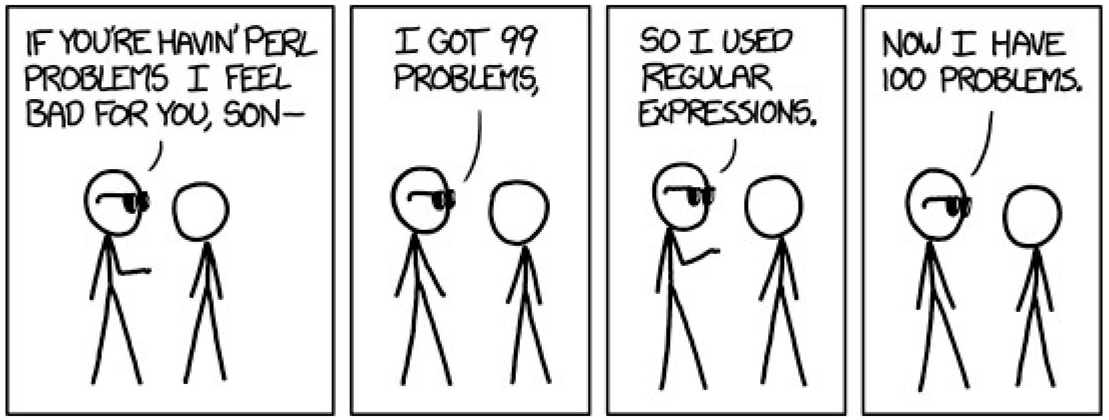
CS 432
Fall 2025

Mike Lam, Professor


IF YOU'RE HAVIN' PERL PROBLEMS I FEEL BAD FOR YOU, SON—
I GOT 99 PROBLEMS,
SO I USED REGULAR EXPRESSIONS.
NOW I HAVE 100 PROBLEMS.

