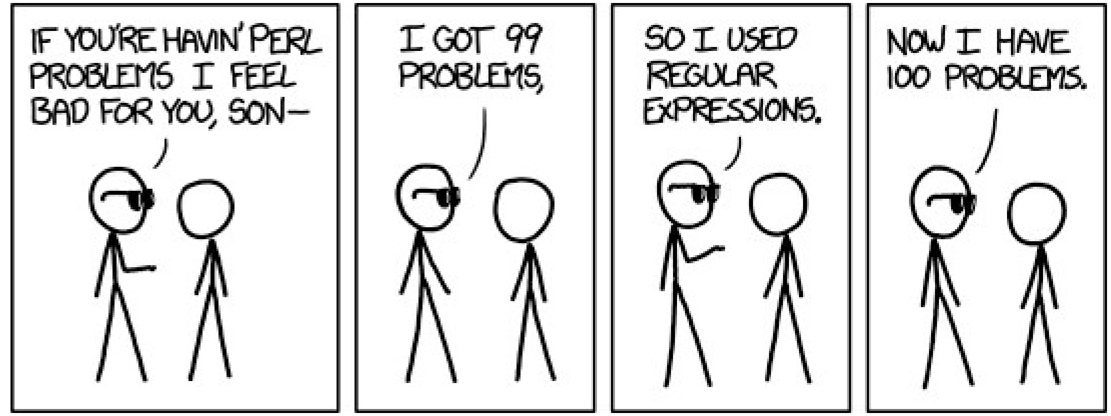
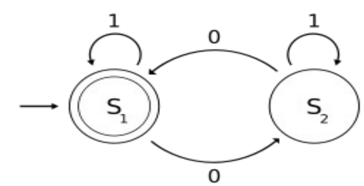
CS 432
Fall 2023

