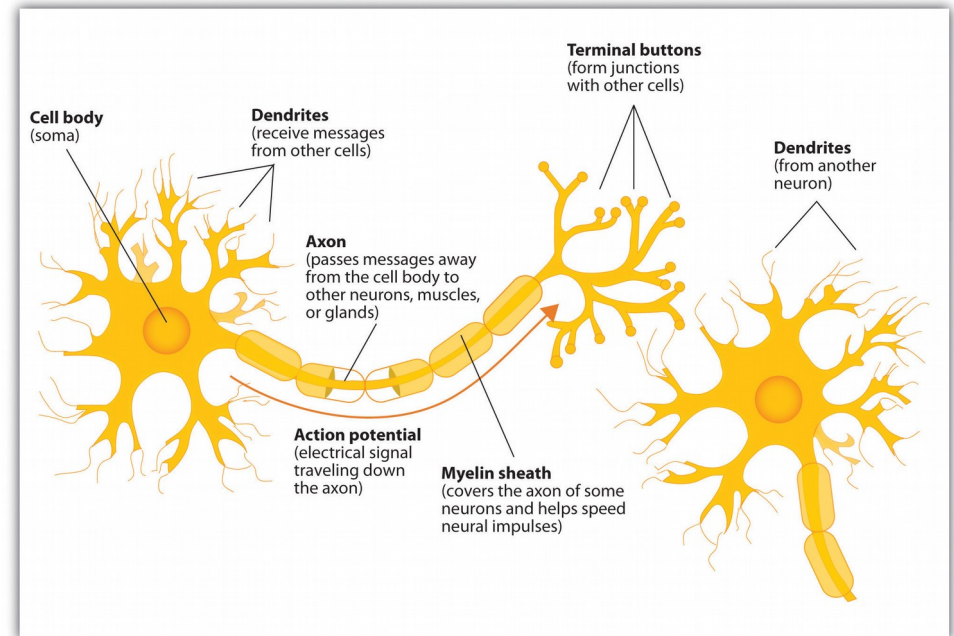


Convolutional Neural Networks

Nathan Sprague

Neurons

- Neurons communicate using discrete electrical signals called “spikes” (or action potentials).
 - Spikes travel along axons.
 - Reach axon terminals.
 - Terminals release neurotransmitters.
 - Postsynaptic neurons respond by allowing current to flow in (or out).
 - If voltage crosses a threshold a spike is created



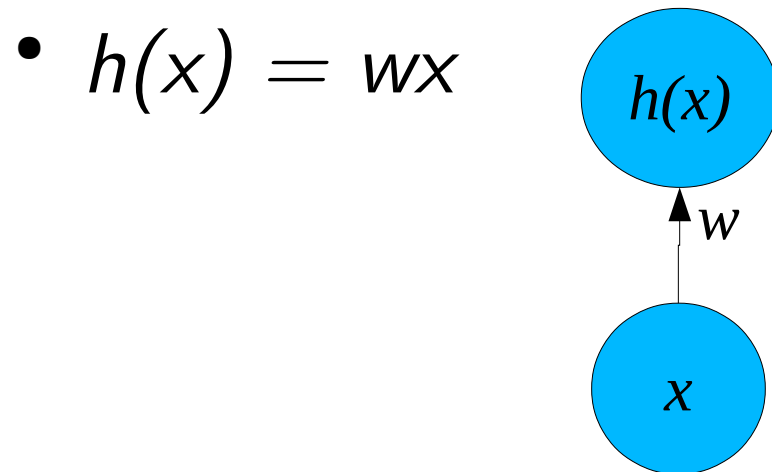
[Beginning Psychology](#) (v. 1.0).

<http://2012books.lardbucket.org/books/beginning-psychology/>

[Creative Commons by-nc-sa 3.0](#)