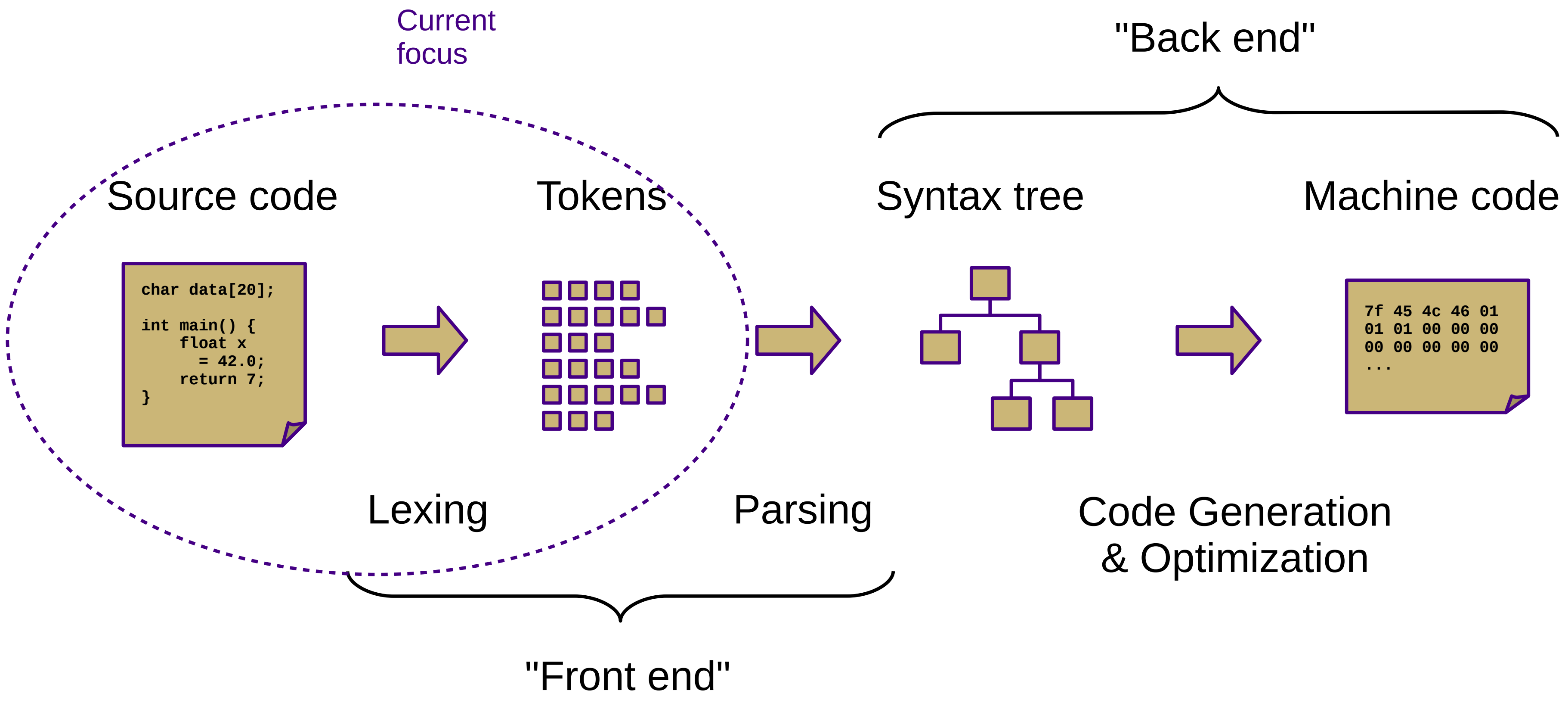
https://xkcd.com/1171/

`a|(bc)*`

$$\begin{tikzcd}
\arrow[r] & S_1 \arrow[loop above, "1"] \arrow[r, bend right, "0"'] & S_2 \arrow[loop above, "1"] \arrow[l, bend right, "0"']
\end{tikzcd}$$

Regular Expressions
and
Finite Automata

Compilation



Lexical Analysis

- Lexemes or tokens: the smallest building blocks of a language's syntax
- Lexing or scanning: the process of separating a character stream into tokens

`total = sum(vals) / n`

`total`	`identifier`
`=`	`equals_op`
`sum`	`identifier`
`(`	`left_paren`
`vals`	`identifier`
`)`	`right_paren`
`/`	`divide_op`
`n`	`identifier`

`char *str = "hi";`

`char`	`keyword`
`*`	`star_op`
`str`	`identifier`
`=`	`equals_op`
`"hi"`	`str_literal`
`;`	`semicolon`

Discussion question

- What is a *language*?

Language

- A language is "a (potentially infinite) set of strings over a finite alphabet"

Discussion question

- How do we describe languages?

xyy
xy
xyyzzz
xyz
xyzz
xyyzz
xyyz
xyzzz
(etc.)

xy
xyy
xyz
xyyz
xyzz
xyyzz
xyzzz
xyyzzz
(etc.)

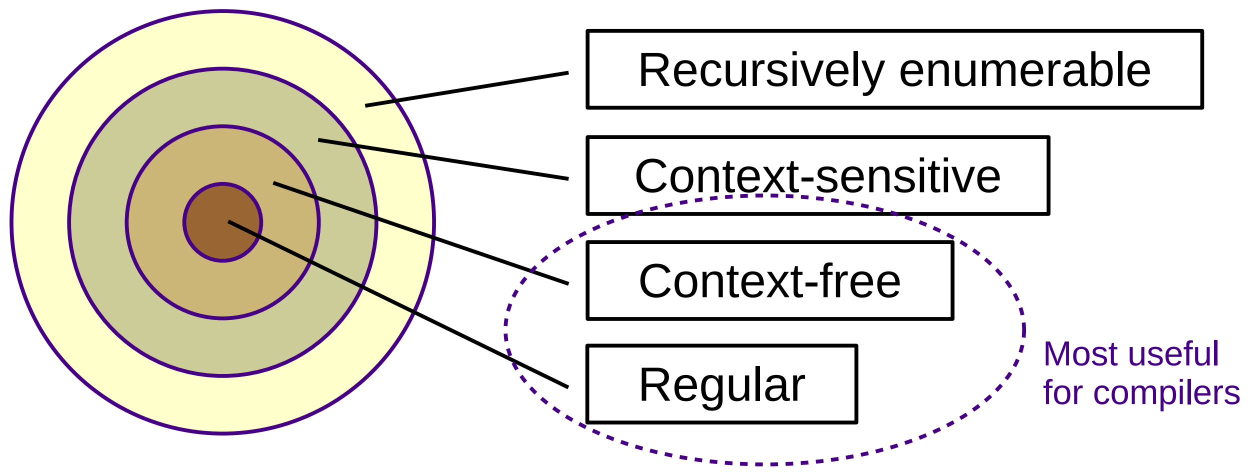
xy	**xyy**
xyz	**xyyz**
xyzz	**xyyzz**
xyzzz	**xyyzzz**
(etc.)	

