CS 432
Fall 2024

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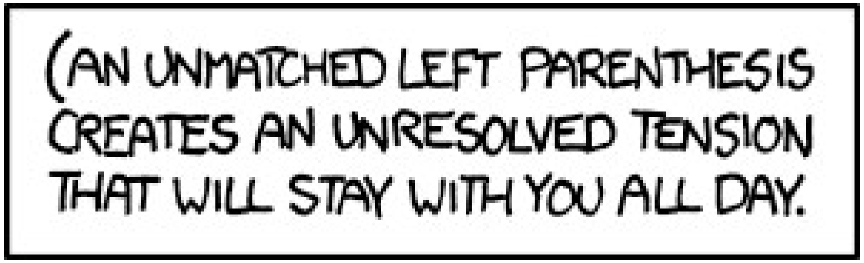
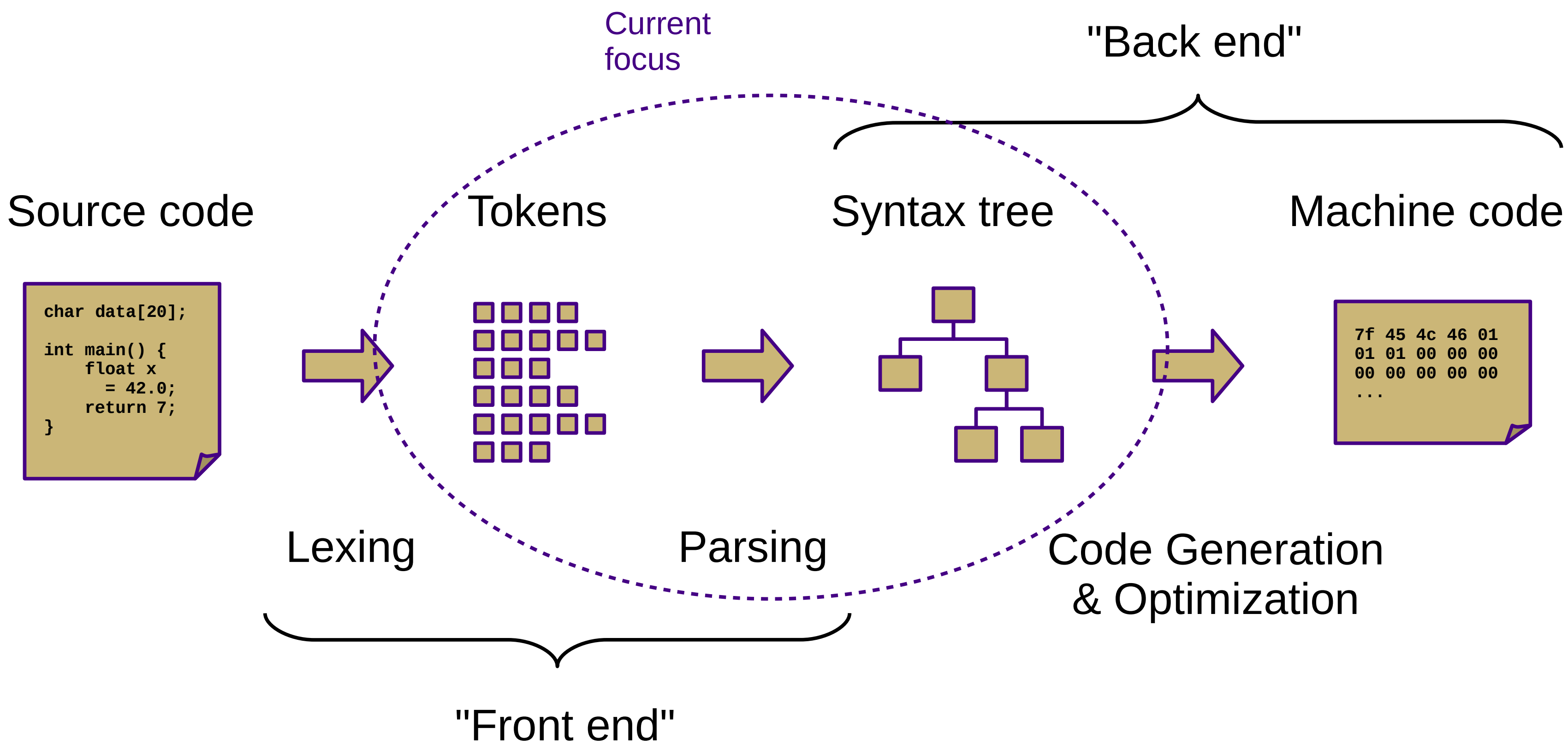


