro, Everest)

c) AsHighAs(Kilimanjaro, Everest) $\wedge$ AsHighAs(BenNevis, Everest) (after two applications).

Question 9.3

Suppose a knowledge base contains just one sentence:

∃x AsHighAs(x, Everest).

Which of the following are legitimate results of applying Existential Instantiation?

a) AsHighAs(Everest, Everest). (not valid, can not introduce new variable not previously introduced)

b) AsHighAs(Kilimanjaro, Everest). (valid)

c)AsHighAs(Kilimanjaro, Everest) ∧ AsHighAs(BenNevis, Everest) (after two applications). (Could be that BenNevis and Kilimanjaro are the same thing, see figure 8.5)

Question 9.4

For each pair of atomic sentences, give the most general unifer if it exists:

a) P(A, B, B), P(x, y, z)

b) Q(y, G(A, B)), Q (G(x,x), y)

c) Older (Father(y), y), Older(Father(x), John)

d) Knows (Father(y), y), Knows(x, x)

Question 9.4

For each pair of atomic sentences, give the most general unifer if it exists:

a) P(A, B, B), P(x, y, z). {x/A, y/B, z/B}

b) Q(y, G(A, B)), Q (G(x,x), y). No solution, x can not bind A, and B)

c) Older (Father(y), y), Older(Father(x), John)

   {y/John, x/John}

d) Knows (Father(y), y),   Knows(x, x)

   No Unifier because the x can not be y and Father(y) at the same time). The so called occurs-check.

Question 9.6

Write down logical representations for the following sentences., suitable for user with Generalized Modus Ponens:

a) Horses, cows, and pigs are mammals.

b) An offspring of a horse is a horse

c) Bluebeard is a horse

c) Bluebeard is Charlie's parent.

Question 9.6

Write down logical representations for the following sentences., suitable for user with Generalized Modus Ponens:

a) Horses, cows, and pigs are mammals.

Horse(x) $\implies$ Mammals(x)

Cows(x) $\implies$ Mammals(x)

Pigs(x) $\implies$ Mammals(x)

An offspring of a horse is a horse

Offspring(x, y) $\land$ Horse(y) $\implies$ Horse(x)

Question 9.6

Bluebeard is a horse

Horse(Bluebeard)

Bluebeard is Charlie's parent.

Parent(Bluebeard, Charlie)

Question 9.7

Given the premise ∀ x ∃ y P(x,y) show that it is not valid to conclude that ∃q P(q,q). Given an example where the first is true and the second is false.

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Given the premise ∀ x ∃ y P(x,y) show that it is not valid to conclude that ∃q P(q,q). Given an example where the first is true and the second is false.

Let P(x,y) be the relation that "x is less than y" over the integers. First is true, second is false