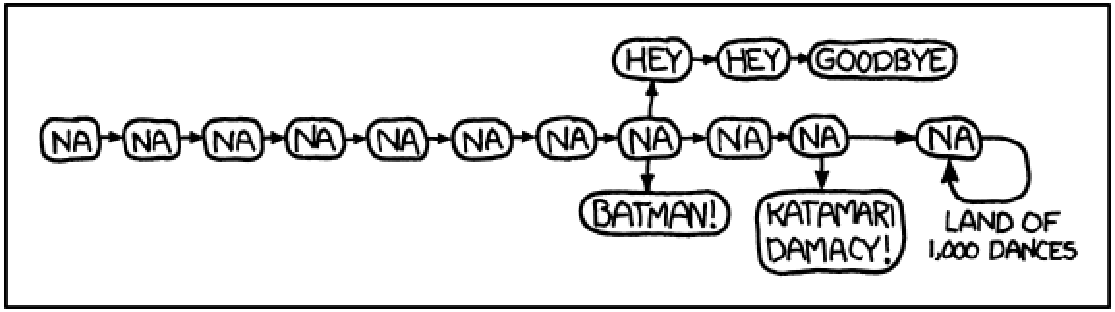
CS 432
Fall 2024

Mike Lam, Professor



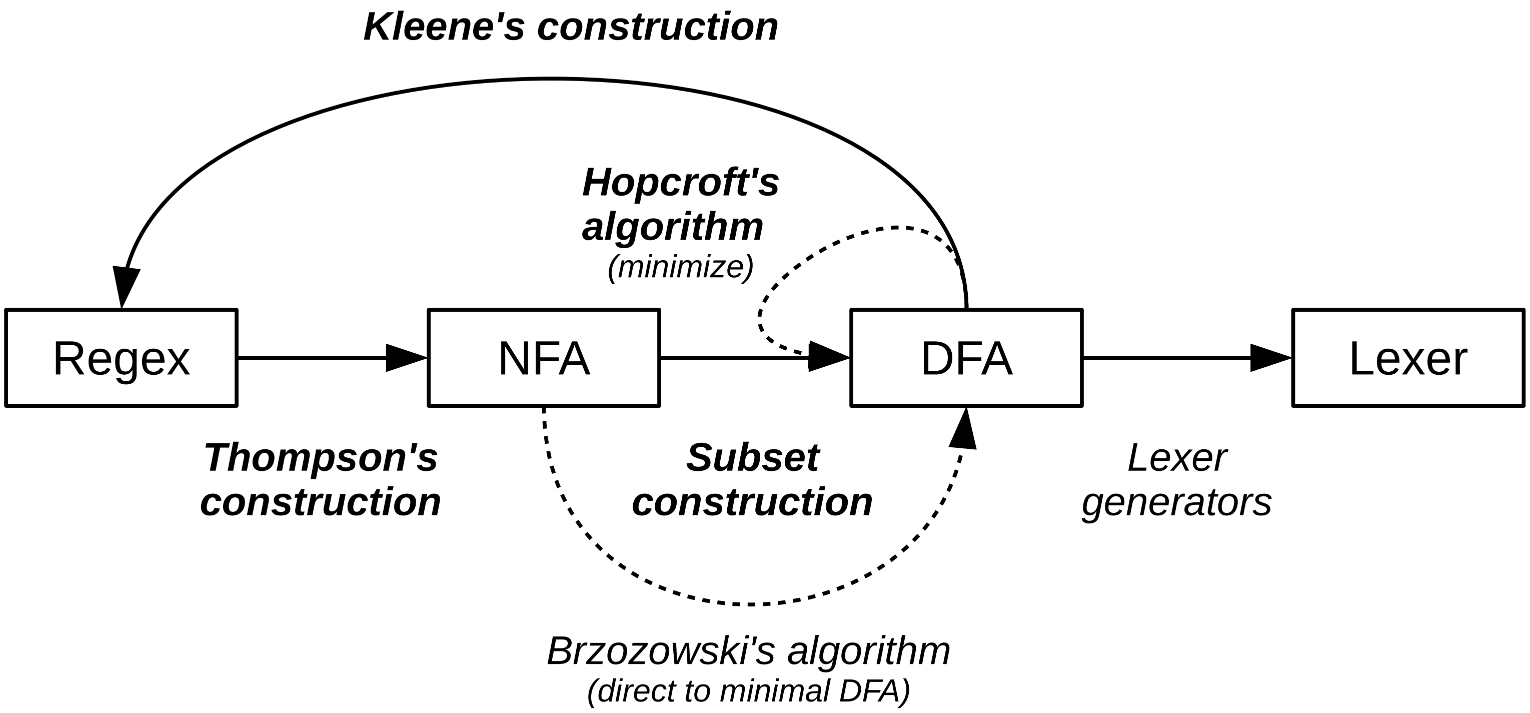
Finite Automata Conversions and Lexing

Finite Automata

- Key result: all of the following have the same expressive power (i.e., they all describe *regular* languages):
  - Regular expressions (REs)
  - Non-deterministic finite automata (NFAs)
  - Deterministic finite automata (DFAs)
- Proof by construction
  - An algorithm exists to convert any RE to an NFA
  - An algorithm exists to convert any NFA to a DFA
  - An algorithm exists to convert any DFA to an RE
  - For every regular language, there exists a minimal DFA
    - Has the fewest number of states of all DFAs equivalent to RE

Finite Automata

- Finite automata transitions:



(dashed lines indicate transitions to a minimized DFA)

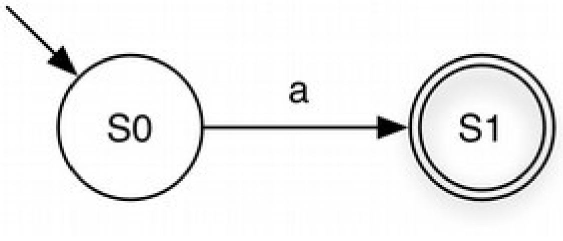
Finite Automata Conversions

- RE to NFA: Thompson's construction
  - Core insight: **inductively** build up NFA using “templates”
  - Core concept: use **null transitions** to build NFA quickly
- NFA to DFA: Subset construction
  - Core insight: DFA nodes represent **subsets** of NFA nodes
  - Core concept: use **null closure** to calculate subsets
- DFA minimization: Hopcroft’s algorithm
  - Core insight: create **partitions**, then keep splitting
- DFA to RE: Kleene's construction
  - Core insight: repeatedly eliminate states by **combining** regexes

Thompson's Construction

- Basic idea: create NFA inductively, bottom-up
  - Base case:
    - Start with individual alphabet symbols (see below)
  - Inductive case:
    - Combine by adding new states and null/epsilon transitions
    - **Templates** for the three basic operations
  - Invariant:
    - The NFA always has exactly one start state and one accepting state