https://xkcd.com/859/

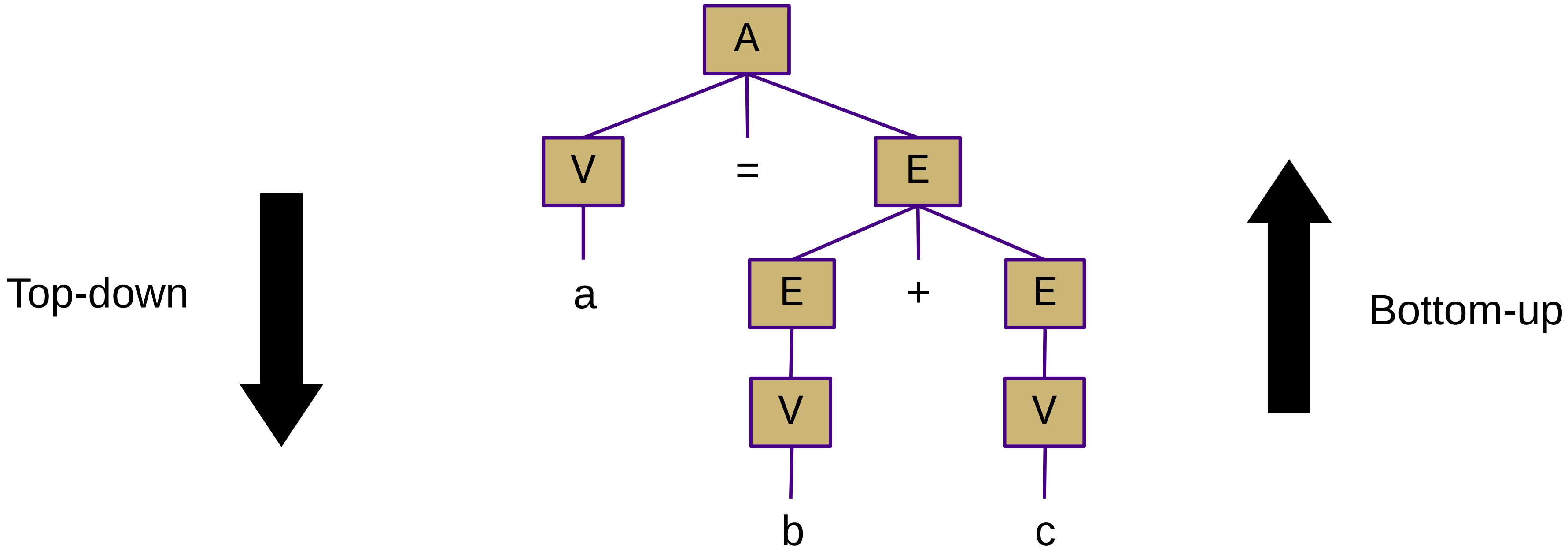
Bottom-Up (LR) Parsing

Compilation



Overview

- Two general parsing approaches
  - Top-down: begin with start symbol (root of parse tree), and gradually expand non-terminals
  - Bottom-up: begin with terminals (leaves of parse tree), and gradually connect using non-terminals



