Name: e-ID:

CS 444:Take Home Quiz (3)
Spring 2019 – James Madison University
Points available: 60

This is an open notes/book quiz that is due by 11:59 pm on Friday, April 19th. All of your work is to be completed individually. You can turn in a written copy to me or scan/submit via email. **PLEASE** write your answers in space provided in the PDF (or print a copy and write it in the space provided), as I will be grading this with Gradescope. **You must** show all your work to get full credit.

Problem 1 (4 pts): Consider the set of all possible five-card poker hands dealt fairly from a standard deck of fifty-two cards.

a) How many atomic events are there in the joint probability distribution (i.e., how many five-card hands are there)? $\binom{52}{5} = 2,598,960$

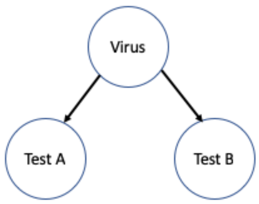
b) What is the probability of each atomic event? $\frac{1}{2{,}598{,}960}$

c) What is the probability of being dealt a royal straight flush (A-K-Q-K-10 all of the same suit)? 4 potential ways to form this hand, so, $\frac{4}{2{,}598{,}960}$

d) What is the probability of being dealt four of a kind? 13 ways to have 4 of a kind, plus any of the remaining (52-4 = 48) cards as the fifth card. Thus, $\frac{13\times 48}{2{,}598{,}960} = \frac{1}{4165}$

Problem 2 (20 pts): Consider two medical tests, A and B, for a virus. Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is in fact not present). Test B is 90% effective at recognizing the virus when it is present, but has a 5% false positive rate. The two tests use **independent** methods to identify the virus. The virus exists in 1% of the population.

a) Draw a Bayesian network (similar to the earthquake burglar problem).



b) If we get a positive result from TestB, is TestA independent? Briefly explain (at most 2 sentences).

No, because the results of TestB change our belief about virus. Knowing something about virus changes our belief in TestA (and thus, they are **not** independent).

c) Say that a person is tested for the virus using only one of the tests, and that test comes back positive. Which test returning positive is more indicative of someone actually having the virus? Justify your answer.

We need to calculate out the probabilities of having the virus for both tests. Let V indicate the have the virus, $T1$ represent that test1 indicates we have the virus, and $T2$ represent that test2 indicates we have the virus. Thus, we want to compute $P(v|t1)$ and $P(v|t2)$.

$P(v|t1) = P\frac{(t2|v)P(v)}{P(t2)}$

$P(t1) = P(t1|v) + P(t1|\neg v) = .95 \times 0.01 + .10 \times 0.99 = 0.1085$

$P\frac{(t1|v)P(v)}{P(t1)} = \frac{0.95 \times 0.01}{0.1085} \approx 0.876$

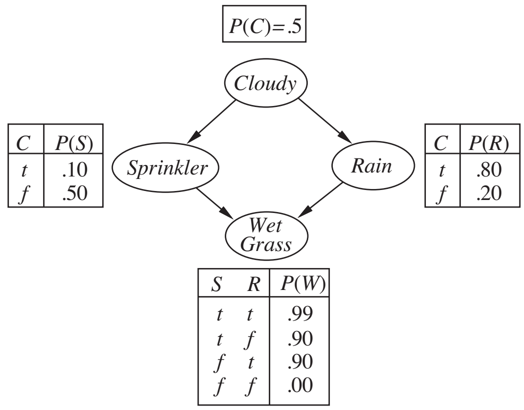
$P(v|t2) = P\frac{(t2|v)P(v)}{P(t2)}$

$P(t2) = P(t2|v) + P(t2|\neg v) = .90 \times 0.01 + .05 \times 0.99 = 0.0585$

$P\frac{(t2|v)P(v)}{P(t2)} = \frac{0.90 \times 0.01}{0.0585} \approx 0.1538$

Test 2 is more indicative of someone having the virus.

Problem 3 (25 pts): A belief network with conditional probabilities is given in the following figure.



The letters C, R, S, and W stand for *Cloudy, Rain, Sprinkler, and Wet Grass*, respectively. Compute the probabilities of $P(W)$ and $P(S|W)$. Notice that these are capital letters. *Hint:* This will require summing out over a few variables.

$P(w) = \sum_c \sum_s \sum_r P(c)P(s|c)P(r|c)P(W|s,r)$

$P(w) = \sum_c P(c) \sum_s P(s|c) \sum_r P(r|c)P(W|s,r)$

$P(w) = P(c)\ \{P(s|c)\ [P(r|c)P(w|s,r) + P(\neg r|c)P(w|s\neg r)]+$
$P(\neg s|c)[P(r|c)P(w|\neg s,r) + P(\neg r|c)P(w|\neg s\neg r)]\}+$
$P(\neg c)\{P(s|\neg c)\ [P(r|\neg c)P(w|s,r) + P(\neg r|\neg c)P(w|s\neg r)]+$
$P(\neg s|\neg c)[P(r|\neg c)P(w|\neg s,r) + P(\neg r|\neg c)P(w|\neg s\neg r)]\}+$

$$\begin{aligned} P(w) = 0.5\{&0.1[0.8*0.99+0.2*0.9]+ \\ &0.9[0.8*0.9+0.2*0.00]\}+ \\ &0.5\{0.5[0.2*0.99+0.8*0.9]+ \\ &0.5[0.2*0.9+0.8*0.00]\} \end{aligned}$$

$$\begin{aligned} P(w) &= 0.5\{0.1[0.972]+0.9[0.72]\}+ \\ &\quad 0.5\{0.5[0.918]+0.5[0]\} \\ P(w) &= 0.5\{0.7452\}+0.5\{0.459\} \\ P(w) &= 0.6021 \qquad P(\neg w) = 1 - P(w) = 0.3979 \end{aligned}$$

$P(S,W) = P(S|W)P(W)$ (via the product rule)
$P(S|W) = \frac{P(S,W)}{P(W)}$
$P(S|W) = \alpha P(S,W) \quad s.t.\ \alpha = \frac{1}{P(W)}$
This tell us that if we want to know $P(S|W)$, we can find an answer that is proportional to this value by solving P(S,W), which is easier. To solve P(S,W) , we will vary S and W to all combinations (4) and sum out over the other variables.