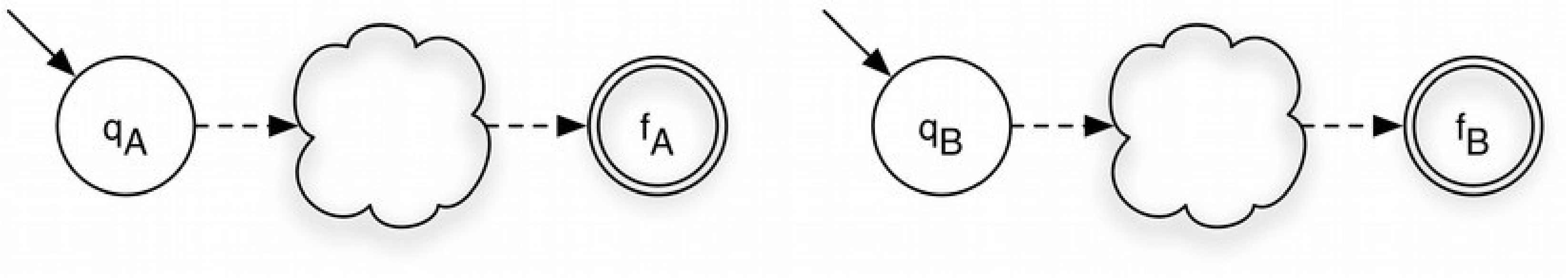
a



Thompson's: Concatenation

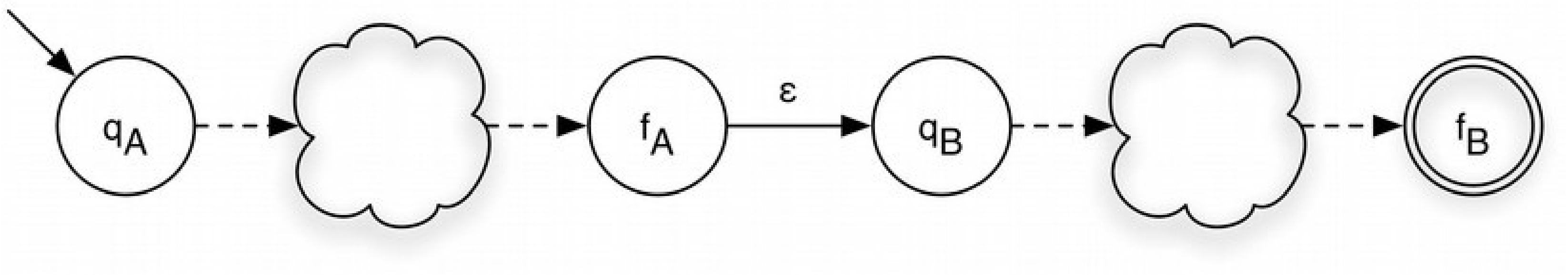
A

B



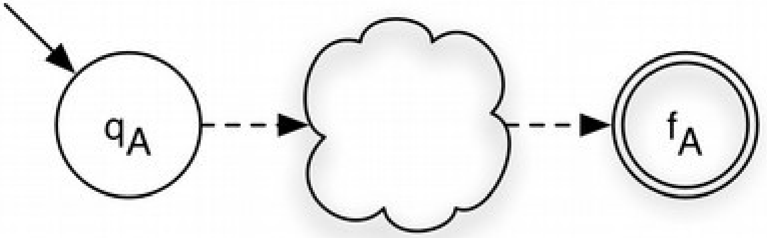
Thompson's: Concatenation

AB

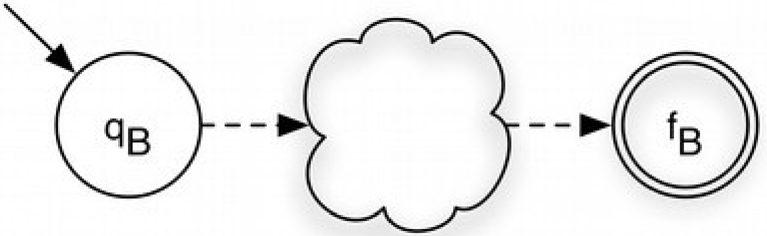


Thompson's: Union

A

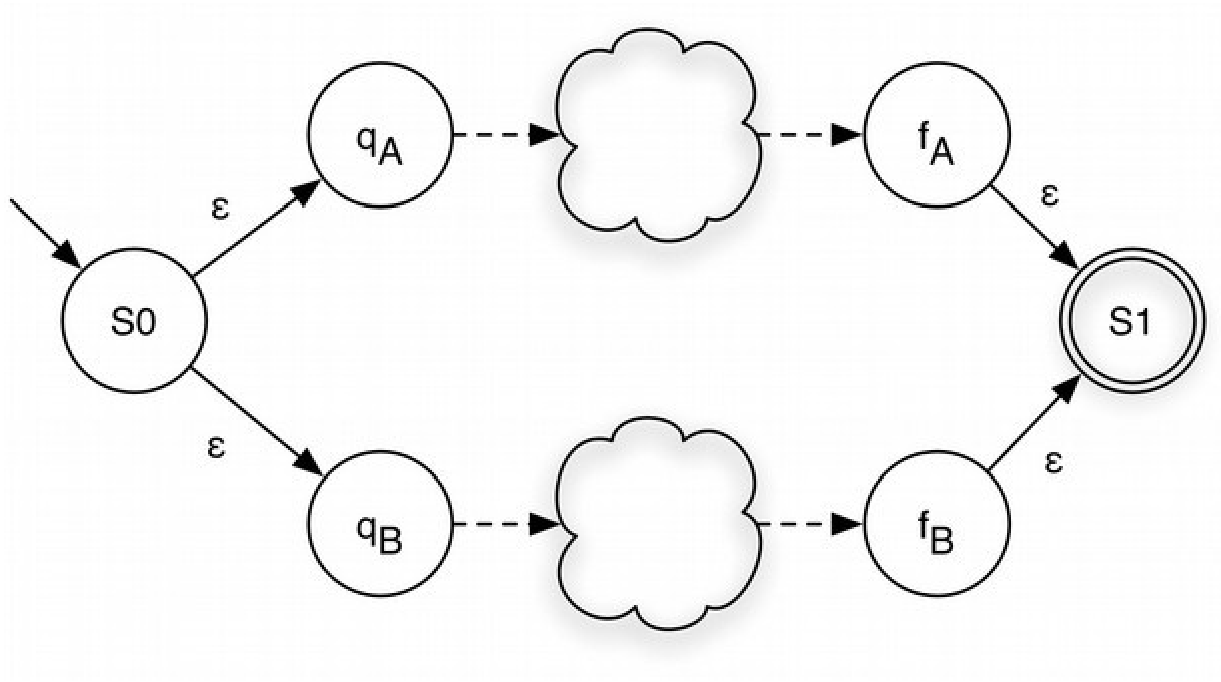


B



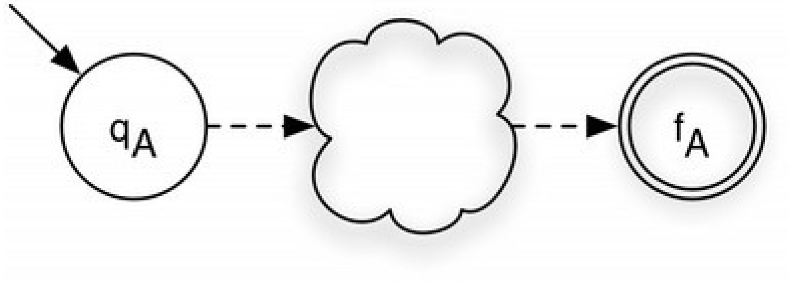
Thompson's: Union

A|B



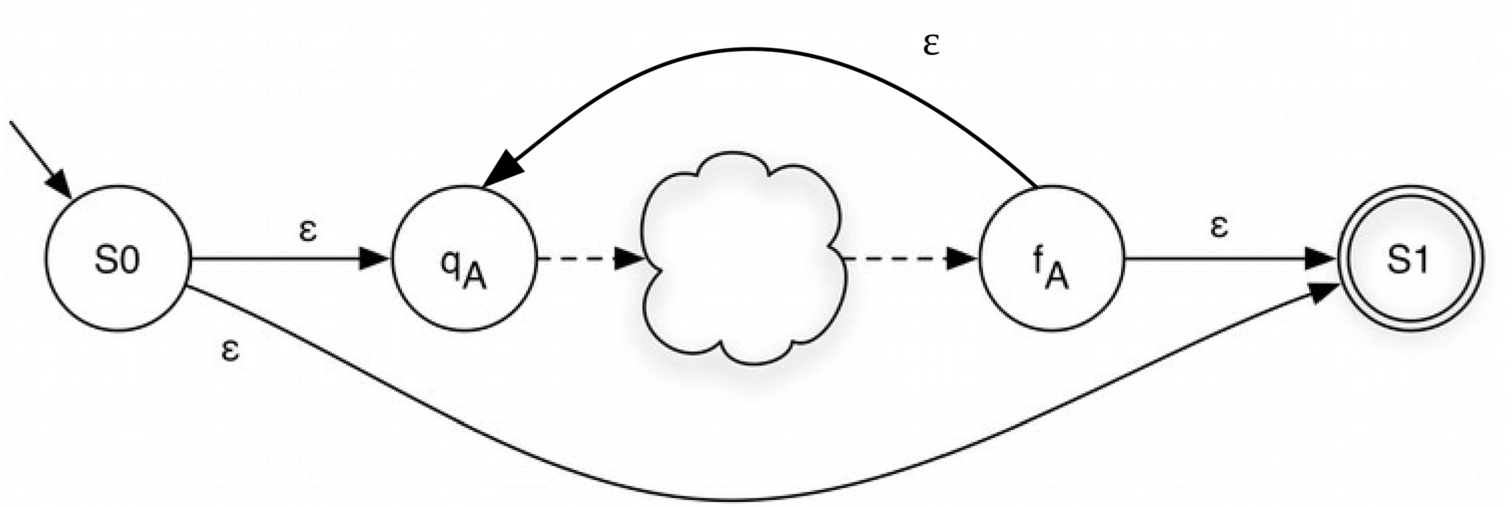
Thompson's: Closure

A



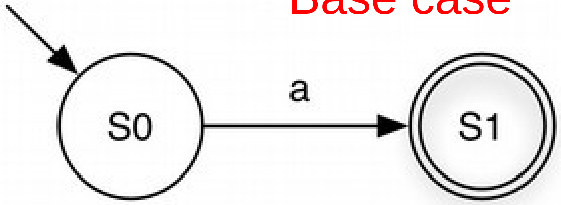
Thompson's: Closure

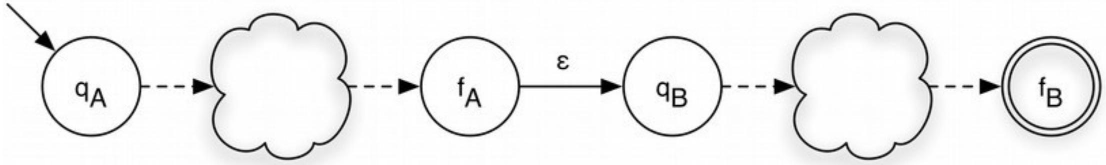
A*