Shift-Reduce Parsing

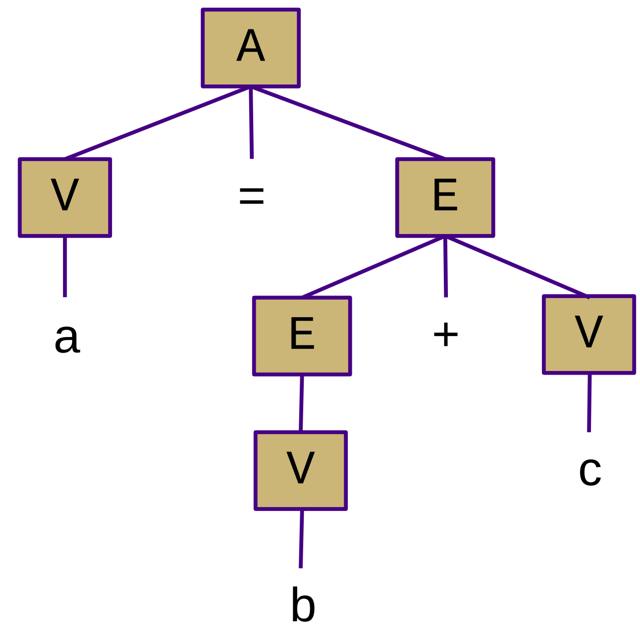
- Top-down (LL) parsers
  - Left-to-right scan, Leftmost derivation
  - Recursive routines, one per non-terminal (*recursive descent*)
  - Implicit stack (system call stack)
  - Requires more restrictive grammars
  - Simpler to understand and possible to hand-code
- Bottom-up (LR) parsers
  - Left-to-right scan, (reverse) Rightmost derivation
  - "Shift"/push terminals and non-terminals onto a stack
  - "Reduce"/pop to replace *handles* with non-terminals
  - Less restrictive grammars
  - Harder to understand and nearly always auto-generated; very efficient!

Shift-Reduce Parsing

-
  - shift 'a'
- $\underline{a}$
  - reduce (V → a)
- V
  - shift '='
- V =
  - shift 'b'
- V = $\underline{b}$
  - reduce (V → b)
- V = $\underline{V}$
  - reduce (E → V)
- V = E
  - shift '+'
- V = E +
  - shift 'c'
- V = E + $\underline{c}$
  - reduce (V → c)
- V = $\underline{E + V}$
  - reduce (E → E + V)
- $\underline{V = E}$
  - reduce (A → V = E)
- A
  - accept

(handles are underlined)

shift = push, reduce = popN



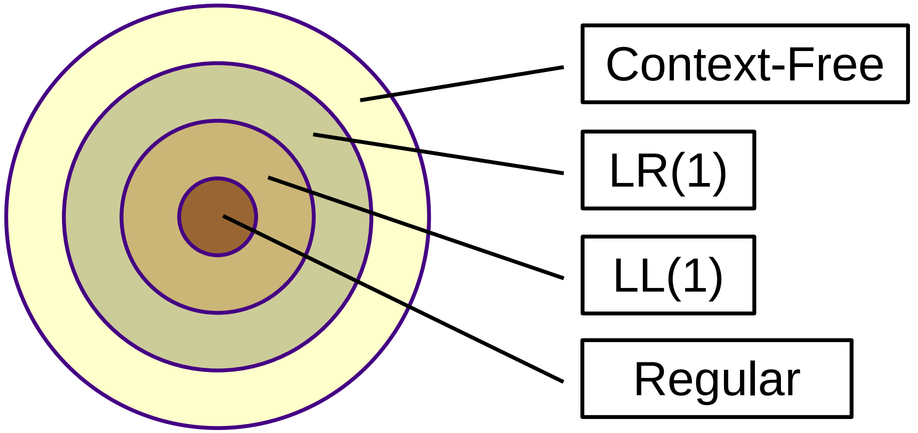
$$
\begin{aligned}
A &\to V = E \\
E &\to E + V \\
  &\mid V \\
V &\to a \mid b \mid c
\end{aligned}
$$

"a = b + c"

LR Parsing

- LR(1) grammars and parsers
  - **L**eft-to-right scan of the input string
  - **R**ightmost derivation
  - **1 symbol** of lookahead
  - Less restricted form of context-free grammar
    - Support for most language features
    - Efficient parsing