$P(s|w) = \alpha \sum_c P(c)P(s|c) \sum_r P(r|c)P(w|r,s)$
$= \alpha\{P(c)P(s|c)\ [P(r|c)P(w|s,r) + P(\neg r|c)P(w|s,\neg r)\ ]+$
$P(\neg c)P(s|\neg c)\ [P(r|\neg c)P(w|s,r) + P(\neg r|\neg c)P(w|\neg r,s)\ ]\ \}$
$= \alpha\{[0.5 * .1[.8 * 0.99 + 0.2 * 0.9)\ ] + [0.5 * 0.5(0.2 * 0.99 + 0.8 * 0.9)\ ]\ \}$
$= \alpha\{[0.5 * .1 * 0.972. + 0.5 * 0.5 * 0.918\ ]\ \}$
$= \alpha\{0.0486 + 0.2295\} = \alpha 0.2781$

$P(\neg s|w) = \alpha \sum_c P(c)P(\neg s|c) \sum_r P(r|c)P(w|r,\neg s)$
$= \alpha\{P(c)P(\neg s|c)\ [P(r|c)P(w|\neg s,r) + P(\neg r|c)P(w|\neg s,\neg r)\ ]+$
$P(\neg c)P(\neg s|\neg c)[P(r|\neg c)P(w|\neg s,r) + P(\neg r|\neg c)P(w|\neg r,\neg s)\ ]\}$
$= \alpha\{0.5 * .9[.8 * 0.9 + 0.2 * 0.0\ ] + 0.5 * 0.5[0.2 * 0.90 + 0.8 * 0.0\ ]\ \}$
$= \alpha\{0.5 * 0.9 * 0.72 + 0.5 * 0.5 * 0.18\ ]\ \}$
$= \alpha\ \{0.324 + 0.045\ \} = \alpha 0.369$

$P(s|w) + P(\neg s|w) = 1$
$\alpha 0.2781 + \alpha 0.369 = 1$
$\alpha(0.2781 + 0.369) = 1$
$\alpha = \frac{1}{0.2781+0.369}$
$\alpha = \frac{1}{0.6471}$
$\alpha = 1.545356205$

$P(s|w) = \alpha 0.2781 \approx 0.43$
$P(\neg s|w) = \alpha 0.369 \approx 0.57$

$P(s|\neg w) = \alpha \sum_c P(c)P(s|c) \sum_r P(r|c)P(\neg w|r,s)$
$= \alpha\{P(c)P(s|c)\ [\ P(r|c)P(\neg w|s,r) + P(\neg r|c)P(\neg w|s,\neg r)\ ]+$
$P(\neg c)P(s|\neg c)[P(r|\neg c)P(\neg w|s,r) + P(\neg r|\neg c)P(\neg w|\neg r,s)\ ]\ \}$
$= \alpha\{0.5 * .1[.8 * 0.01 + 0.2 * 0.1\ ]+$
$0.5 * 0.5[0.2 * 0.01 + 0.8 * 0.1]\ \}$
$= \alpha\{0.5 * .1 * 0.028 + 0.5 * 0.5 * 0.082\ \}$
$= \alpha\{0.0014 + 0.0205\ \} = \alpha 0.0219$

$P(\neg s|\neg w) = \alpha \sum_c P(c)P(\neg s|c) \sum_r P(r|c)P(\neg w|r,\neg s)$
$= \alpha\{P(c)P(\neg s|c)[P(r|c)P(\neg w|\neg s,r) + P(\neg r|c)P(\neg w|\neg s,\neg r)\ ]+$
$P(\neg c)P(\neg s|\neg c)[P(r|\neg c)P(\neg w|\neg s,r) + P(\neg r|\neg c)P(\neg w|\neg r,\neg s)\ ]\}$
$= \alpha\{0.5 * .9 * [.8 * 0.10 + 0.2 * 1]\ \}+$
$\{0.5 * 0.5[0.2 * 0.10 + 0.8 * 1\ ]\ \}$
$= \alpha\{0.5 * 0.1 * 0.28 + 0.5 * 0.50.82\ \}$
$= \alpha\{0.0205 + 0.126\ \} = \alpha 0.1465$

$P(s|\neg w) + P(\neg s|\neg w) = 1$
$\alpha 0.0219 + \alpha 0.1465 = 1$
$\alpha(0.0219 + 0.1465) = 1$
$\alpha = \frac{1}{0.0219+0.1465}$
$\alpha = \frac{1}{0.22995}$
$\alpha = 5.93824228$

$P(s|\neg w) = \alpha 0.0219 \approx 0.13$
$P(\neg s|\neg w) = \alpha 0.1465 \approx 0.87$

Problem 4 (11 pts): Consider the query $P(Rain|Sprinkler = true, WetGrass = true)$ and how we can use Gibbs sampling to answer this question. (Chapter 14 material, not Chapter 15).

a) How many states does the Markov chain have?

b) Calculate out the probabilities for $q \to q' \ \ \forall q, q'$ for all states.