Language description

- Ways to describe languages
  - Ad-hoc prose
    - “A single ‘x’ followed by one or two ‘y’s followed by any number of ‘z’s”
  - Formal regular expressions (current focus)
    - x(y|yy)z*
  - Formal grammars (in two weeks)
    - $A \rightarrow x\ B\ C$
    - $B \rightarrow y \mid y\ y$
    - $C \rightarrow z\ C \mid \varepsilon$

Languages

Chomsky Hierarchy of Languages



- Alphabet:
  - Σ = { finite set of all characters }
- Language:
  - L = { potentially infinite set of sequences of characters from Σ }

Aside: Alphabets

- In this class, we will mostly stick to ASCII characters
  - This is mostly for simplicity
  - However, it is a very American-focused character set
  - (It's in the name! American Standard Code for Information Interchange)
- Many modern languages support Unicode for variable and function names
  - This covers most major world alphabets
  - But keywords and core library function names are usually in English, creating barriers for non-native English speakers
  - If you are interested in an alternative, check out Hedy, a multi-lingual programming language

print hello
打印 你好
قول مرحبا

Regular expressions

- Regular expressions describe regular languages
  - Can also be thought of as generalized search patterns
- Three basic recursive operations:
  - Alternation: **A|B** — Lowest precedence
  - Concatenation: **AB** *or* **A␣B**
  - ("Kleene") Closure: **A*** — Highest precedence

Additionally: ε is a regex that matches the empty string

- Extended constructs:
  - Character sets/classes: **[0-9] ≡ [0...9] ≡ 0|1|2|3|4|5|6|7|8|9**
  - Repetition / positive closure: **A^2 ≡ AA** **A^3 ≡ AAA** **A^+ ≡ AA***
  - Grouping: **(A|B)C ≡ AC|BC**

These are not covered extensively in your textbook!

Regular expressions

- Symbols with special meaning in regular expressions must be "escaped" to match the actual symbol
  - E.g., **a\*** matches an "a" followed by an asterisk ("*")
  - This is not usually necessary inside a character class
    - E.g., **a[*]** ≡ **a\***
- Alternation of character classes can be condensed
  - E.g., **[a-z]|[A-Z]** ≡ **[a-zA-Z]**
- Starting a character class with a caret ("^") forms the complement
  - E.g., **[^abc]** matches any character that is **NOT** "a", "b", or "c"
  - Outside a character class, **^** matches the beginning of a string and **$** matches the end of a string (this is not the case in your textbook)

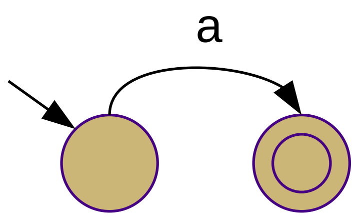
Discussion question

- How would you implement regular expressions?
  - Given a regular expression and a string, how would you tell whether the string belongs to the language described by the regular expression?

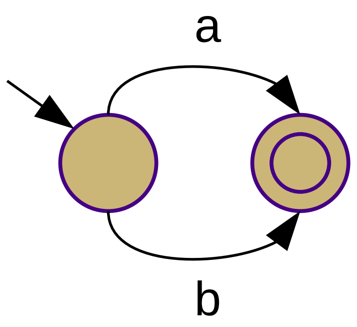
Lexical Analysis

- Implemented using state machines (finite automata)
  - Set of states with a single start state
  - Transitions between states on inputs (w/ implicit dead states)
  - Some states are final or accepting

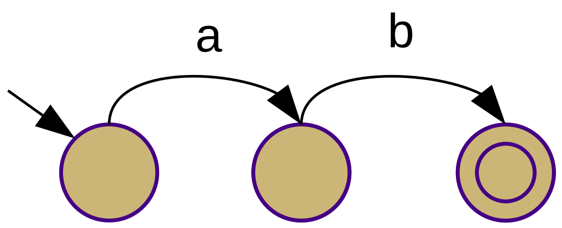
a



a|b

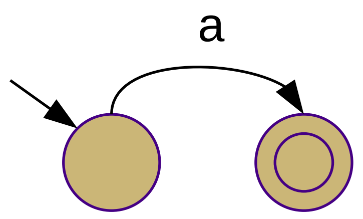


ab

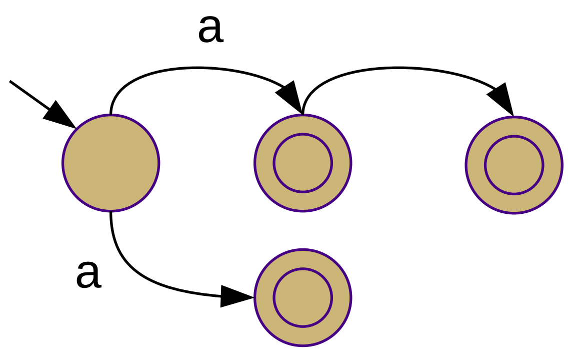


Lexical Analysis

- Deterministic vs. non-deterministic
  - Non-deterministic: multiple possible states for given sequence
  - One edge from each state per character (deterministic)
    - Might lead to implicit “dead state” w/ self-loop on all characters
  - Multiple edges from each state per character (non-deterministic)
  - “Empty” or ε-transitions (non-deterministic)