Context-Free Hierarchy



LR Parser Variants

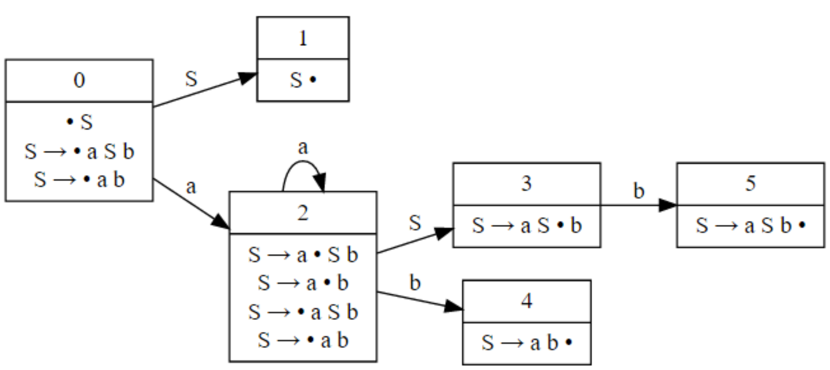
- LR(k) – multiple lookaheads (not necessary)
- LR(1) – single lookahead (*EAC covers this!*)
  - Very general and very powerful
  - Lots of item sets; tedious to construct by hand
  - Overkill for most practical languages
- LALR(1) – special case of LR(1) that merges some states
  - Less powerful, but easier to manage
- **SLR(1)** – special case of LR(1) w/o explicit lookahead (*Dragon book covers this!*)
  - Uses **FOLLOW** sets to disambiguate
  - Even less powerful, but much easier to understand
  - Slightly counterintuitive: all LR(1) languages have SLR(1) grammars
    - So SLR(1) is sufficiently general for our purposes
    - Use LR(0) item sets and generate SLR(1) ACTION/GOTO tables
- LR(0) – no lookahead
  - Severely restricted; most "interesting" grammars aren't LR(0)

LR Parsing

- Creating an LR parser (pushdown automaton)
  - Build item sets from grammar productions
    - An item uses a dot (•) to represent parser status: "A → a • S b"
      - Dots on the left end: "possibilities"
      - Dots in the middle: "partially-complete"
      - Dots on the right end: "complete"
    - Item sets represent multiple parser states (build by taking closure)
      - Similar to NFA state collections in subset construction
  - Build **ACTION** / **GOTO** tables
    - Encodes stack and transition decisions (like δ in FA)
    - **ACTION**(state, terminal) = { *shift/push*, *reduce/pop*, *accept* }
    - **GOTO**(state, non-terminal) = state

LR(0) Item Sets

- LR(0) item sets and automaton
  - Begin with an item representing "• S" or "S' → • S"
    - "S" is the start symbol of the grammar
    - The latter is an augmented grammar
    - The Dragon book uses it; the online tool doesn't
  - Take the closure to add more items if the dot lies immediately to the left of a non-terminal
    - Added items are non-kernel items, denoted here in blue
  - Form new sets by "moving the dot" (and take the closure)
  - Convert to finite automaton for recognizing handles by adding transitions
    - Each set becomes a state
    - "Moving the dot" = state transition + stack push



S → a S b
  | a b

I_0: • S
S → • a S b
S → • a b

I_1: S •

I_2: S → a • S b
S → a • b
S → • a S b
S → • a b

I_3: S → a S • b

I_4: S → a b •

I_5: S → a S b •

SLR(1) Tables

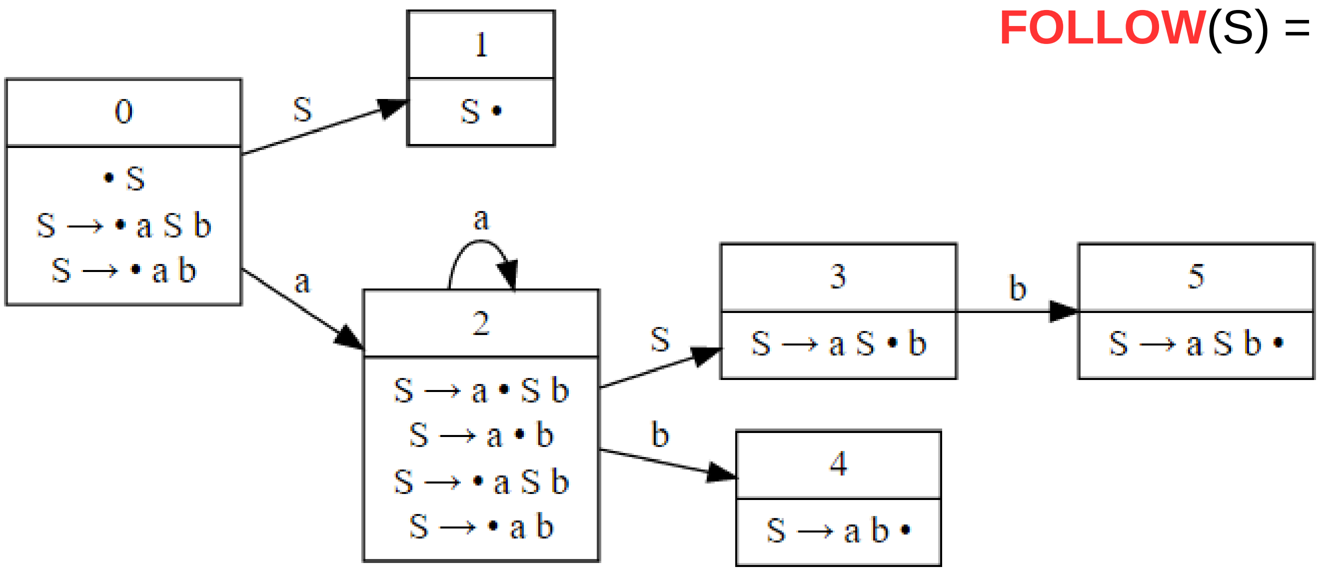
- Create **ACTION** and **GOTO** tables
  - For each item set i
    - If an item matches **A → β • c γ**
      - **ACTION**(i, c) = "shift" to corresponding item set ("move the dot")
    - If an item matches **A → β •**
      - **ACTION**(i, x) = "reduce A → β" for all x in **FOLLOW**(A) (“backtrack in FA”)
    - If an item matches **A → β • B γ**
      - **GOTO**(i, B) = corresponding item set ("move the dot")
  - **ACTION**({S •}, $) = "accept"
  - Any empty **ACTION** entry = “error” (usually left blank)