Linear Regression – The Neural View

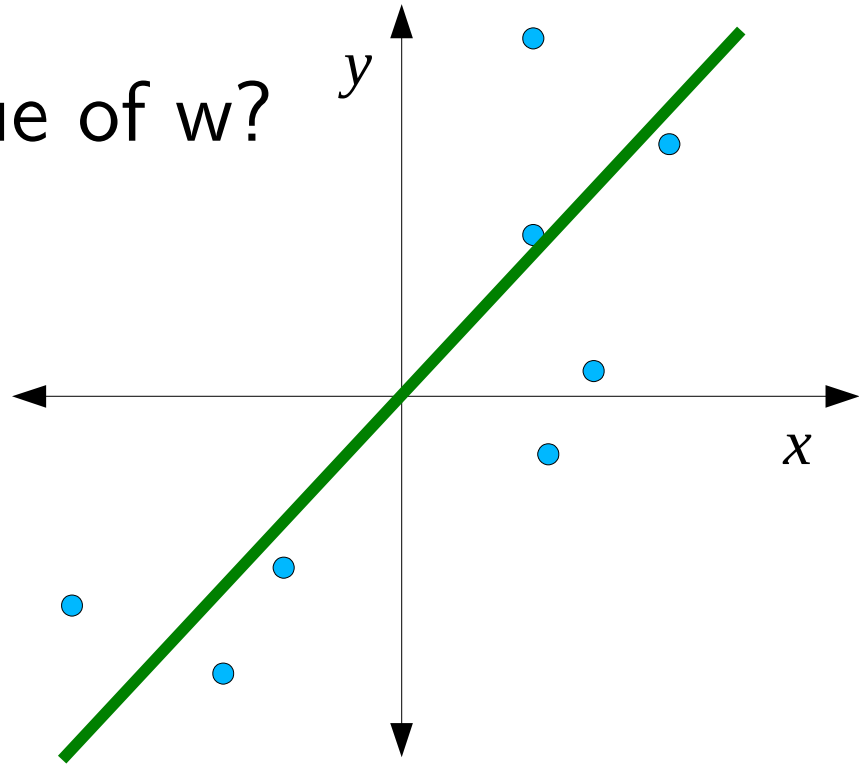
- input = x , desired output = y , weight = w .



- We are given a set of inputs, and a corresponding set of outputs, and we need to choose w .
- What's going on geometrically?

Lines

- $h(x) = wx$ is the equation of a line with a y intercept of 0.
- What is the best value of w ?
- How do we find it?

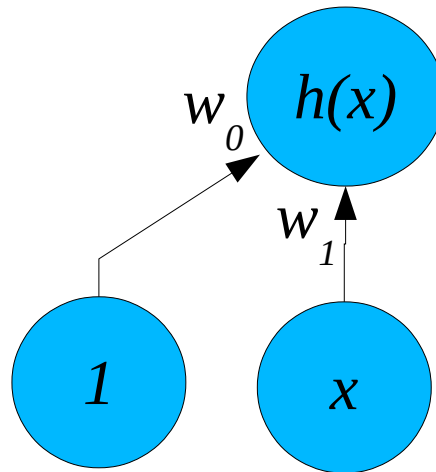


Bias Weights

- We need to use the general equation for a line:

$$h(x) = w_1x + w_0$$

- This corresponds to a new neural network with one additional weight, and an input fixed at 1.



Error Metric

- Sum squared error (y is the desired output):

$$Error_E = \sum_{e \in E} \frac{1}{2} (y_e - h(\mathbf{x}_e))^2$$

- The goal is to find a w that minimizes E .
How?

Gradient Descent



<http://en.wikipedia.org/wiki/>

File:Glacier_park1.jpg

Attribution-Share Alike 3.0 Unported

Gradient Descent

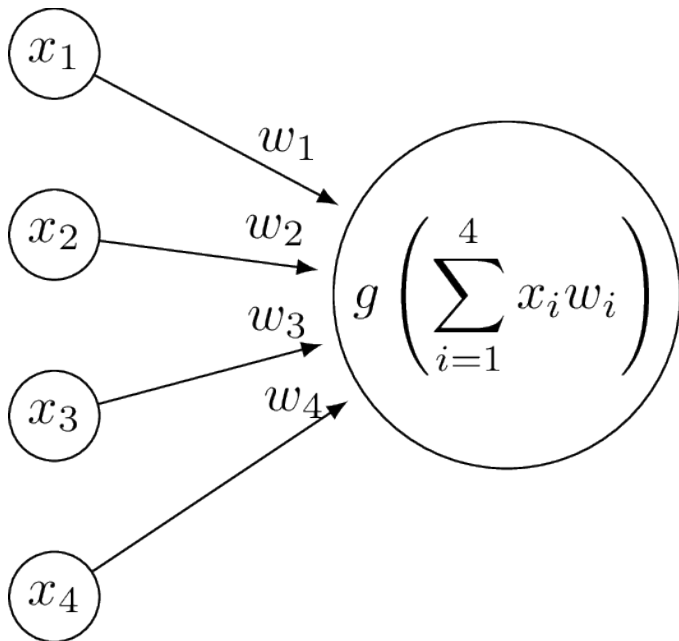
- One possible approach (maximization):
 - 1) take the derivative of the function: $f'(w)$
 - 2) guess a value of w : \hat{w}
 - 3) move \hat{w} a little bit according to the derivative:

$$\hat{w} \leftarrow \hat{w} - \eta f'(\hat{w})$$

- 4) goto 3, repeat.

