# Graph Traversal

## Section 4.1–4.2

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Oct 30, 2015

# Reminder

**“Portfolio 7” due in two weeks**

- Submit four new problems (seven total)
- You may replace/improve the other three

**See Port3 feedback in Canvas!**

- Avoid problems from the first three weeks
- Double check formatting, style, spacing, etc.

# LaTeX tips

- Use dollar signs for math mode (use `$n$` for $n$)
- If you want to say $n^{th}$ occurrence: `$n^{th}$`
- Use tilde for abbreviations (e.g., `Dr.~Mayfield`)
- Add `tabsize=4` to lstdefinestyle (or use spaces)
- Use `--` for – (n-dash) and `---` for — (m-dash)
- Curvy quotes are ``different'' on the left/right
- `lstlisting style` should match the language
- Don't forget to change the date (on first page)

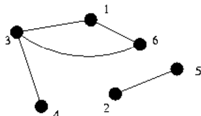
# Section 4.1–4.2

The following slides are by Steven Halim

https://sites.google.com/site/stevenhalim/home/material

# Graph Terms – Quick Review

- Vertices/Nodes
- Edges
- Un/Weighted
- Un/Directed
- In/Out Degree
- Self-Loop/Multiple Edges (Multigraph) vs Simple Graph
- Sparse/Dense
- Path, Cycle
- Isolated, Reachable
- (Strongly) Connected Component
- Sub Graph
- Complete Graph
- Directed Acyclic Graph
- Tree/Forest
- Euler/Hamiltonian Path/Cycle
- Bipartite Graph



Depth-First Search (DFS)

Breadth-First Search (BFS)

- Reachability
- Finding Connected Components
- Flood Fill
- Topological Sort
- Finding Cycles (Back Edges)
- Finding Articulation Points & Bridges
- Finding Strongly Connected Components

# GRAPH TRAVERSAL ALGORITHMS

# Graph Traversal Algorithms

- Given a graph, we want to <u>traverse</u> it!
- There are 2 major ways:
  - Depth First Search (DFS)
    - Usually implemented using recursion
    - More natural
    - Most frequently used to traverse a graph
  - Breadth First Search (BFS)
    - Usually implemented using queue (+ map), use STL
    - Can solve special case* of "shortest paths" problem!

# Depth First Search – Template

- $O(V + E)$ if using Adjacency List
- $O(V^2)$ if using Adjacency Matrix

```
typedef pair<int, int> ii; typedef vector<ii> vi;

void dfs(int u) { // DFS for normal usage
  printf(" %d", u); // this vertex is visited
  dfs_num[u] = DFS_BLACK; // mark as visited
  for (int j = 0; j < (int)AdjList[u].size; j++) {
    ii v = AdjList[u][j]; // try all neighbors v of vertex u
    if (dfs_num[v.first] == DFS_WHITE) // avoid cycle
      dfs(v.first); // v is a (neighbor, weight) pair
  }
}
```

# Breadth First Search (using STL)

- Complexity: also O(V + E) using Adjacency List

```
map<int, int> dist; dist[source] = 0;
queue<int> q; q.push(source); // start from source

while (!q.empty()) {
  int u = q.front(); q.pop(); // queue: layer by layer!
  for (int j = 0; j < (int)AdjList[u].size(); j++) {
    ii v = AdjList[u][j]; // for each neighbours of u
    if (!dist.count(v.first)) {
      dist[v.first] = dist[u] + 1; // unvisited + reachable
      q.push(v.first); // enqueue v.first for next steps
    }
  }
}
```

# 1st Application: Connected Components

- DFS (and BFS) can find connected components
  - A call of dfs(**u**) visits only vertices connected to **u**

```
int numComp = 0;
dfs_num.assign(V, DFS_WHITE);
REP (i, 0, V - 1) // for each vertex i in [0..V-1]
  if (dfs_num[i] == DFS_WHITE) { // if not visited yet
    printf("Component %d, visit", ++numComp);
    dfs(i); // one component found
    printf("\n");
  }
printf("There are %d connected components\n", numComp);
```

# Graph Traversal Comparison

- DFS
- Pros:
  - Slightly easier to code
  - Use less memory
- Cons:
  - Cannot solve SSSP on unweighted graphs

- BFS
- Pros:
  - Can solve SSSP on unweighted graphs (discussed later)
- Cons:
  - Slightly longer to code
  - Use more memory

BFS (unweighted)

Dijkstra's (non –ve cycle)

Bellman Ford's (may have –ve cycle), not discussed

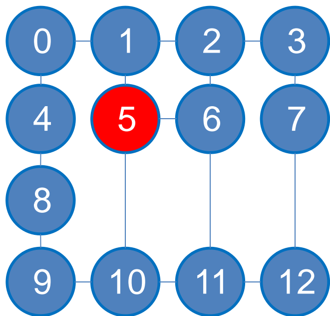
Floyd Warshall's (all-pairs

# SHORTEST PATHS

# BFS for **Special Case** SSSP

- SSSP is a classical problem in Graph theory:
  - Find shortest paths from **one source** to the rest^
- Special case: UVa 336 (A Node Too Far)
- Problem Description:
  - Given an **un-weighted** & un-directed Graph, a starting vertex ***v***, and an integer TTL
  - Check how many nodes are un-reachable from ***v*** or has distance > TTL from ***v***
    - i.e. length(**shortest_path**(v, node)) > TTL