Deterministic (DFA)



Non-deterministic (NFA)

Deterministic finite automata

- Formal definition

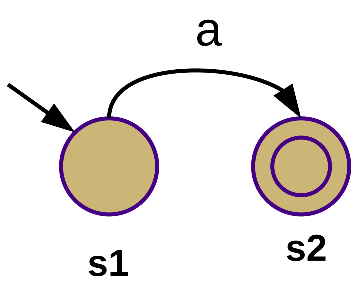
  S: set of states

  Σ: alphabet (set of characters)

  δ: transition function: (S, Σ) → S

  s_0: start state

  S_A: accepting/final states



$S = \{ s1, s2 \}$
$\Sigma = \{ a \}$
$\delta = \{ (s1, a \to s2), (s2, a \to \emptyset) \}$
$s_0 = s1$
$S_A = \{ s2 \}$

- Acceptance algorithm

```
$s := s_0$
for each input $c$:
    $s := \delta(s,c)$
return $s \in S_A$
```

Alternative δ representation:

	a
s1	s2
s2	Ø

Non-deterministic finite automata

- Formal Definition
  - S, Σ, s_0, and S_A same as DFA
  - δ: (S, Σ ∪ {ε}) → [S]
  - ε-closure: all states reachable from s via ε-transitions
    - Formally: ε-closure(s) = {s} ∪ { t ∈ S | (s, ε) → t ∈ δ }
    - Extended to sets by union over all states in set

- Acceptance algorithm

```
$T$ := $\varepsilon$-$closure(s_0)$
**for each input** $c$:
    $N$ := {}
    **for each** $s$ **in** $T$:
        $N$ := $N \cup \varepsilon$-$closure(\delta(s,c))$
    $T$ := $N$
**return** $|T \cap S_A| > 0$
```

Summary

DFAs

- S: set of states
- Σ: alphabet (set of characters)
- δ: transition function: (S, Σ) → S
- s_0: start state
- S_A: accepting/final states

accept():

```
$s := s_0$
**for each input** $c$:
    $s := \delta(s,c)$
**return** $s \in S_A$
```

NFAs

- δ may return a set of states
- δ may contain ε-transitions

accept():

```
$T := \varepsilon\text{-}closure(s_0)$
**for each input** $c$:
    $N := \{\}$
    **for each** $s$ **in** $T$:
        $N := N \cup \varepsilon\text{-}closure(\delta(s,c))$
    $T := N$
**return** $|T \cap S_A| > 0$
```

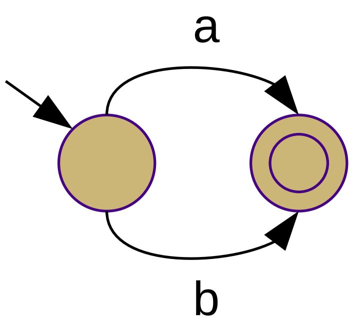
Equivalence

- A regular expression and a finite automaton are equivalent if they recognize the same language
  - Same applies between different REs and between different FAs
- Regular expressions, NFAs, and DFAs all describe the same set of languages
  - "Regular languages" from Chomsky hierarchy
- Next week, we will learn how to convert between them

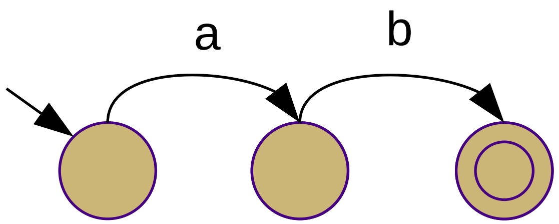
Lexical Analysis

- Examples:

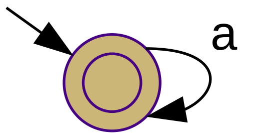
a|b



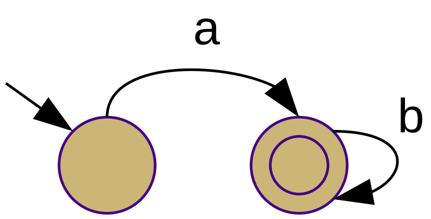
ab



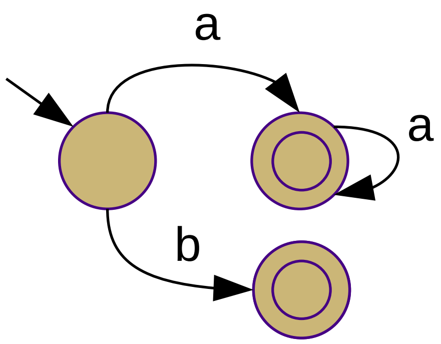
a*



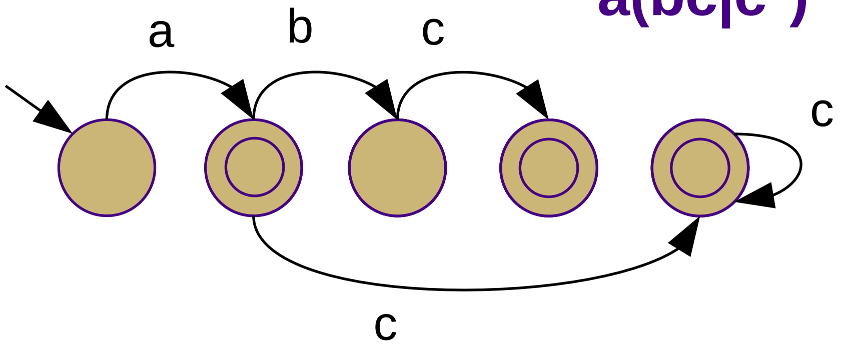
ab*



aa*|b



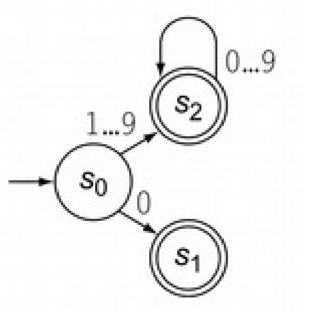
a(bc|c*)



Examples

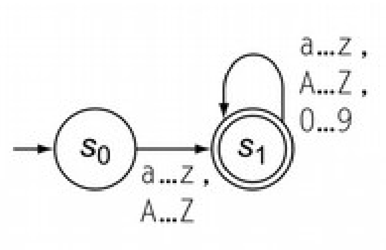
Unsigned integers

$0 \mid [1 \ldots 9]\,[0 \ldots 9]^*$



Identifiers

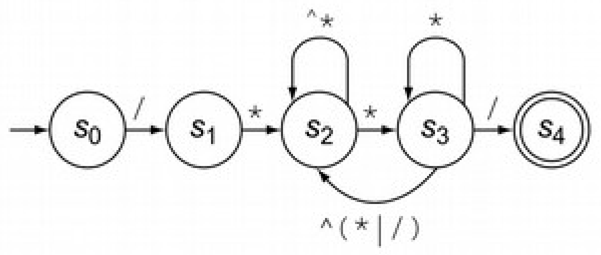
$([A \ldots Z] \mid [a \ldots z])\ ([A \ldots Z] \mid [a \ldots z] \mid [0 \ldots 9])^*$



Multi-line comments

$/ * (\hat{}* \mid *^{+}\,\hat{}/)^*\ */$

uses a caret for negation outside a character class



Exercise

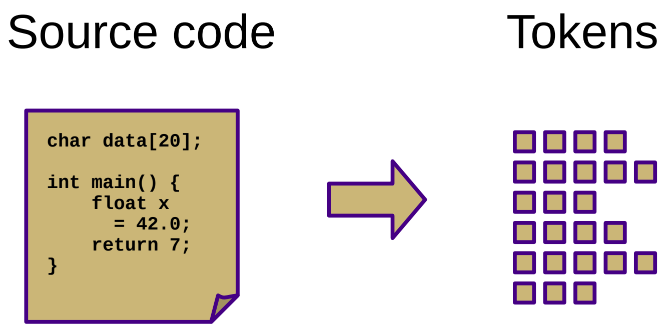
- Construct state machines for the following regular expressions:

x*yz*　　**1(1|0)***　　**1(10)***　　**(a|b|c)(ab|bc)**

(dd*.d*)|(d*.dd*)　　← ε-transitions may make this one slightly easier

Application

- P1: Use POSIX regular expressions to tokenize Decaf files
  - Process the input one line at a time
  - Generally, create one regex per token type
    - Each regex begins with “^” (only match from beginning)
    - Prioritize regexes and try each of them in turn
    - When you find a match, extract the matching text
    - Repeat until no match is found or the input is consumed
  - Less efficient than an auto-generated lexer
    - However, it is simpler to understand
    - Our approach to P2 will be similar



```
char data[20];

int main() {
    float x
      = 42.0;
    return 7;
}
```