Example

S → a S b
   | a b

	ACTION			GOTO
State	a	b	$	S
0	shift(2)			1
1			accept	
2	shift(2)	shift(4)		3
3		shift(5)		
4		reduce($S \to$ a b)	reduce($S \to$ a b)	
5		reduce($S \to$ a S b)	reduce($S \to$ a S b)	

FOLLOW(S) = { b, $ }



SLR(1) Parsing

- **Push** state 0 onto the stack
- Repeat until next action is `accept` or `error`:
  - Look up next action in **ACTION** table
    - Row is the current state (top of stack)
    - Column is the next input (terminal or $)
  - If action is `shift(s)`:
    - **Push** state s onto stack
    - Consume one token from input
  - If action is `reduce(`**A → β**`)`:
    - **Pop** one state for each terminal or non-terminal in **β**
    - Look up next state in **GOTO** table and **push** it onto the stack
      - Row is the current state (top of stack, *after* popping **β**)
      - Column is **A** (newly-reduced non-terminal)

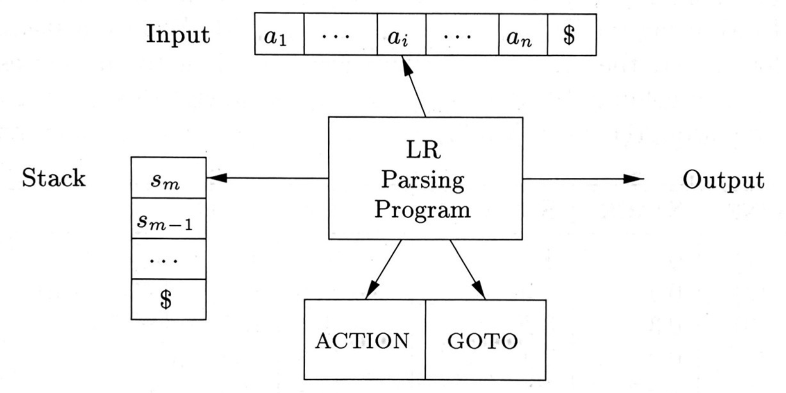


Figure 4.35: Model of an LR parser

Example

S → a S b
  | a b

		ACTION		GOTO
State	a	b	$	S
0	shift(2)			1
1			accept	
2	shift(2)	shift(4)		3
3		shift(5)		
4		reduce($S \rightarrow$ a b)	reduce($S \rightarrow$ a b)	
5		reduce($S \rightarrow$ a S b)	reduce($S \rightarrow$ a S b)	

(Dragon Book version)

Parsing for "a a b b":

Stack	Symbols	Input	Action
$ **0**	$	**a** a b b $	shift(2)
$ 0 **2**	$ a	**a** b b $	shift(2)
$ 0 2 **2**	$ a a	**b** b $	shift(4)
$ 0 2 2 **4**	$ a a b	**b** $	reduce(S → a b)
$ 0 2 **3**	$ a S	**b** $	shift(5)
$ 0 2 3 **5**	$ a S b	**$**	reduce(S → a S b)
$ 0 **1**	$ S	**$**	accept