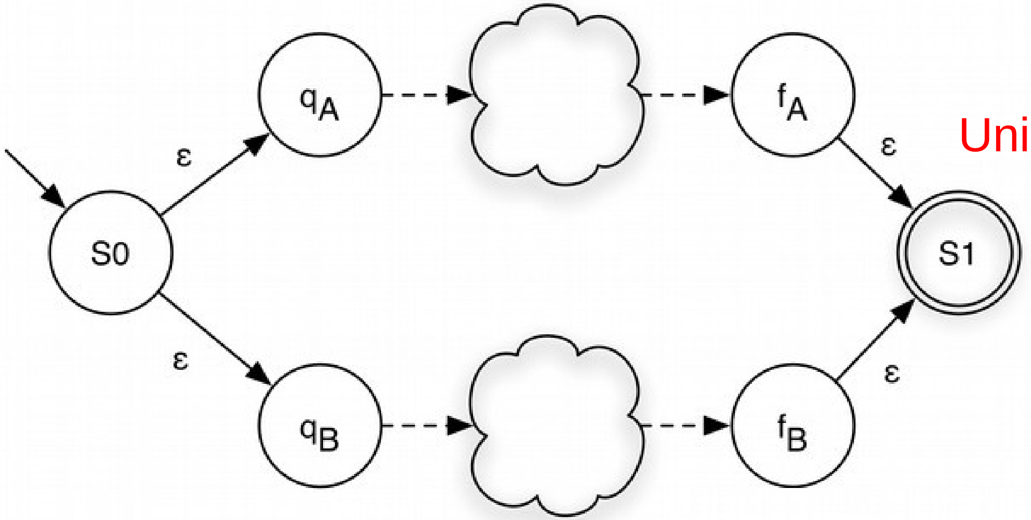
Thompson's Construction

Base case



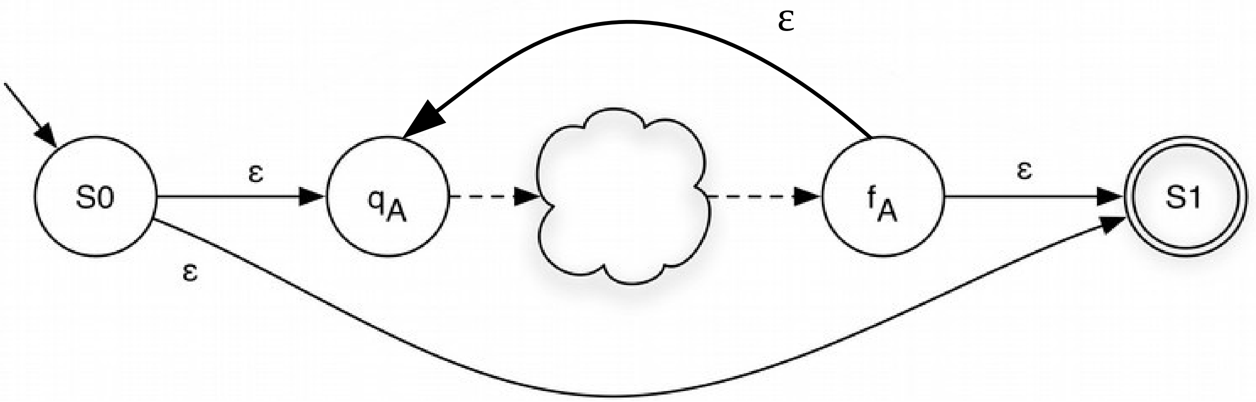


Concatenation



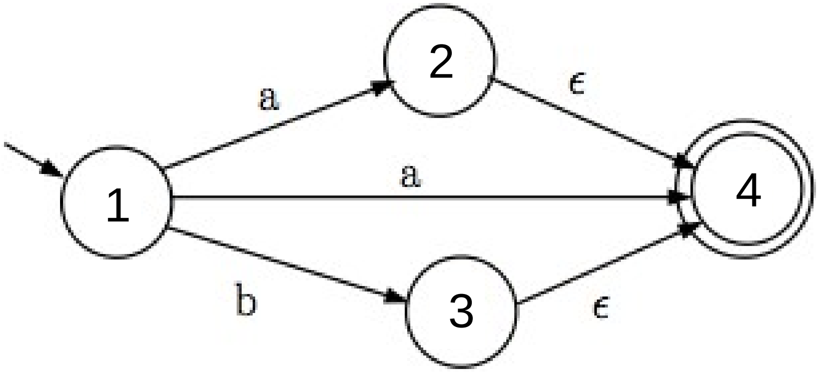
Union

Closure



Subset construction

- Basic idea: create DFA incrementally
  - Each DFA state represents a subset of NFA states
  - Use null closure operation to “collapse” null/epsilon transitions
  - Null closure: all states reachable via epsilon transitions
    - Essentially: where can we go “for free?”
    - Formally: ε-closure(s) = $\{s\} \cup \{ t \in S \mid (s,\varepsilon \rightarrow t) \in \delta \}$