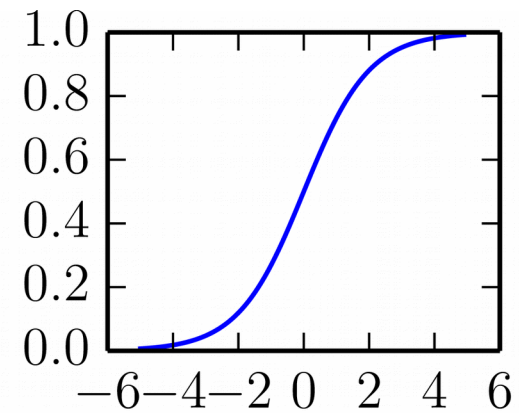
Full “Neuron”

Neuron

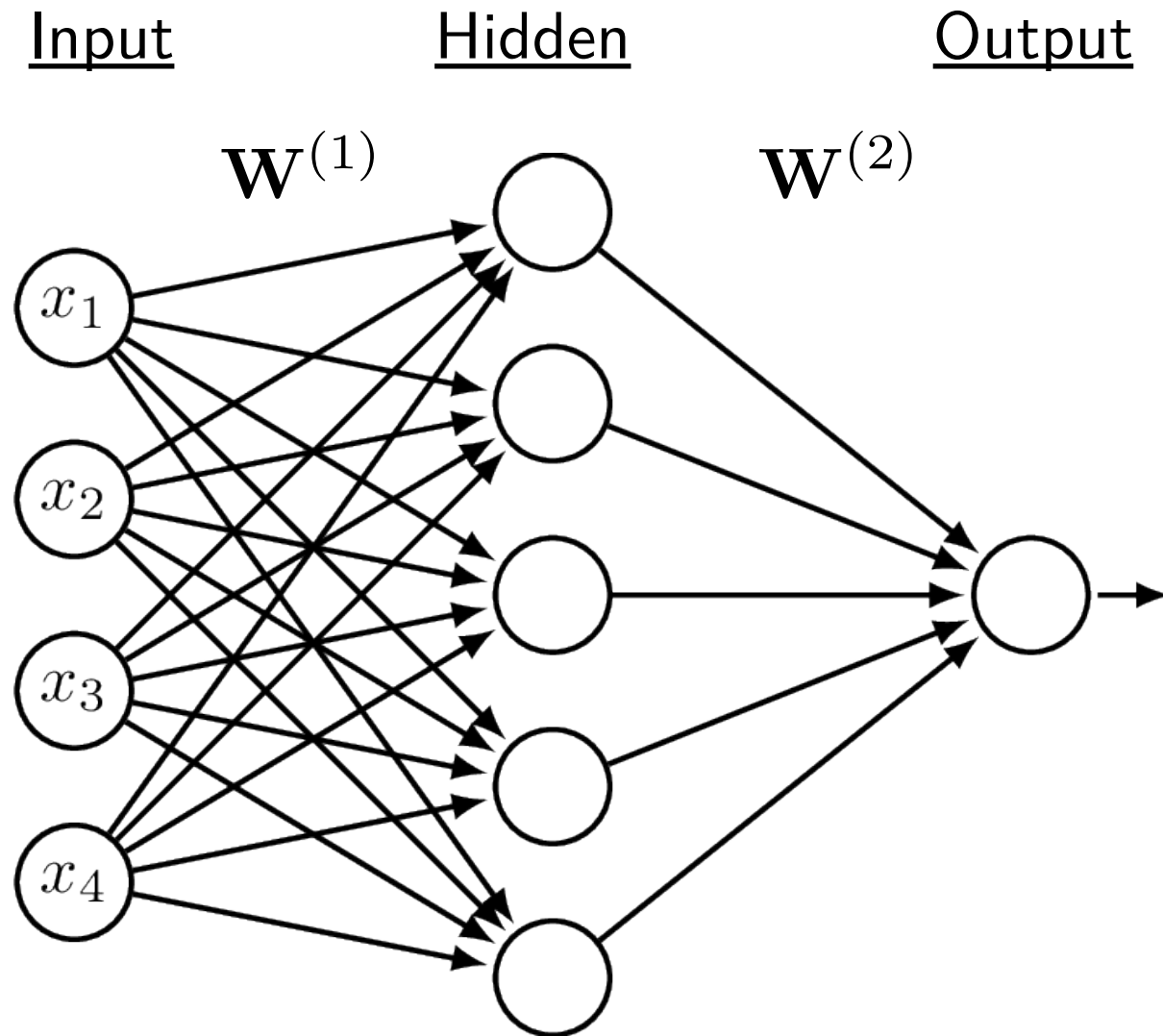


Non-linearity

$$g(a) = \frac{1}{1 + e^{-a}}$$



















Multi-Layer Networks

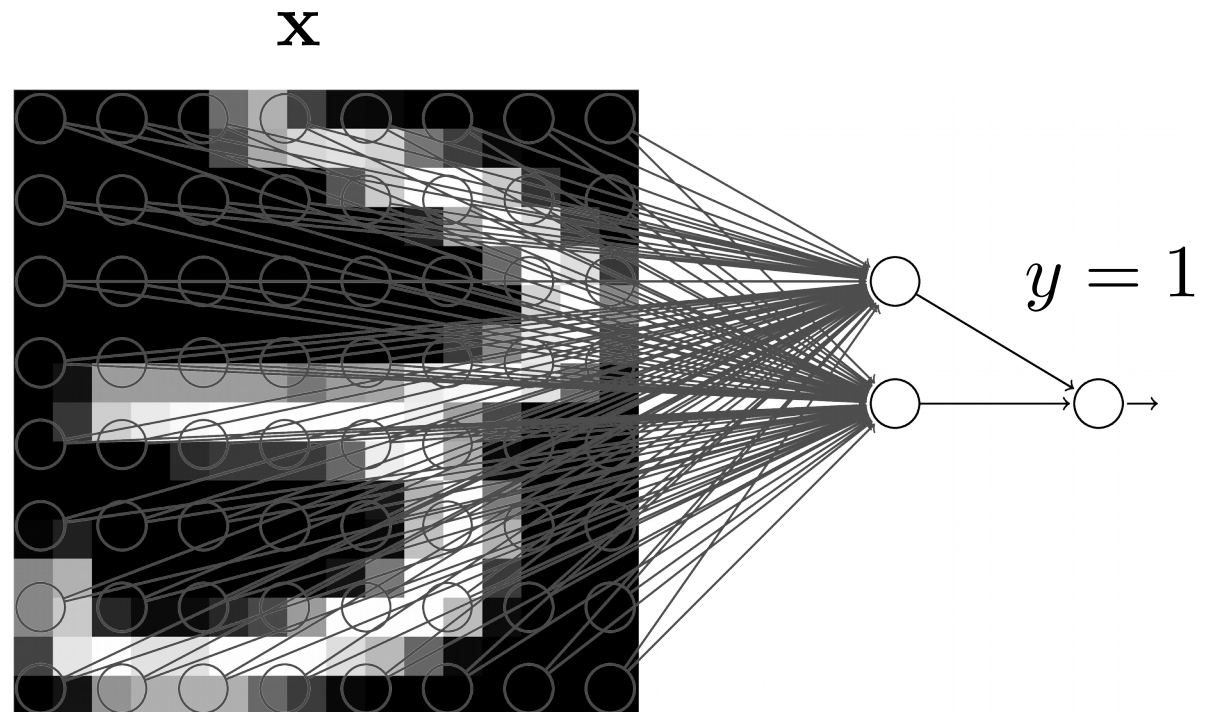


Neural Network Example

Training Data

\mathbf{x}	y
	→ 1
	→ 1
	→ 0
	→ 1
	→ 1
	→ 0
	→ 0
	→ 1
	→ 0
	→ 0
	→ 1
	→ 1
	→ 1
	→ 0
	→ 0
	→ 1
⋮	

Network



Backpropagation

- Activation at the output layer:

$$a_k = o \left(\sum_j w_{j,k}^{(2)} g \left(\sum_i w_{i,j}^{(1)} x_i \right) \right)$$

- Here o is the activation function at the output layer. Units at the input layer are indexed with i , hidden with j and output with k .
- Error metric, assuming multiple output units:

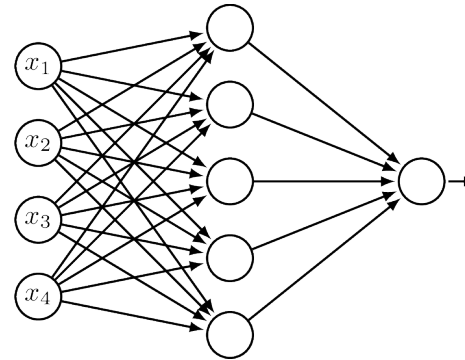
$$Error = \frac{1}{k} \sum_k (y_k - a_k)^2$$

- Now just compute $\frac{\partial Error}{\partial w_{j,k}^{(2)}}$ and $\frac{\partial Error}{\partial w_{i,j}^{(1)}}$.

