Graphs and Trees

Section 2.4

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acm International Collegiate Programming Contest

IBM event sponsor

Due today!

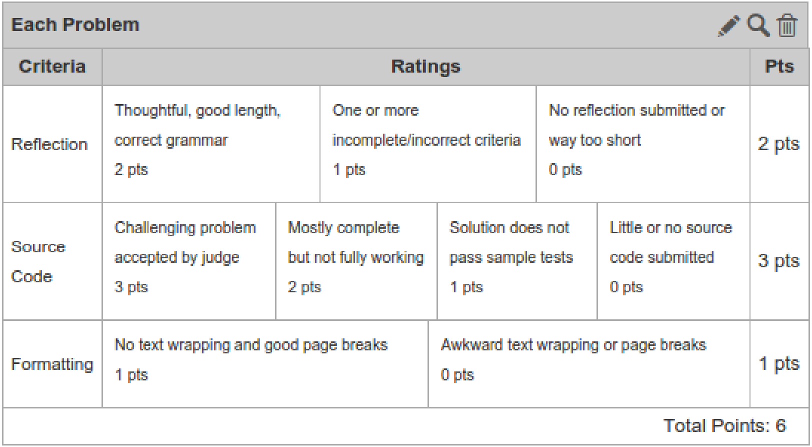
Your portfolio must be done in LaTeX

- See `portfolio.tex` in Template Files
  - In Texmaker, press F1 *twice* to build
  - The rest should be self-explanatory :)
- Write your name, email, today's date
- Fill in the table on Page 2 (rows 1–3)

For each problem you submit:

- Write 1–2 paragraphs of reflection
- Format your code neatly for printing
  - There should be no text wrapping
  - Double check the page boundaries

Portfolio rubric



Each Problem

Criteria	Ratings				Pts
Reflection	Thoughtful, good length, correct grammar 2 pts	One or more incomplete/incorrect criteria 1 pts	No reflection submitted or way too short 0 pts		2 pts
Source Code	Challenging problem accepted by judge 3 pts	Mostly complete but not fully working 2 pts	Solution does not pass sample tests 1 pts	Little or no source code submitted 0 pts	3 pts
Formatting	No text wrapping and good page breaks 1 pts	Awkward text wrapping or page breaks 0 pts			1 pts
					Total Points: 6

- Note: 3 problems = 18 points possible

Today's Problems

What is a graph?

Graphs 101

Collection of nodes (vertices) and edges (links)

http://en.wikipedia.org/wiki/Graph_(mathematics)

- Can be *directed* or *undirected*
- Can be *weighted* or *unweighted*

Some examples

- Driving directions (cities and roads)
- Social networks (people and friendships)
- The Internet (hosts and connections)
- World Wide Web (URLs and hyperlinks)

Representing graphs

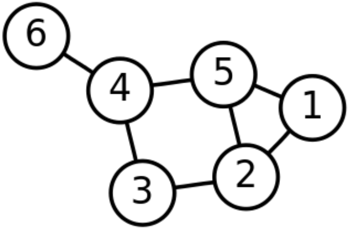
Check out http://visualgo.net/

- Started in 2011 by Dr. Steven Halim
- "VisuAlgo is like a 24/7 copy of himself."

visualising data structures and algorithms through animation

See Graph Data Structures

- Switch to "Tutorial Mode"



Adjacency matrix (easier)

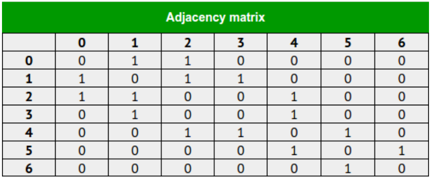
2D array of integers

- 0=no edge, 1=edge
- Or other values (if *weighted*)
- Symmetric if undirected

```
// V is the number of vertices
int AdjMat[V][V];                   // C++
int[][] AdjMat = new int[V][V];     // Java
```

Good for small, dense graphs

- small: $V < 1000$
- dense: not many zeros



Adjacency matrix

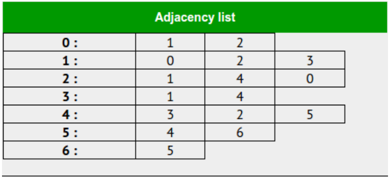
	0	1	2	3	4	5	6
0	0	1	1	0	0	0	0
1	1	0	1	1	0	0	0
2	1	1	0	0	1	0	0
3	0	1	0	0	1	0	0
4	0	0	1	1	0	1	0
5	0	0	0	0	1	0	1
6	0	0	0	0	0	1	0