  - Simulates running all possible paths through the NFA



Null closure of 1 = { 1 }
Null closure of 2 = { 2, 4 }
Null closure of 3 =
Null closure of 4 =

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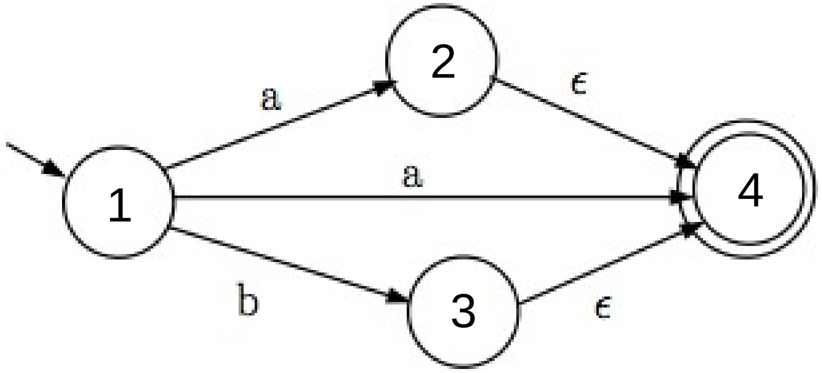

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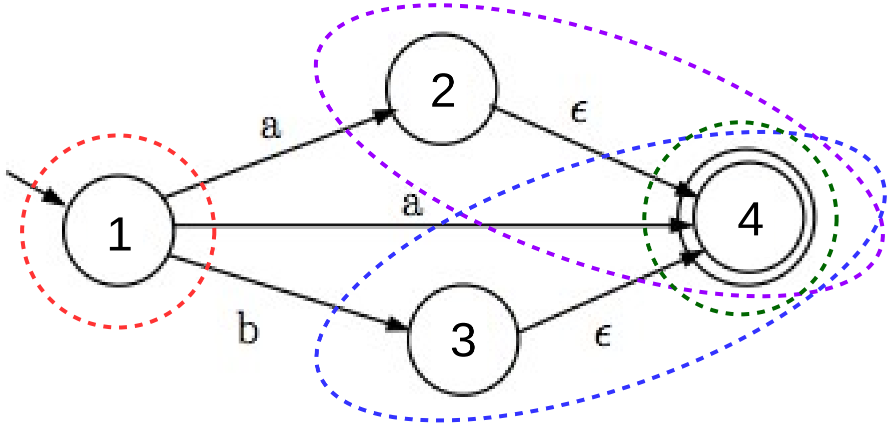

Null closure of 1 = { 1 }
Null closure of 2 = { 2, 4 }
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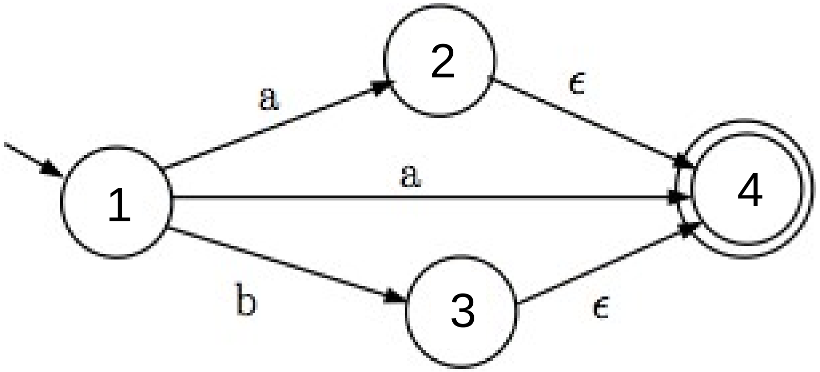
Null closure of 4 = { 4 }