Example

$S \rightarrow a\ S\ b$
$\quad\ \mid\ a\ b$

	ACTION			GOTO
State	a	b	$	S
0	shift(2)			1
1			accept	
2	shift(2)	shift(4)		3
3		shift(5)		
4		reduce($S \rightarrow$ a b)	reduce($S \rightarrow$ a b)	
5		reduce($S \rightarrow$ a S b)	reduce($S \rightarrow$ a S b)	

Parsing for "a a b b":

(cleaner version w/ goto)

Stack	Input	Action	Goto
0	**a** a b b $	shift(**2**)	-
0 **2**	**a** b b $	shift(**2**)	-
0 2 **2**	**b** b $	shift(**4**)	-
0 2 2 **4**	**b** $	reduce(**S → a b**)	**3**
0 2 **3**	**b** $	shift(**5**)	-
0 2 3 **5**	**$**	reduce(**S → a S b**)	**1**
0 **1**	**$**	accept	

LR Conflicts

- Shift/reduce
  - Can be resolved by always shifting or by grammar modification
- Reduce/reduce
  - Requires grammar modification to fix

```
A -> V = E
E -> E + V
E -> V
V -> a | b | c
```

Shift/reduce conflict in LR(0)

```
A -> x A x
A ->
```

Shift/reduce conflict (all LR)

```
A -> B | C
B -> x
C -> x
```

Reduce/reduce conflict (all LR)

Observation: none of these languages are LL(1) either!

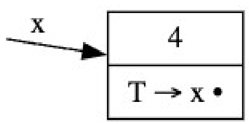
LR examples

- This grammar is LR(0):

  S → a T a

  | a T b

  T → x



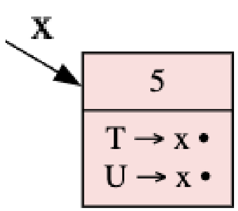
LR(0) Parsing Table

State	a	b	x	$	S	T
0	shift(2)				1	
1	accept	accept	accept	accept		
2			shift(4)			3
3	shift(5)	shift(6)				
4	reduce($T \to$ x)	reduce($T \to$ x)	reduce($T \to$ x)	reduce($T \to$ x)		
5	reduce($S \to$ a T a)	reduce($S \to$ a T a)	reduce($S \to$ a T a)	reduce($S \to$ a T a)		
6	reduce($S \to$ a T b)	reduce($S \to$ a T b)	reduce($S \to$ a T b)	reduce($S \to$ a T b)		

LR examples

- This grammar is SLR(1) but not LR(0):

$$S \rightarrow a\ T\ a$$
$$\quad |\ a\ U\ b$$
$$T \rightarrow x$$
$$U \rightarrow x$$



FOLLOW(T) = { a }
FOLLOW(U) = { b }

LR(0) Parsing Table

State	a	b	x	$	S	T	U
0	shift(2)				1		
1	accept	accept	accept	accept			
2			shift(5)			3	4
3	shift(6)						
4		shift(7)					
5	reduce($T \rightarrow$ x) reduce($U \rightarrow$ x)	reduce($T \rightarrow$ x) reduce($U \rightarrow$ x)	reduce($T \rightarrow$ x) reduce($U \rightarrow$ x)	reduce($T \rightarrow$ x) reduce($U \rightarrow$ x)			
6	reduce($S \rightarrow$ a T a)	reduce($S \rightarrow$ a T a)	reduce($S \rightarrow$ a T a)	reduce($S \rightarrow$ a T a)			
7	reduce($S \rightarrow$ a U b)	reduce($S \rightarrow$ a U b)	reduce($S \rightarrow$ a U b)	reduce($S \rightarrow$ a U b)			

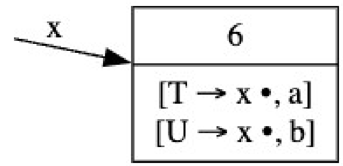
SLR(1) Parsing Table

State	a	b	x	$	S	T	U
0	shift(2)				1		
1				accept			
2			shift(5)			3	4
3	shift(6)						
4		shift(7)					
5	reduce($T \rightarrow$ x)	reduce($U \rightarrow$ x)					
6				reduce($S \rightarrow$ a T a)			
7				reduce($S \rightarrow$ a U b)			