Backpropagation Algorithm

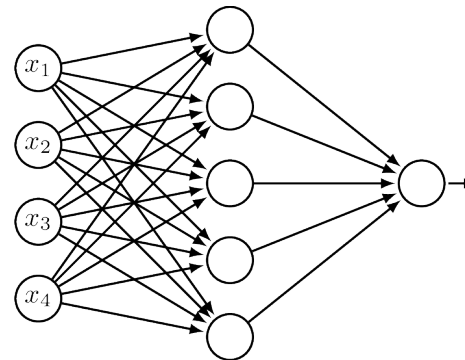
- Forward Pass:

Activation 



- Backward Pass:

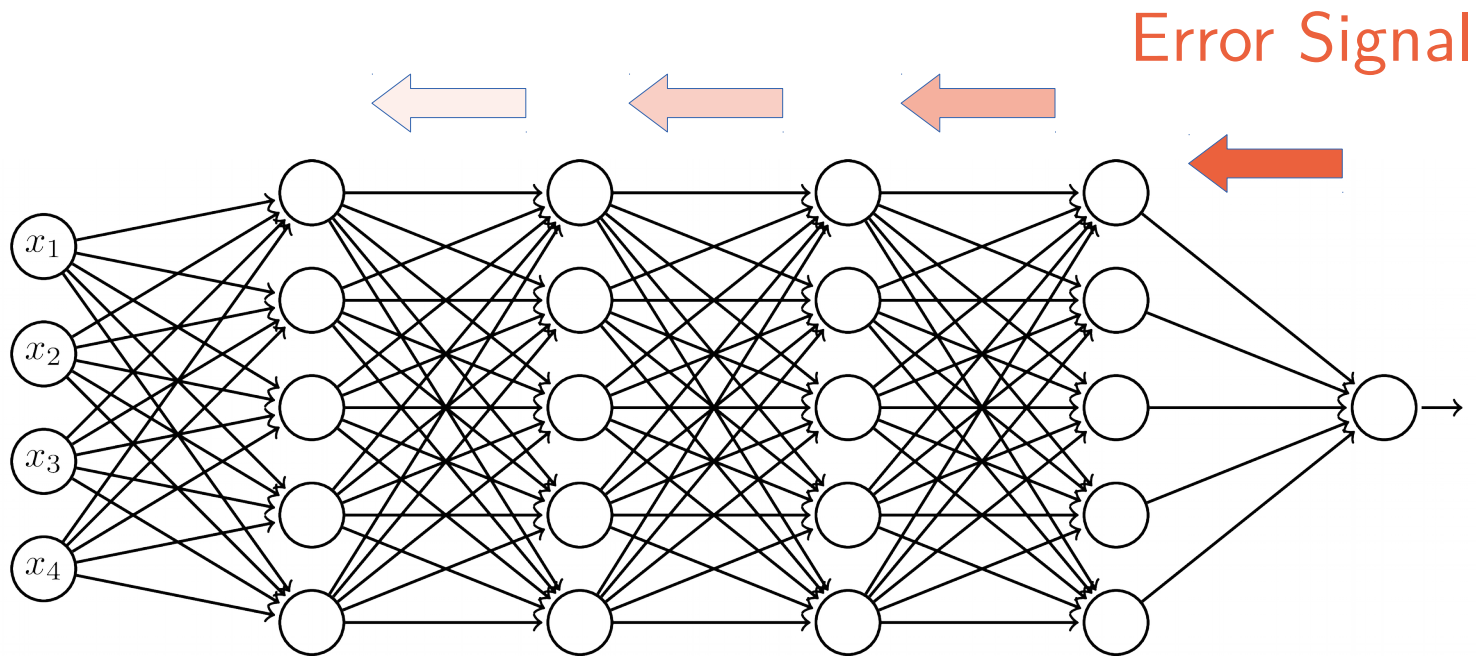
 Error Signal



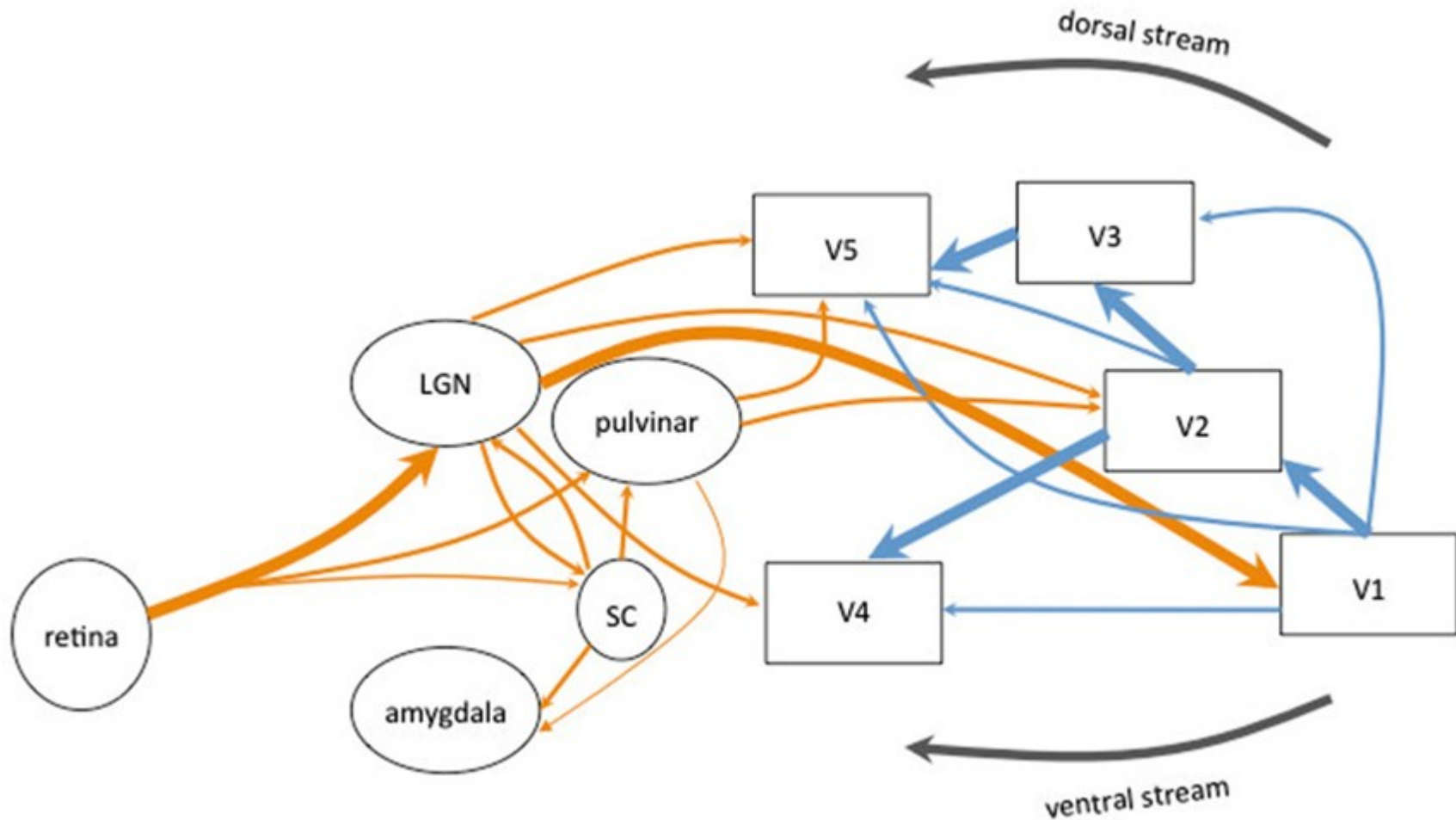
Backpropagation: Some Good News

- Calculating partial derivatives is tedious, but mechanical
- Modern neural network libraries perform **automatic differentiation**
 - Tensorflow
 - PyTorch
- The programmer just needs to specify the network structure and the loss function – No need to explicitly write code for performing weight updates