Mike Lam, Professor



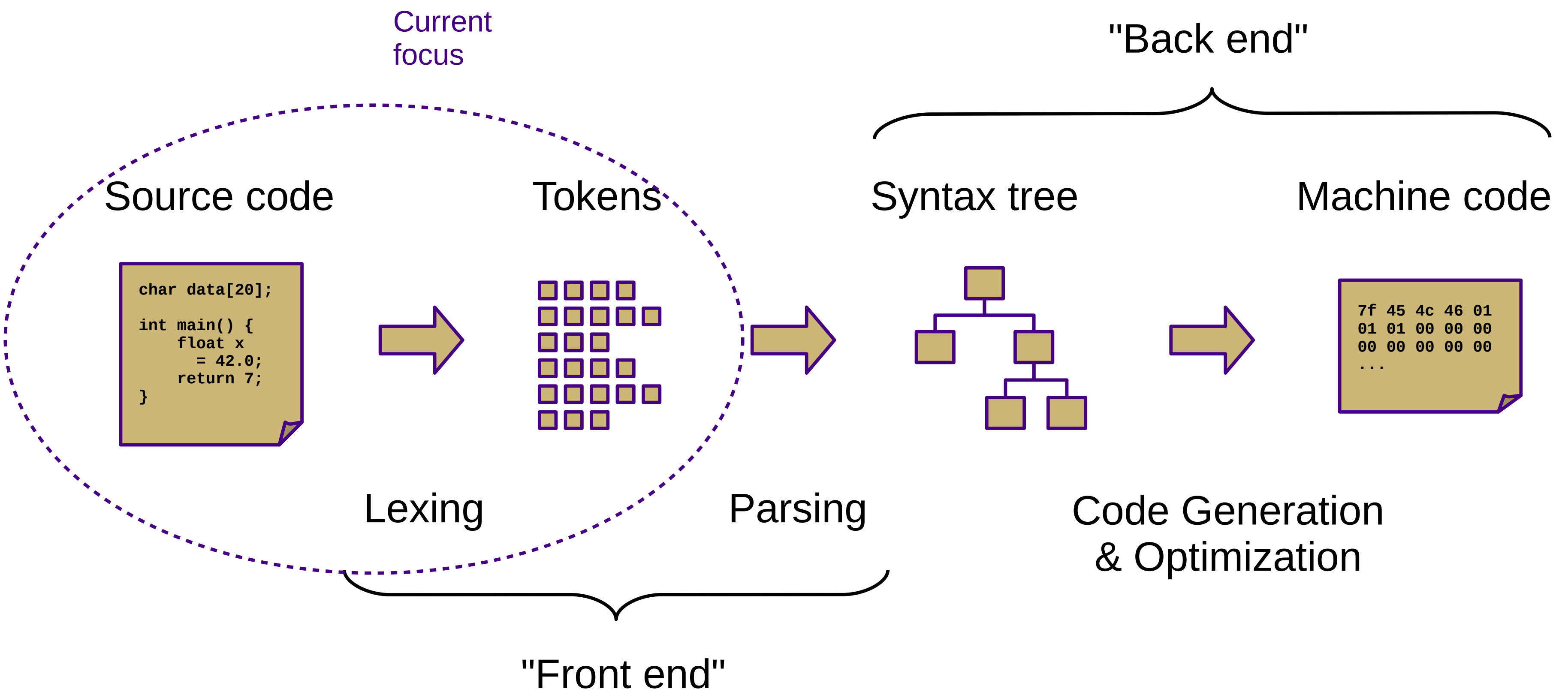
https://xkcd.com/1171/

`a|(bc)*`



Regular Expressions
and
Finite Automata

Compilation



Lexical Analysis

- Lexemes or tokens: the smallest building blocks of a language's syntax
- Lexing or scanning: the process of separating a character stream into tokens

```
total = sum(vals) / n

total      identifier
=          equals_op
sum        identifier
(          left_paren
vals       identifier
)          right_paren
/          divide_op
n          identifier
```

```
char *str = "hi";

char       keyword
*          star_op
str        identifier
=          equals_op
"hi"       str_literal
;          semicolon
```

Discussion question

- What is a *language*?

Language

- A language is "a (potentially infinite) set of strings over a finite alphabet"

Discussion question

- How do we describe languages?

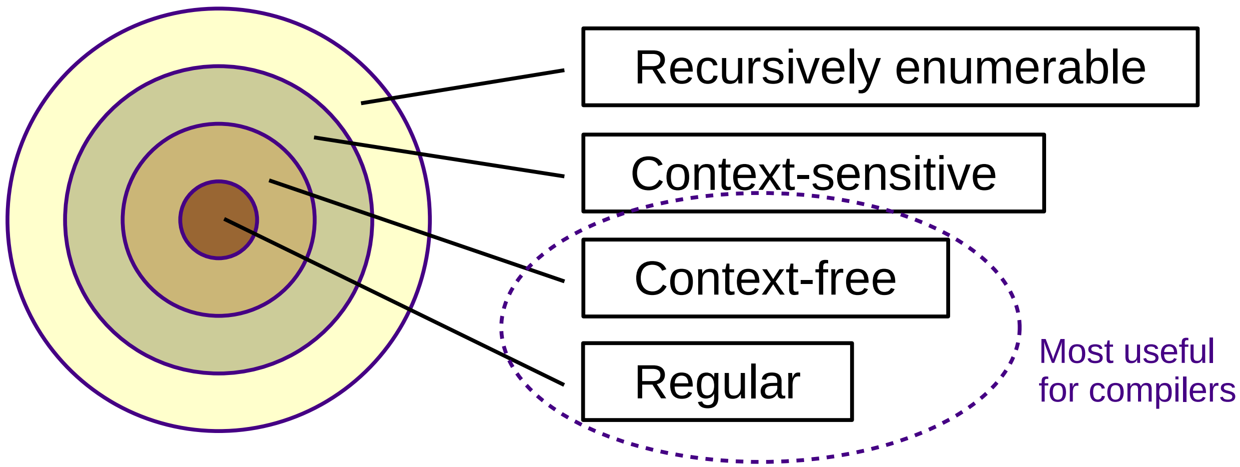
xyy	xy	xy	xyy
xy	xyy	xyz	xyyz
xyyzzz	xyz	xyzz	xyyzz
xyz	xyyz	xyzzz	xyyzzz
xyzz	xyzz	(etc.)	
xyyzz	xyyzz		
xyyz	xyzzz		
xyzzz	xyyzzz		
(etc.)	(etc.)		