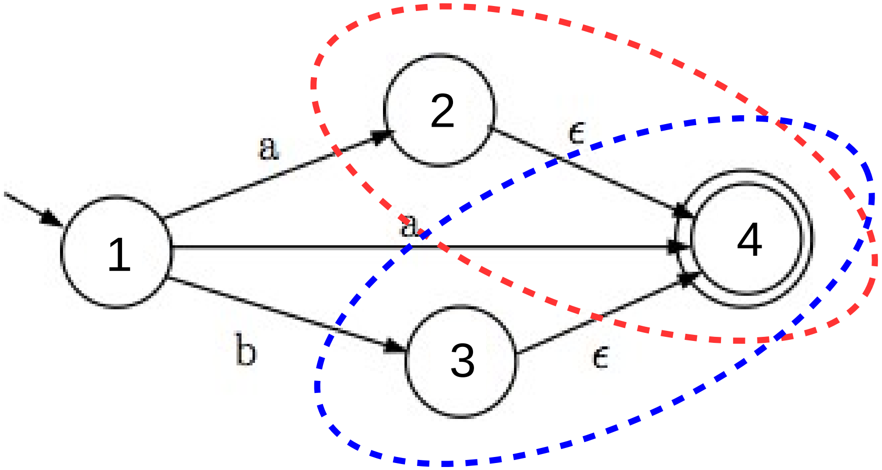
Formal Algorithm

```
SubsetConstruction($S, \Sigma, s_0, S_A, \delta$):
    $t_0 := \varepsilon\text{-}closure(s_0)$
    $S' := \{ t_0 \}$        $S'_A := \varnothing$     $W := \{ t_0 \}$
    while $W \neq \varnothing$:
        choose $u$ in $W$ and remove it from $W$
        for each $c$ in $\Sigma$:
            $t := \varepsilon\text{-}closure(\delta(u,c))$
            $\delta'(u,c) = t$
            if $t$ is not in $S'$ then
                add $t$ to $S'$ and $W$
                add $t$ to $S'_A$ if any state in $t$ is also in $S_A$
    return $(S', \Sigma, t_0, S'_A, \delta')$
```