- The computational cost for the backward pass is not much more than the cost for the forward pass

Vanishing Gradients



Human Visual System



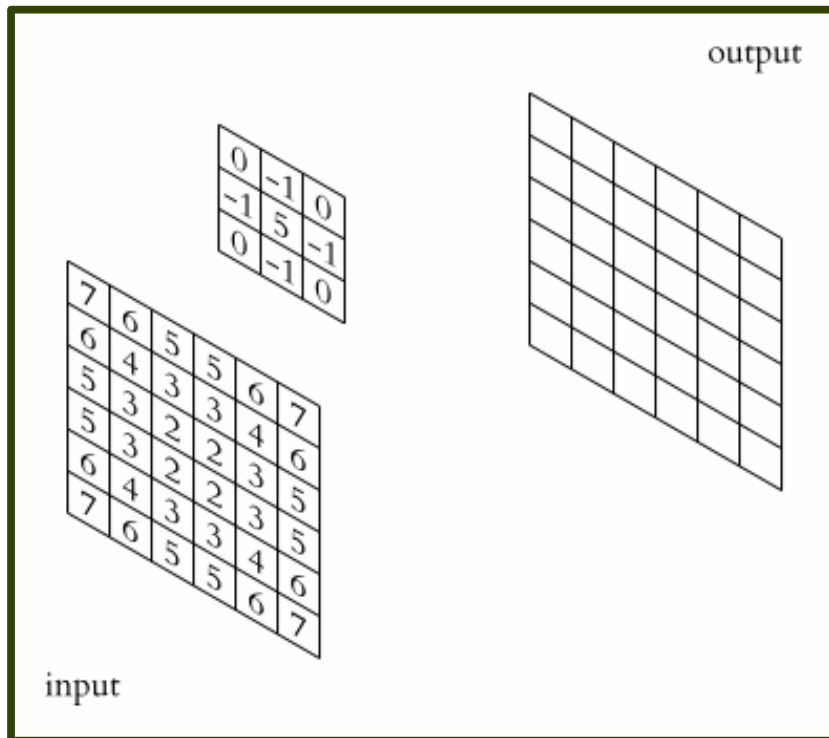
Urbanski, Marika, Olivier A. Coubar, and Clémence Bourlon. "Visualizing the blind brain: brain imaging of visual field defects from early recovery to rehabilitation techniques." *Neurovision: Neural bases of binocular vision and coordination and their implications in visual training programs* (2014).