Language description

- Ways to describe languages
  - Ad-hoc prose
    - “A single ‘x’ followed by one or two ‘y’s followed by any number of ‘z’s”
  - Formal regular expressions (current focus)
    - x(y|yy)z*
  - Formal grammars (in two weeks)
    - A → x B C
    - B → y | y y
    - C → z C | ε

Languages

Chomsky Hierarchy of Languages



- Alphabet:
  - Σ = { finite set of all characters }
- Language:
  - L = { potentially infinite set of sequences of characters from Σ }

Regular expressions

- Regular expressions describe regular languages
  - Can also be thought of as generalized search patterns

- Three basic recursive operations:
  - Alternation: **A|B** — Lowest precedence
  - Concatenation: **AB** *or* **A␣B**
  - ("Kleene") Closure: **A*** — Highest precedence

Additionally: **ε** is a regex that matches the empty string

- Extended constructs:
  - Character sets/classes: **[0-9]** ≡ **[0...9]** ≡ **0|1|2|3|4|5|6|7|8|9**
  - Repetition / positive closure: A^2 ≡ **AA** A^3 ≡ **AAA** A^+ ≡ **AA***
  - Grouping: **(A|B)C** ≡ **AC|BC**

These are not covered extensively in your textbook!

Regular expressions

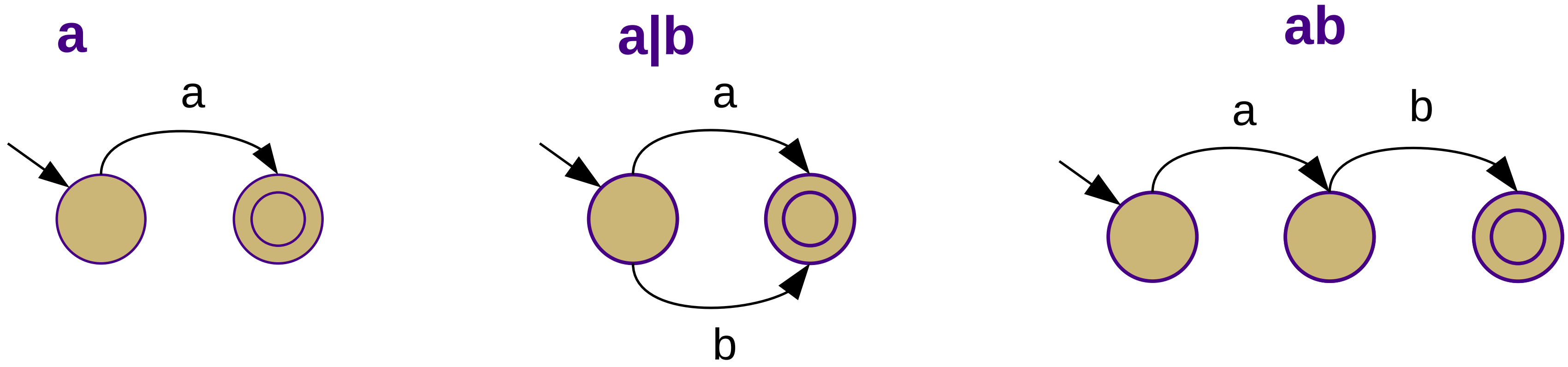
- Symbols with special meaning in regular expressions must be “escaped” to match the actual symbol
  - E.g., **a*** matches an “a” followed by an asterisk (“*”)
  - This is not usually necessary inside a character class
    - E.g., **a[*]** ≡ **a***
- Alternation of character classes can be condensed
  - E.g., **[a-z]|[A-Z]** ≡ **[a-zA-Z]**
- Starting a character class with a caret (“^”) forms the complement
  - E.g., **[^abc]** matches any character that is **NOT** “a”, “b”, or “c”
  - Outside a character class, **^** matches the beginning of a string and **$** matches the end of a string

Discussion question

- How would you implement regular expressions?
  - Given a regular expression and a string, how would you tell whether the string belongs to the language described by the regular expression?

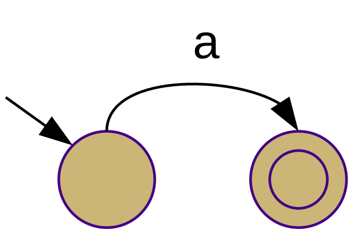
Lexical Analysis

- Implemented using state machines (finite automata)
  - Set of states with a single start state
  - Transitions between states on inputs (w/ implicit dead states)
  - Some states are final or accepting