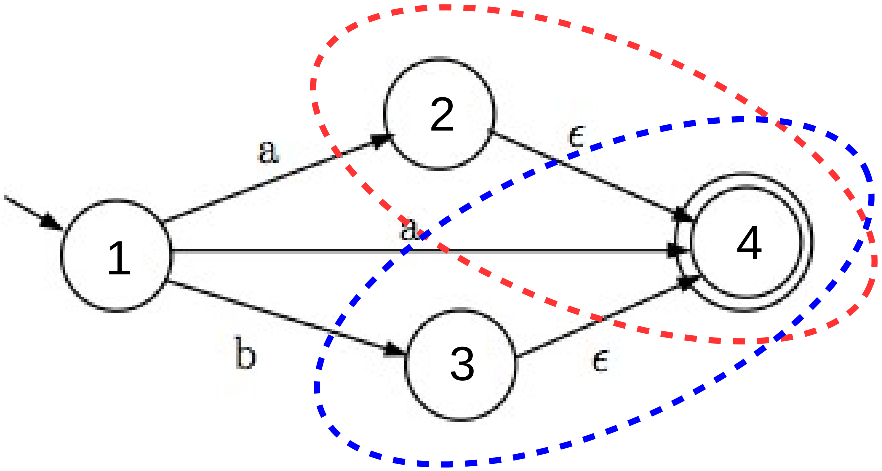
Subset Example

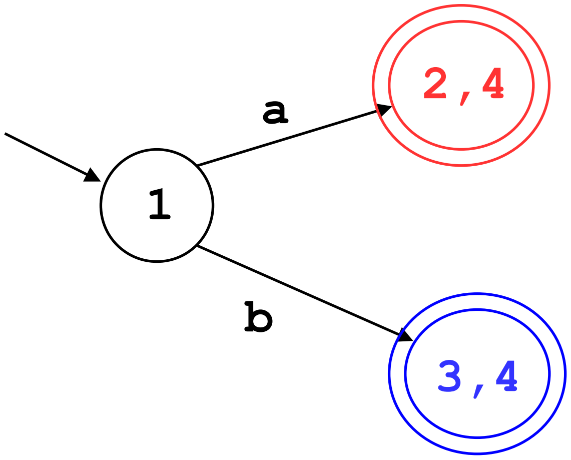


Subset Example

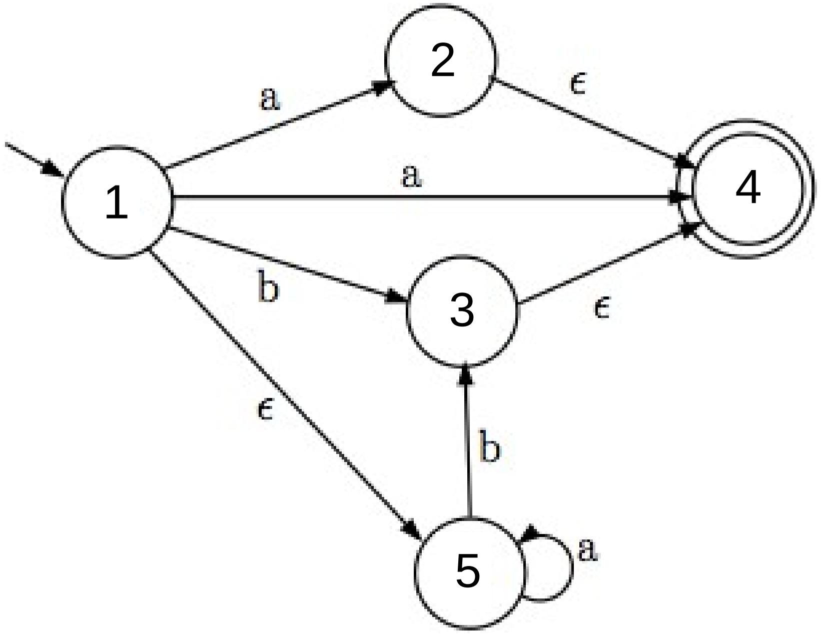


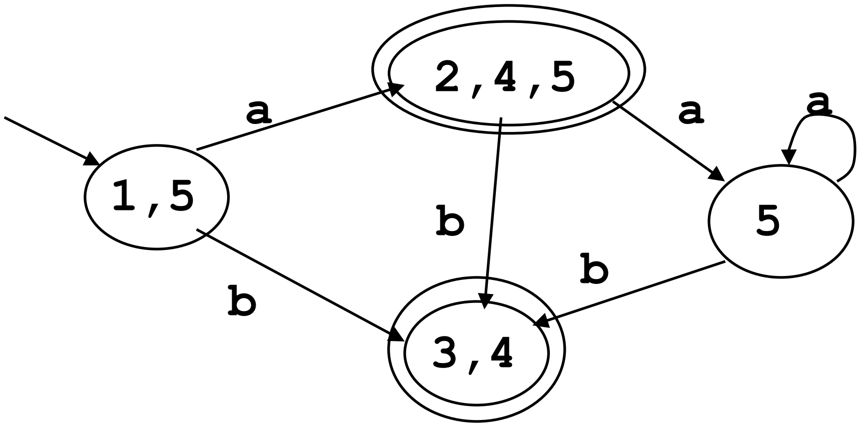
Subset Example





Subset Example



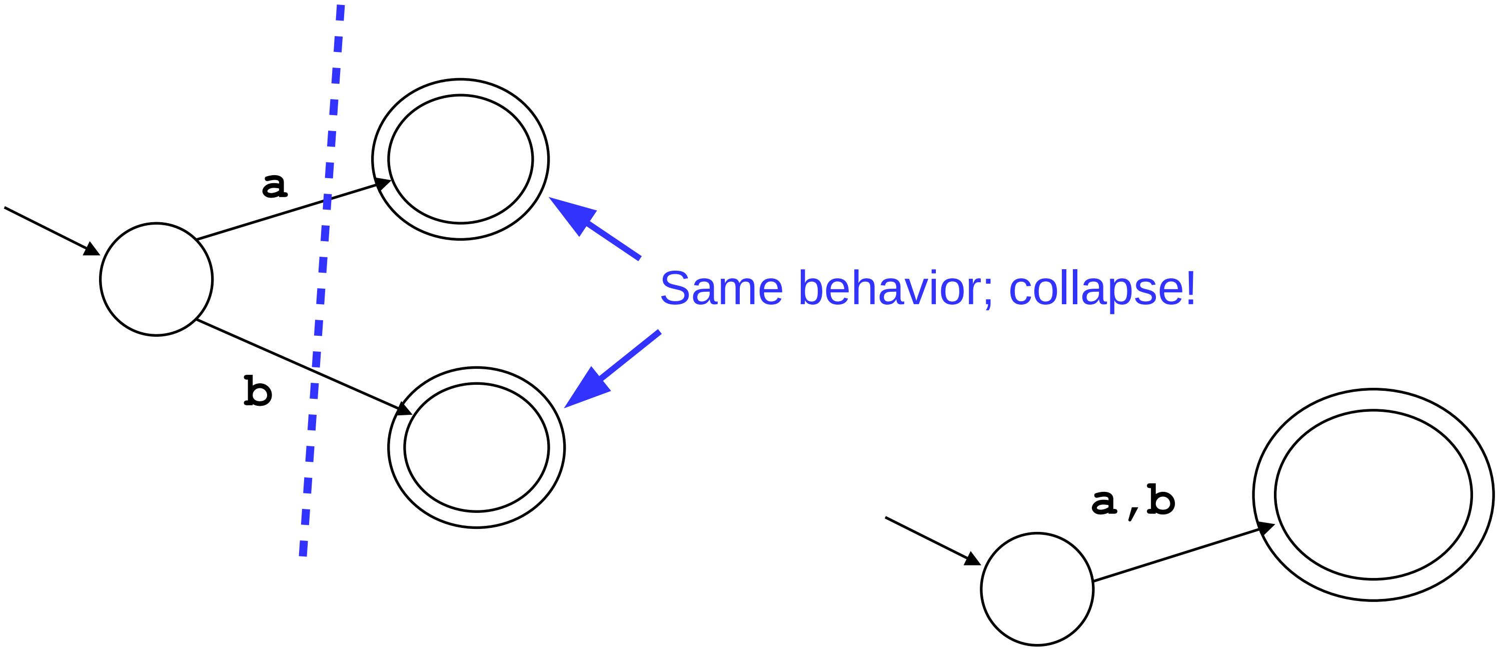


Algorithms

- Subset construction is a fixed-point algorithm
  - Textbook: “Iterated application of a monotone function”
  - Basically: A loop that is mathematically guaranteed to terminate at some point
  - When it terminates, some desirable property holds
    - In the case of **subset construction**: the NFA has been converted to a DFA
    - In the case of **DFA minimization** (up next): the DFA has the smallest number of states possible

Hopcroft’s DFA Minimization

- Split into two partitions (final & non-final)
- Keep splitting a partition while there are states with differing behaviors
  - Two states transition to differing partitions on the same symbol
  - Or one state transitions on a symbol and another doesn’t
- When done, each partition becomes a single state



Same behavior; collapse!

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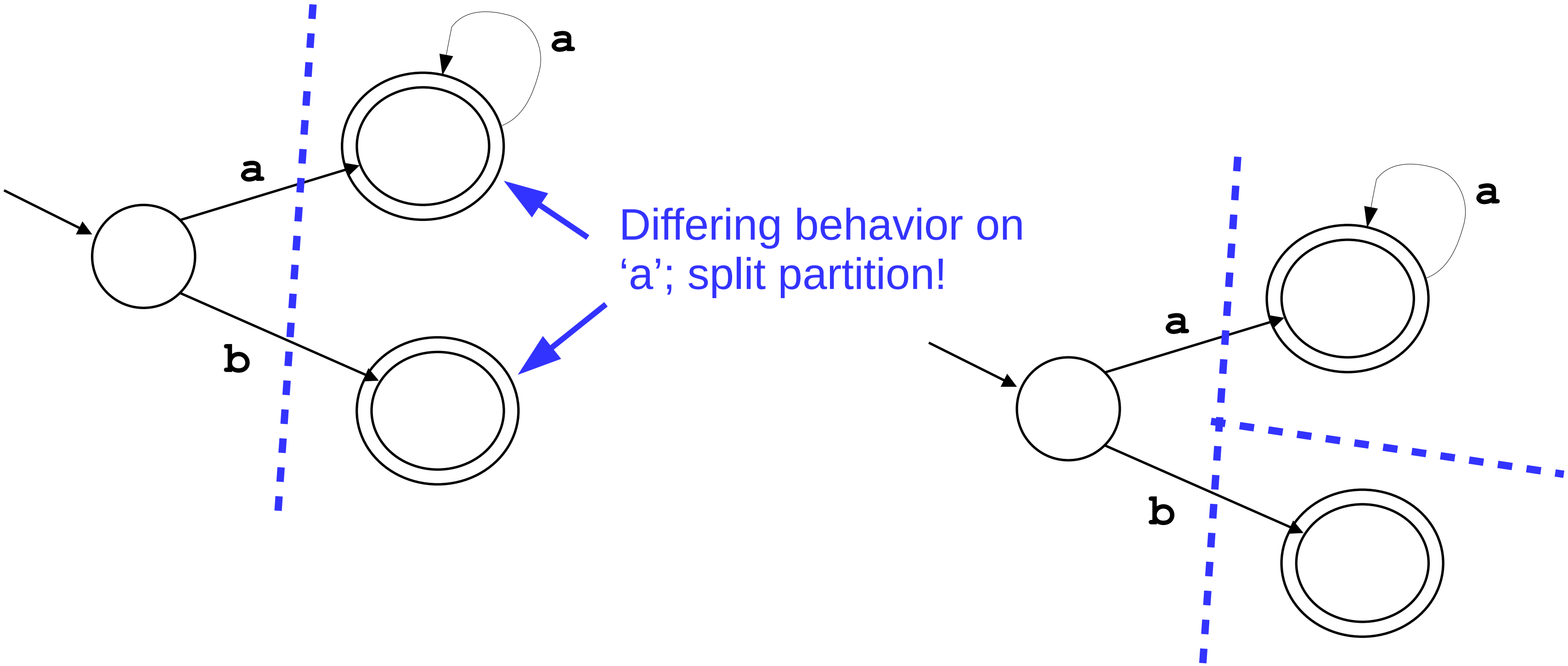
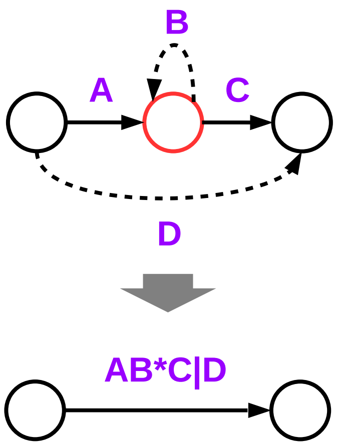