Convolutional Neural Networks

- Convolutional neural networks use the same trick of learning layers of localized features...
- CNN's were actually being used by Yann Lecun at Bell Labs around 1990

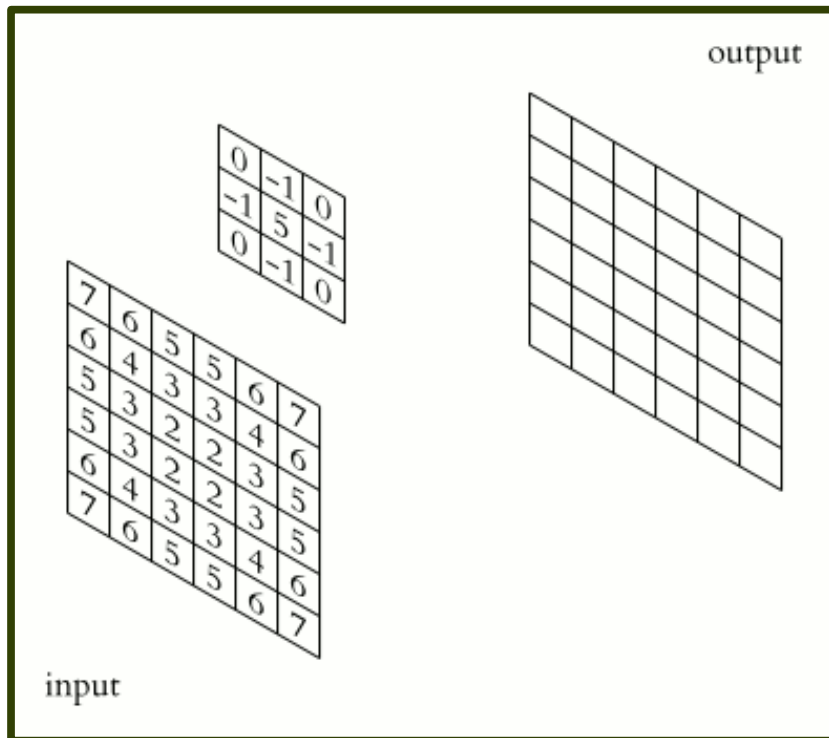
Convolutions

Grayscale Image
1 convolutional filter

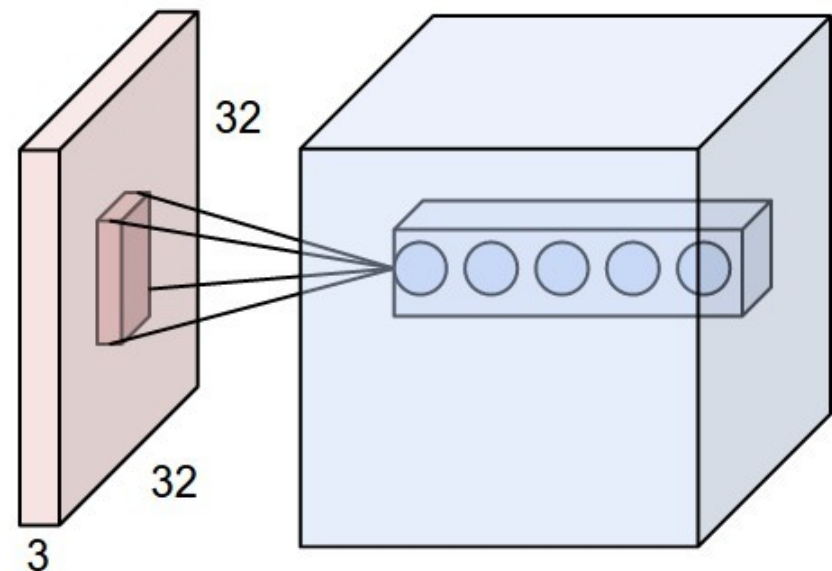


Convolutions

Grayscale Image
1 convolutional filter

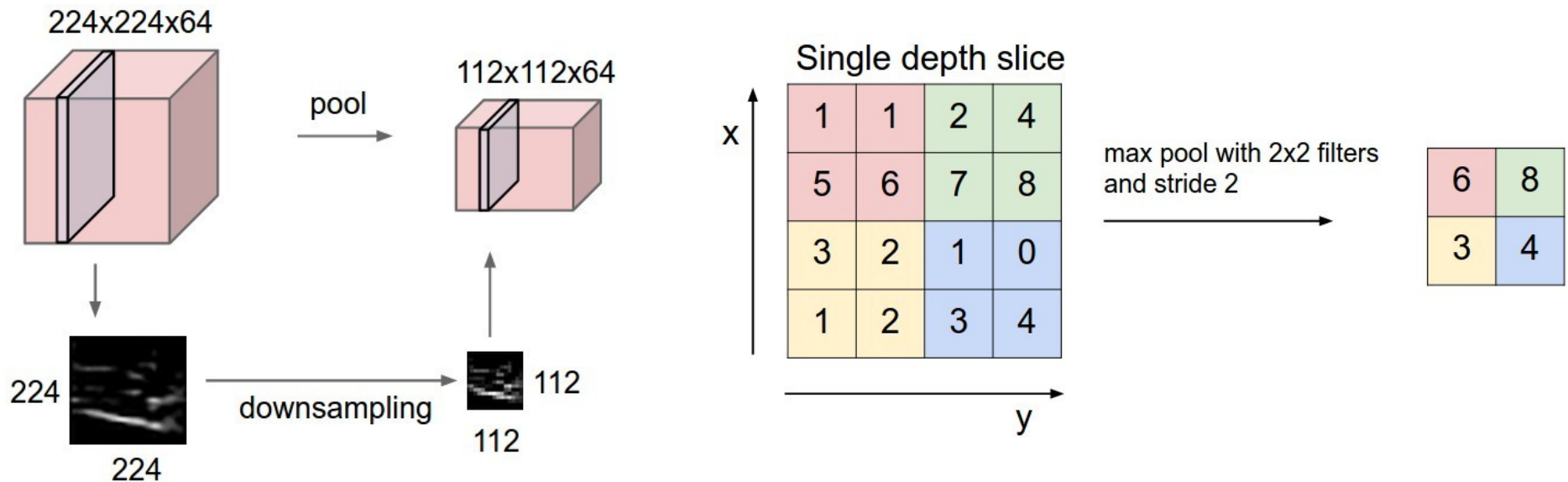


Color Image
5 convolutional filters



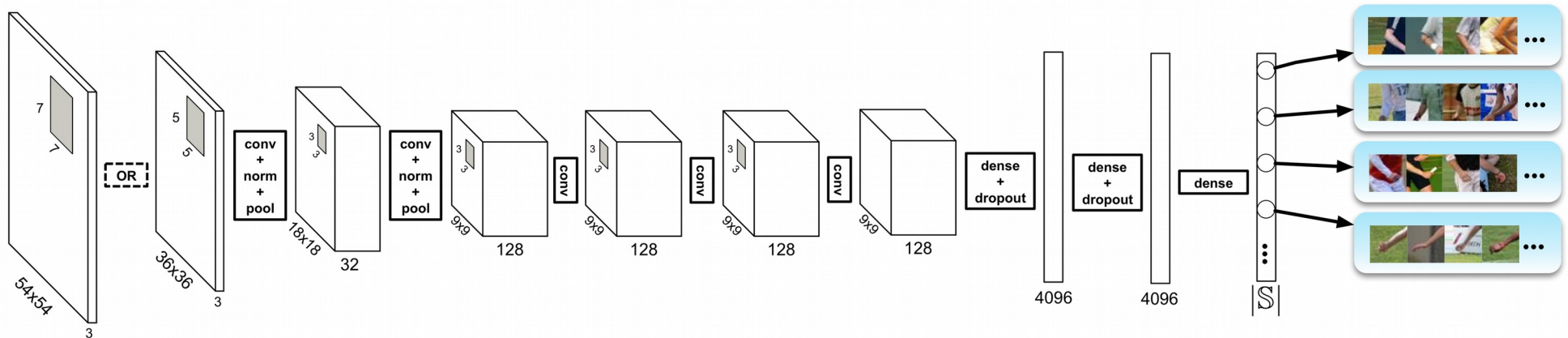
Pooling Layers

- Pooling layers down-sample the filter outputs to
 - Reduce dimensionality and computational requirements
 - Increase the spatial extent of subsequent filters



Complete Network

- A “traditional” CNN is composed of convolutional layers, each followed by non-linearities, followed by pooling layers, with a dense (non-convolutional) layer at the end:



Chen, Xianjie, and Alan L. Yuille. "Articulated pose estimation by a graphical model with image dependent pairwise relations." Advances in Neural Information Processing Systems. 2014.