

# CS354

Nathan Sprague

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What will probability allow us to do?

- 1 Update our existing beliefs on the basis of new sensor data
- 2 Combine multiple (conflicting) sources of information
- 3 Combine uncertain predictive models with noisy sensor data to obtain better state estimates than either source alone could provide

Today we will focus 1.

# Probability Notation

- Probability Functions/Distributions:
  - $P(A)$  is a function that maps from all possible values of  $A$  to the probability of the corresponding event.
  - Examples:
    - $P(A = \textit{true}) = .9$   
 $P(A = \textit{false}) = .1$
    - $P(B = \textit{red}) = .8$   
 $P(B = \textit{blue}) = .1$   
 $P(B = \textit{green}) = .1$

# Sample Spaces and Joint Probability Distributions

- Sample space is the set of all possible outcomes.
- The full joint probability distribution assigns a probability to each element of the sample space:
  - $S$  - Squished,  $U$  - Under falling Piano

| $S$ | $U$ | $P(S, U)$ |
|-----|-----|-----------|
| T   | T   | .008      |
| T   | F   | .002      |
| F   | T   | .001      |
| F   | F   | .989      |

# Conditional Probability

- $P(A | B)$  Expresses the probability of assignments to  $A$  given assignments to  $B$ .
  - $P(\text{SQUISHED} = \text{true}) = .01$
  - $P(\text{SQUISHED} = \text{true} | \text{UNDER\_PIANO} = \text{true}) \approx .89$

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

# Bayes Rule

$$P(A | B) = \frac{P(B | A)P(A)}{P(B)}$$

Very handy for updating our beliefs on the basis of evidence.

# Bayes Rule Example

- Robot is in a simple four room maze, rooms are labeled a-d.
- Initially, we think he is most likely to be in the left half,  
 $P(X = a) = .4$ ,  $P(X = b) = .4$ , ...

|    |    |    |    |
|----|----|----|----|
| a  | b  | c  | d  |
| .4 | .4 | .1 | .1 |

# Bayes Rule Example

- Robot has a sensor designed to tell him what room he is in.
- Sensor is not perfect: only 80% likely to report he is in the correct room. 20% of the time the sensor is off by one. (Errors at the edge wrap around.)
- Distribution of sensor readings when robot is in  $a$ :

|     |     |     |     |
|-----|-----|-----|-----|
| $a$ | $b$ | $c$ | $d$ |
| .8  | .1  | 0   | .1  |

- In probability notation, where  $X$  is the position and  $Z$  is sensor reading.
  - $P(Z = a \mid X = a) = .8$
  - $P(Z = b \mid X = a) = .1$
  - $P(Z = c \mid X = a) = 0$
  - $P(Z = d \mid X = a) = .1$



# Bayes Rule Example

- Given that we have a sensor model, Baye's rule enables us to update our prior beliefs based on sensor input:

$$P(X | Z) = \frac{P(Z | X)P(X)}{P(Z)}$$

# Bayes Rule Example

- Let's calculate  $P(X = a | Z = b)$

$$P(X = a | Z = b) = \frac{P(Z = b | X = a)P(X = a)}{P(Z = b)}$$

- $P(Z = b | X = a) = .1$  (From our sensor model)
- $P(X = a) = .4$  (Our prior)
- $P(Z = b)$  (??)

# Bayes Rule Example

To calculate  $P(Z = b)$ , we can use the total probability theorem:

$$P(Z) = \sum_i^N P(X = x_i)P(Z | X = x_i)$$

We can also treat  $P(Z)$  as an unknown constant,

$$P(X | Z) = \eta P(Z | X)P(X)$$

and set it to whatever value makes  $P(X | Z)$  sum to 1. The two approaches are equivalent.

# Bayes Rule Example

Back to work...

$$P(X = a | Z = b) = \frac{P(Z = b | X = a)P(X = a)}{P(Z = b)}$$
$$= \eta \times .1 \times .4 = .04\eta$$

Similarly:

$$P(X = b | Z = b) = \eta \times .8 \times .4 = .32\eta$$

$$P(X = c | Z = b) = \eta \times .1 \times .1 = .01\eta$$

$$P(X = d | Z = b) = \eta \times 0 \times .1 = 0$$

# Bayes Rule Example

Therefore, after our sensor reading, the updated distribution over possible robot locations is:

| a         | b         | c         | d |
|-----------|-----------|-----------|---|
| $.04\eta$ | $.32\eta$ | $.01\eta$ | 0 |

We know the robot is *somewhere*, so we know that:

$$.04\eta + .32\eta + .01\eta = 1$$

$$\eta = \frac{1}{.04 + .32 + .01} = 1/.37 \approx 2.70$$

# Bayes Rule Example

Finally, we have an updated belief about the robot location:

| a    | b    | c    | d |
|------|------|------|---|
| .108 | .865 | .027 | 0 |

We may use this as our new prior, and incorporate additional sensor readings.