Differing behavior on ‘a’; split partition!

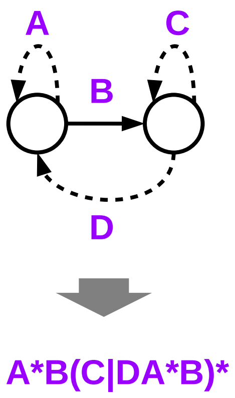
Kleene's Construction

- Replace edge labels with REs
  - "a" → "a" and "a,b" → "a|b"
- Eliminate states by combining REs
  - See pattern below; apply pairwise around each state to be eliminated
  - Repeat until only one or two states remain
- Build final RE
  - One state with "A" self-loop → "A*"
  - Two states: see pattern below

Eliminating states:



Combining final two states:

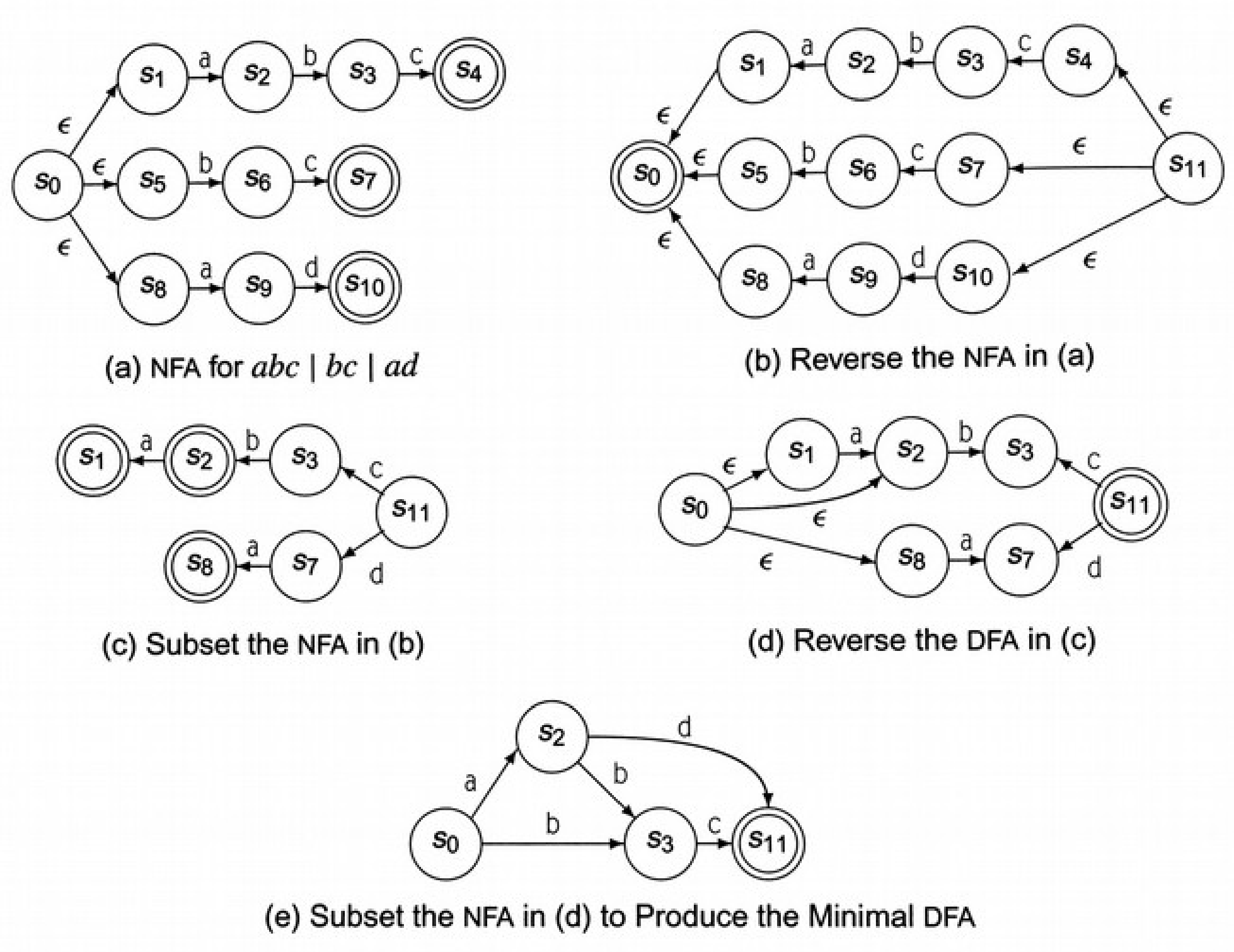


Brzozowski’s Algorithm

- Direct NFA → minimal DFA conversion
- Sub-procedures:
  - Reverse(n): invert all transitions in NFA n, adding a new start state connected to all old final states
  - Subset(n): apply subset construction to NFA n
  - Reach(n): remove any part of NFA n unreachable from start state
- Apply them all in order two times to get minimal DFA
  - First time eliminates duplicate suffixes
  - Second time eliminates duplicate prefixes
  - MinDFA(n) = Reach(Subset(Reverse(Reach(Subset(Reverse(n))))))
  - Potentially easier to code than Hopcroft’s algorithm

Brzozowski's Algorithm

- MinDFA(n) = Reach(Subset(Reverse(Reach(Subset(Reverse(n))))))



(a) NFA for *abc* | *bc* | *ad*

(b) Reverse the NFA in (a)

(c) Subset the NFA in (b)

(d) Reverse the DFA in (c)

(e) Subset the NFA in (d) to Produce the Minimal DFA

■ **FIGURE 2.19** Minimizing a DFA with Brzozowski's Algorithm.