Adjacency list (more useful)

Vector of vector of ints

- Stores a list of neighbors

```
// unweighted graph
typedef vector<int> vi;
vector<vi> AdjList;
```



Adjacency list

0 :	1	2	
1 :	0	2	3
2 :	1	4	0
3 :	1	4	
4 :	3	2	5
5 :	4	6	
6 :	5		

Or, vector of vector of pairs

- Stores neighbors and weights

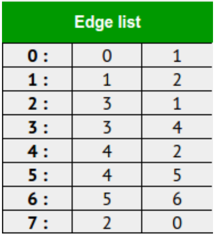
```
// weighted graph
typedef pair<int, int> ii;
typedef vector<ii> vii;
vector<vii> AdjList;
```

Edge list (uncommon)

Sorted list of edges (useful for some algorithms)

```
// unweighted
typedef pair<int, int> ii;
vector<ii> EdgeList;

// weighted
vector< pair<int, ii> > EdgeList;
```



Edge list		
0 :	0	1
1 :	1	2
2 :	3	1
3 :	3	4
4 :	4	2
5 :	4	5
6 :	5	6
7 :	2	0

C++ tip

- nested templates can't use << or >>
- you need a space for it to compile

Adjacency list (more useful)

In Java, you probably should just write some simple wrapper classes and be prepared to use them.

- Examples at Open Data Structures

One issue: Java doesn't have built-in pairs. Possible work-around:

```java
public class Pair<F, S> {
    public final F first;
    public final S second;

    public Pair(F first, S second) {
        this.first = first;
        this.second = second;
    }
}
```

Today's Problems

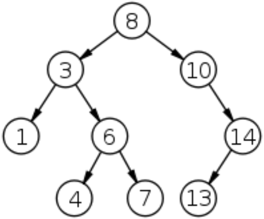
What is a tree?

Trees 101

Graph with no cycles / one path between any two nodes

http://en.wikipedia.org/wiki/Tree_(graph_theory)

A **binary** tree contains nodes with a maximum of two children (left and right).



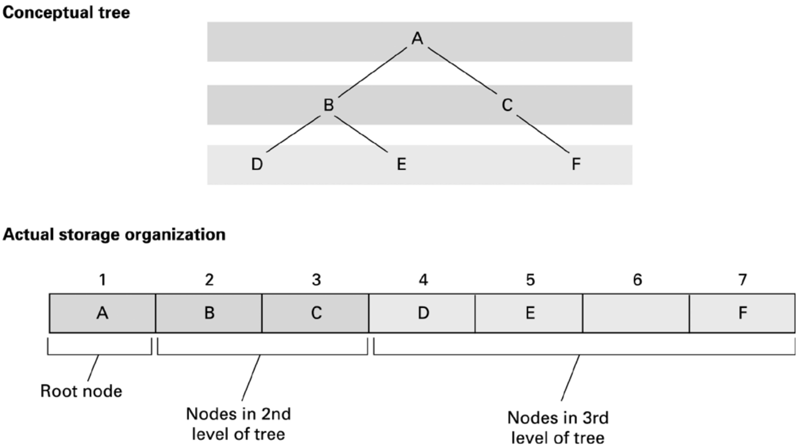
Implementation

Traditional binary tree implementation that we won't use:

```
class Node
{
public:
    int value;
    Node *left;
    Node *right;
};

int main()
{
    Node *root = new Node();
    root->value = 8;
    root->left = NULL;
    root->right = NULL;
}
```

Figure 8.17 **A tree stored without pointers**



Conceptual tree
A
B
C
D
E
F
Actual storage organization

1	2	3	4	5	6	7
A	B	C	D	E		F

Root node
Nodes in 2nd level of tree
Nodes in 3rd level of tree

Binary tree

Store the root at index 1

```
// n is index of a node
#define left(n)   ((n)*2)           // left subchild
#define right(n)  ((n)*2+1)         // right subchild
#define parent(n) ((n)/2)           // parent

// array of max nodes
int tree[MAX] = {0};
```

Ideas for today's contest

See the book for more details!

- Union-Find Disjoint Sets
    - UVa 11503 Virtual Friends
- Segment Tree
    - UVa 11235 Frequent Values

Implicit structures

Some graphs don't need to be generated

- Navigating a 2D grid (e.g., chessboard)
- Determine edges with simple rules, e.g.:
    - (1,2), (2,3), . . . , (n-1, n), (n, 1)
    - All (u, v) such that $u + v$ is prime

Some trees don't need to be generated

- UVa 11350 Stern-Brocot Tree
- Construct new nodes as you go