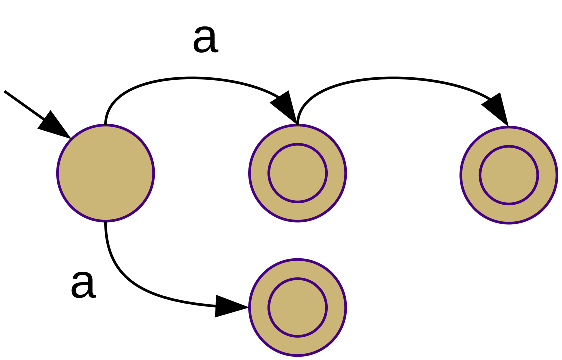


Lexical Analysis

- Deterministic vs. non-deterministic
  - Non-deterministic: multiple possible states for given sequence
  - One edge from each state per character (deterministic)
    - Might lead to implicit “dead state” w/ self-loop on all characters
  - Multiple edges from each state per character (non-deterministic)
  - “Empty” or ε-transitions (non-deterministic)



Deterministic (DFA)



Non-deterministic (NFA)

Deterministic finite automata

- Formal definition

  S: set of states

  Σ: alphabet (set of characters)

  δ: transition function: (S, Σ) → S

  s_0: start state

  S_A: accepting/final states

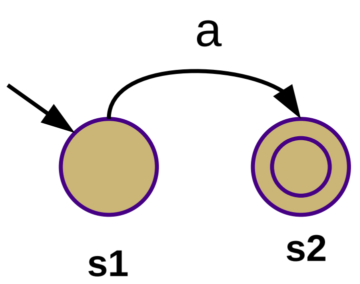
- Acceptance algorithm

  $s := s_0$

  for each input c:

  $\quad s := \delta(s,c)$

  return $s \in S_A$



S = { s1, s2 }
Σ = { a }
δ = { (s1, a → s2), (s2, a → Ø) }
s_0: = s1

S_A = { s2 }

Alternative δ representation:

	a
s1	s2
s2	Ø

Non-deterministic finite automata

- Formal Definition
  - S, Σ, s_0, and S_A same as DFA
  - δ: (S, Σ ∪ {ε}) → [S]
  - ε-closure: all states reachable from s via ε-transitions
    - Formally: ε-closure(s) = {s} ∪ { t ∈ S | (s, ε)→t ∈ δ }
    - Extended to sets by union over all states in set

- Acceptance algorithm

T := *ε-closure*(*s₀*)

for each input *c*:

    N := {}

    for each *s* ***in*** *T*:

        N := *N* ∪ *ε-closure*(*δ*(*s*,*c*))

    T := *N*

return $|T \cap S_A| > 0$

Summary

DFAs

- S: set of states
- Σ: alphabet (set of characters)
- δ: transition function: (S, Σ) → S
- s_0: start state
- S_A: accepting/final states

accept():

$s := s_0$

for each input c:

$s := \delta(s,c)$

return $s \in S_A$

NFAs

- δ may return a set of states
- δ may contain ε-transitions
- δ may contain transitions to multiple states on a symbol

accept():

$T := \varepsilon\text{-}closure(s_0)$

for each input c:

$N := \{\}$

for each s ***in*** T:

$N := N \cup \varepsilon\text{-}closure(\delta(s,c))$

$T := N$

return $|T \cap S_A| > 0$

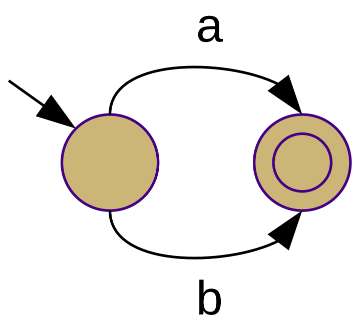
Equivalence

- A regular expression and a finite automaton are equivalent if they recognize the same language
  - Same applies between different REs and between different FAs
- Regular expressions, NFAs, and DFAs all describe the same set of languages
  - "Regular languages" from Chomsky hierarchy
- Next week, we will learn how to convert between them

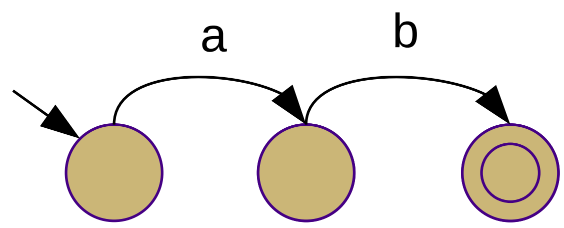
Lexical Analysis

- Examples:

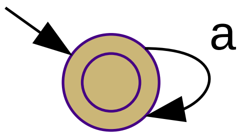
a|b



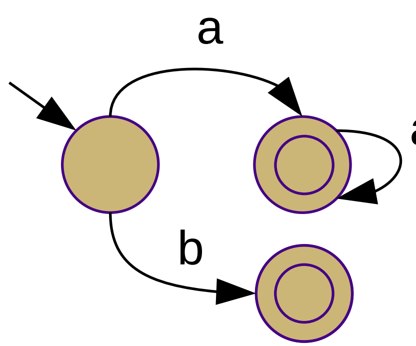
ab



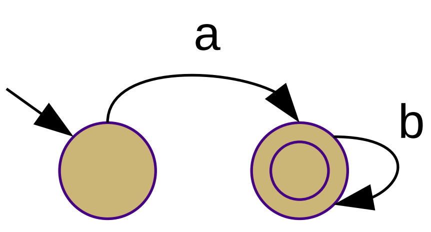
a*



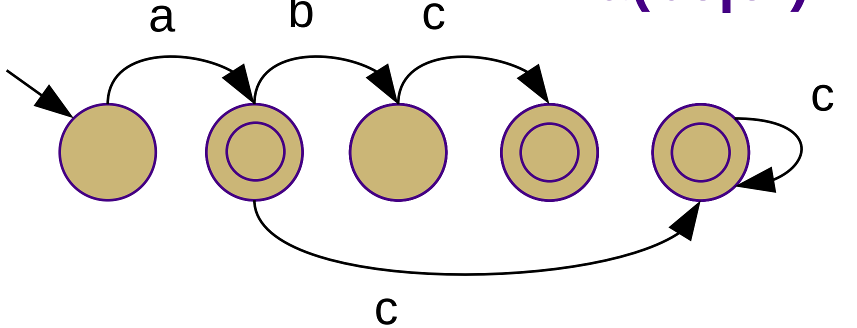
aa*|b



ab*



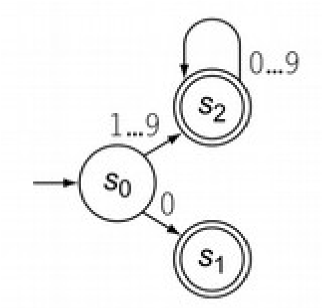
a(bc|c*)



Examples

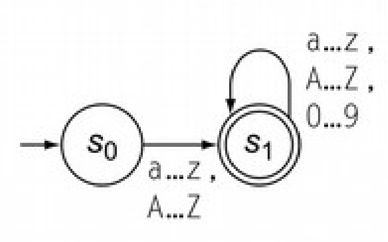
Unsigned integers

$$0 \mid [1 \ldots 9]\,[0 \ldots 9]^*$$



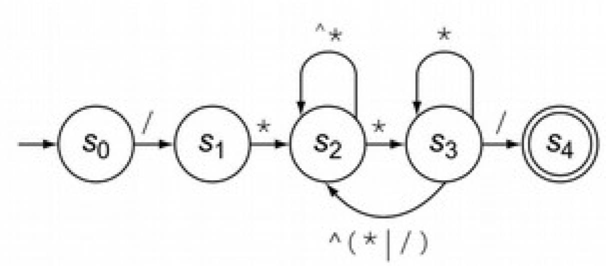
Identifiers

$$([A \ldots Z] \mid [a \ldots z])\ \ ([A \ldots Z] \mid [a \ldots z] \mid [0 \ldots 9])^*$$



Multi-line comments

$$/ * \,(\,\hat{}* \mid *^{+}\,\hat{}/\,)^*\ */$$



Exercise

- Construct state machines for the following regular expressions:

x*yz* **1(1|0)*** **1(10)*** **(a|b|c)(ab|bc)**

(dd*.d*)|(d*.dd*) ← ε-transitions may make this one slightly easier

Application

- P1: Use POSIX regular expressions to tokenize Decaf files
  - Process the input one line at a time
  - Generally, create one regex per token type
    - Each regex begins with “^” (only match from beginning)
    - Prioritize regexes and try each of them in turn
    - When you find a match, extract the matching text
    - Repeat until no match is found or the input is consumed
  - Less efficient than an auto-generated lexer
    - However, it is simpler to understand
    - Our approach to P2 will be similar