LR examples

- This grammar is LALR(1) but not SLR(1):

S → a T a
  | a U b
  | b T b
T → x
U → x



FOLLOW(T) = { a, b }
FOLLOW(U) = { b }

SLR(1) Parsing Table

State	a	b	x	$	S	T	U
0	shift(2)	shift(3)			1		
1				accept			
2			shift(6)			4	5
3			shift(8)			7	
4	shift(9)						
5		shift(10)					
6	reduce(T → **x**)	reduce(T → **x**) reduce(U → **x**)					
7		shift(11)					
8	reduce(T → **x**)	reduce(T → **x**)					
9				reduce(S → **a** T **a**)			
10				reduce(S → **a** U **b**)			
11				reduce(S → **b** T **b**)			

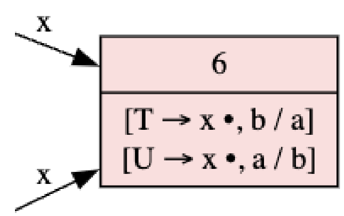
LALR(1) Parsing Table

State	a	b	x	$	S	T	U
0	shift(2)	shift(3)			1		
1				accept			
2			shift(6)			4	5
3			shift(8)			7	
4	shift(9)						
5		shift(10)					
6	reduce(T → **x**)	reduce(U → **x**)					
7		shift(11)					
8		reduce(T → **x**)					
9				reduce(S → **a** T **a**)			
10				reduce(S → **a** U **b**)			
11				reduce(S → **b** T **b**)			

LR examples

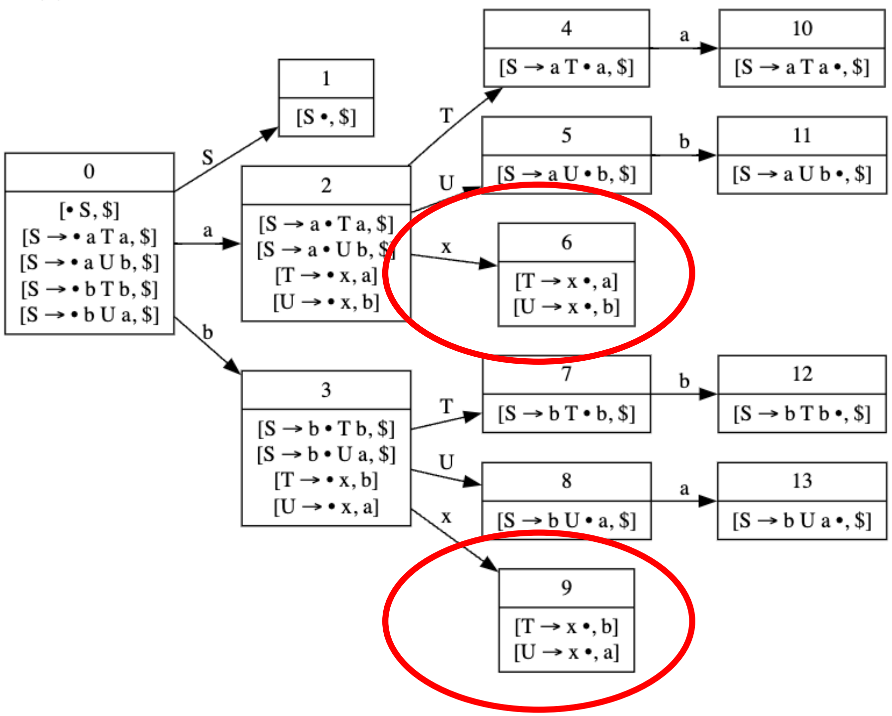
- This grammar is LR(1) but not LALR(1):

S → a T a
  | a U b
  | b T b
  | b U a
T → x
U → x



FOLLOW(T) = { a, b }
FOLLOW(U) = { a, b }

LR(1) Automaton



LR(1) Parsing Table

...

6	reduce($T \to \mathbf{x}$)	reduce($U \to \mathbf{x}$)
7		shift(12)
8	shift(13)	
9	reduce($U \to \mathbf{x}$)	reduce($T \to \mathbf{x}$)

...