Example from EAC (p.76)

Summary and Review

DFAs

- S: set of states
- Σ: alphabet (set of characters)
- δ: transition function: (S, Σ) → S
- s_0: start state
- S_A: accepting/final states

accept():

```
$s := s_0$
**for each input** $c$:
    $s := \delta(s,c)$
**return** $s \in S_A$
```

NFAs

- δ may return a set of states
- δ may contain ε-transitions

accept():

```
$T := \varepsilon\text{-}closure(s_0)$
**for each input** $c$:
    $N := \{\}$
    **for each** $s$ **in** $T$:
        $N := N \cup \varepsilon\text{-}closure(\delta(s,c))$
    $T := N$
**return** $|T \cap S_A| > 0$
```

Summary and Review

- RE to NFA: Thompson's construction
  - Core insight: **inductively** build up NFA using “templates”
  - Core concept: use **null transitions** to build NFA quickly
- NFA to DFA: Subset construction
  - Core insight: DFA nodes represent **subsets** of NFA nodes
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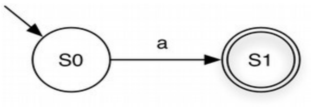
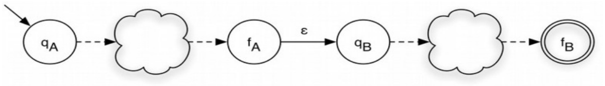
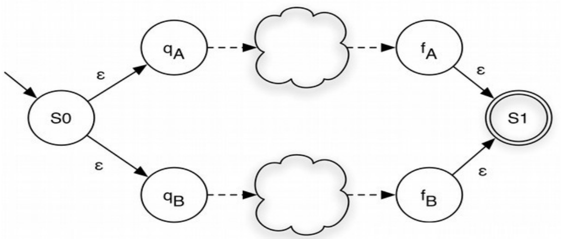
  - Core insight: create **partitions**, then keep splitting
- DFA to RE: Kleene's construction
  - Core insight: repeatedly eliminate states by **combining** regexes

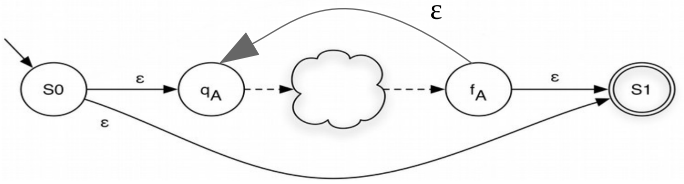
NFA/DFA complexity

- What are the time and space requirements to...
  - Build an NFA?
  - Run an NFA?
  - Build a DFA?
  - Run a DFA?

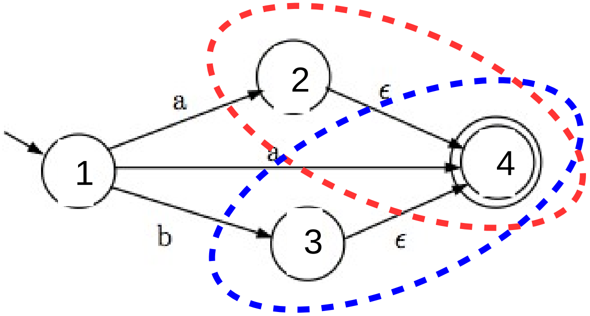


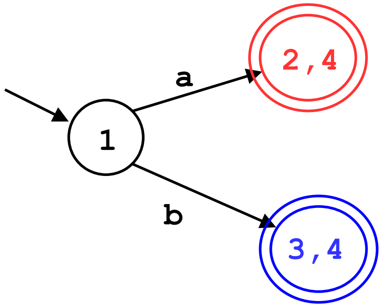






aa*|b





NFA/DFA complexity

- Thompson's construction
  - At most two new states and four transitions per regex character
  - Thus, a linear space increase with respect to the # of regex characters
  - Constant # of operations per increase means linear time as well
- NFA execution
  - Proportional to both NFA size and input string size (multiplicatively)
  - Must track multiple simultaneous “current” states
- Subset construction
  - Potential exponential state space explosion
  - A *n*-state NFA could require up to 2^n DFA states
  - However, this rarely happens in practice
- DFAs execution
  - Proportional to input string size only (only track a single “current” state)

NFA/DFA complexity

- NFAs build quicker (linear) but run slower
  - Better if you will only run the FA a few times
  - Or if you need features that are difficult to implement with DFAs
- DFAs build slower but run faster (linear)
  - Better if you will run the FA many times (like in a compiler)

	NFA	**DFA**
Build time	$O(m)$	$O(2^m)$
Run time	$O(m \times n)$	$O(n)$

m = length of regular expression
n = length of input string

Lexing/Scanning w/ DFAs

- One approach:
  - Combine all regexes and build one DFA
  - Run DFA on input until there is no outgoing edge on a character
    - If current state is accepting, generate token and restart
    - Otherwise, back up to most recent accepting state then generate token and restart (if no accepting states were passed, report error)
- Another approach (P1):
  - Build a DFA for each regex
  - Run each DFA in sequence in priority order on input until there is no outgoing edge on the next character
    - If current state is accepting, generate token and restart
    - Otherwise, run the next DFA (if no more DFAs, report error)

Lexers

- Auto-generated
  - Table-driven: generic scanner, auto-generated tables
  - Direct-coded: hard-code transitions using jumps
  - Common tools: lex/flex and similar
- Hand-coded
  - Better I/O performance (i.e., buffering)
  - More efficient interfacing w/ other phases
  - This is what we'll do for P1

Handling Keywords

- Issue: keywords are valid identifiers
- Option 1: Embed into NFA/DFA
  - Separate regex for keywords
  - Easier/faster for generated scanners
- Option 2: Use lookup table
  - Scan as identifier then check for a keyword
  - Easier for hand-coded scanners
  - (Thus, this is probably easier for P1)

Handling Whitespace

- Issue: whitespace is usually ignored
  - Write a regex and remove it before each new token
- Side effect: some results are counterintuitive
  - Is this a valid token? “`3abc`”
  - For now, it’s actually two!
  - We’ll reject this sequence later in the parsing phase

Escaped characters

- Issue: some characters must be escaped in regular expressions
  - E.g., “+” or “*”
- Complication: C strings also have escape codes!
  - So you’ll need “\\+” or “\\*”
  - And “\\\\” for recognizing a slash!