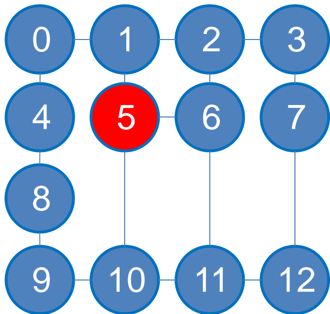
# Example (1)

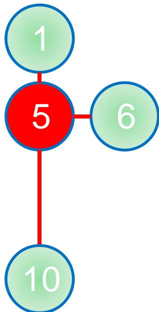



Q = {5}

D[5] = 0

# Example (2)

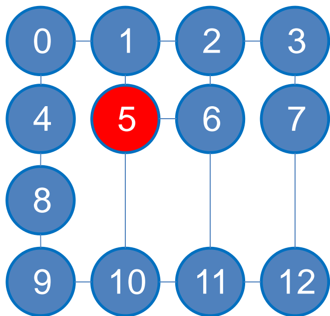


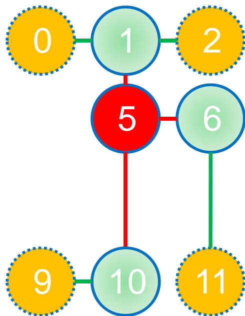




Q = {5}
Q = {1, 6, 10}

D[5] = 0
D[1] = D[5] + 1 = 1
D[6] = D[5] + 1 = 1
D[10] = D[5] + 1 = 1

# Example (3)

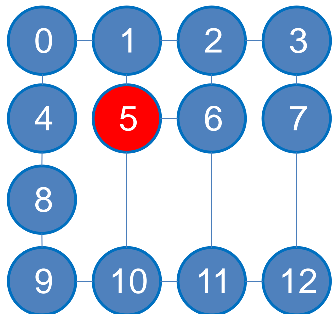


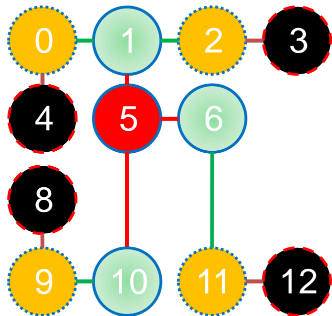




Q = {5}
Q = {1, 6, 10}
Q = {6, 10, **0, 2**}
Q = {10, 0, 2, **11**}
Q = {0, 2, 11, **9**}

D[5] = 0
D[1] = D[5] + 1 = 1
D[6] = D[5] + 1 = 1
D[10] = D[5] + 1 = 1
D[0] = D[1] + 1 = 2
D[2] = D[1] + 1 = 2
D[11] = D[6] + 1 = 2
D[9] = D[10] + 1 = 2

# Example (4)

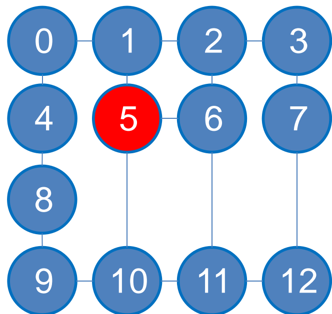


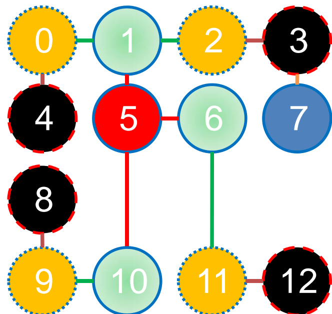




Q = {5}
Q = {1, 6, 10}
Q = {6, 10, **0, 2**}
Q = {10, 0, 2, **11**}
Q = {0, 2, 11, **9**}
Q = {2, 11, 9, **4**}
Q = {11, 9, 4, **3**}
Q = {9, 4, 3, **12**}
Q = {4, 3, 12, **8**}

D[5] = 0
D[1] = D[5] + 1 = 1
D[6] = D[5] + 1 = 1
D[10] = D[5] + 1 = 1
D[0] = D[1] + 1 = 2
D[2] = D[1] + 1 = 2
D[11] = D[6] + 1 = 2
D[9] = D[10] + 1 = 2
D[4] = D[0] + 1 = 3
D[3] = D[2] + 1 = 3
D[12] = D[11] + 1 = 3
D[8] = D[9] + 1 = 3

# Example (5)




Q = {5}
Q = {1, 6, 10}
Q = {6, 10, **0, 2**}
Q = {10, 0, 2, **11**}
Q = {0, 2, 11, **9**}
Q = {2, 11, 9, **4**}
Q = {11, 9, 4, **3**}
Q = {9, 4, 3, **12**}
Q = {4, 3, 12, **8**}
Q = {3, 12, 8}
Q = {12, 8, **7**}
Q = {8, 7}
Q = {7}
Q = {}

D[5] = 0
D[1] = D[5] + 1 = 1
D[6] = D[5] + 1 = 1
D[10] = D[5] + 1 = 1
D[0] = D[1] + 1 = 2
D[2] = D[1] + 1 = 2
D[11] = D[6] + 1 = 2
D[9] = D[10] + 1 = 2
D[4] = D[0] + 1 = 3
D[3] = D[2] + 1 = 3
D[12] = D[11] + 1 = 3
D[8] = D[9] + 1 = 3
D[7] = D[3] + 1 = 4

This is the **BFS = SSSP** ☺ **spanning tree** when BFS is started from vertex 5