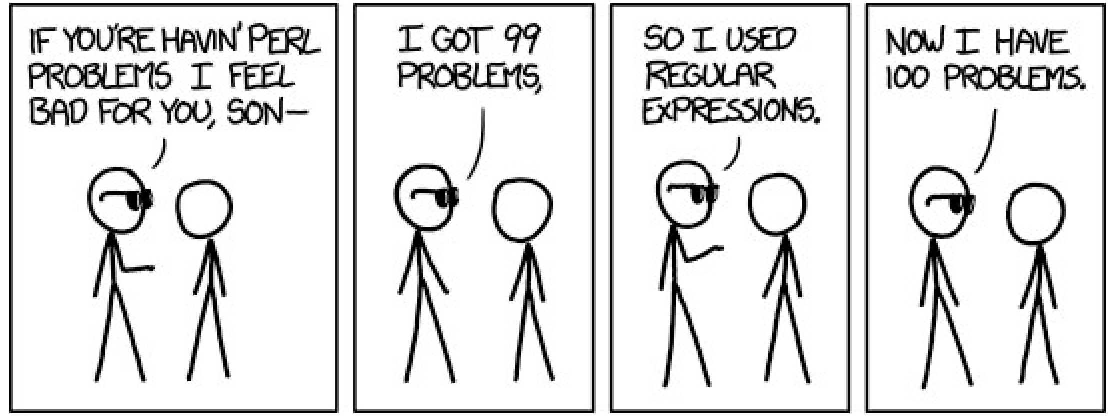
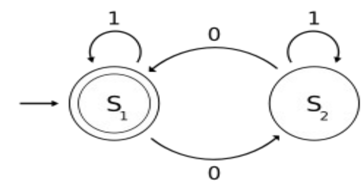
# CS 432
# Fall 2018

Mike Lam, Professor

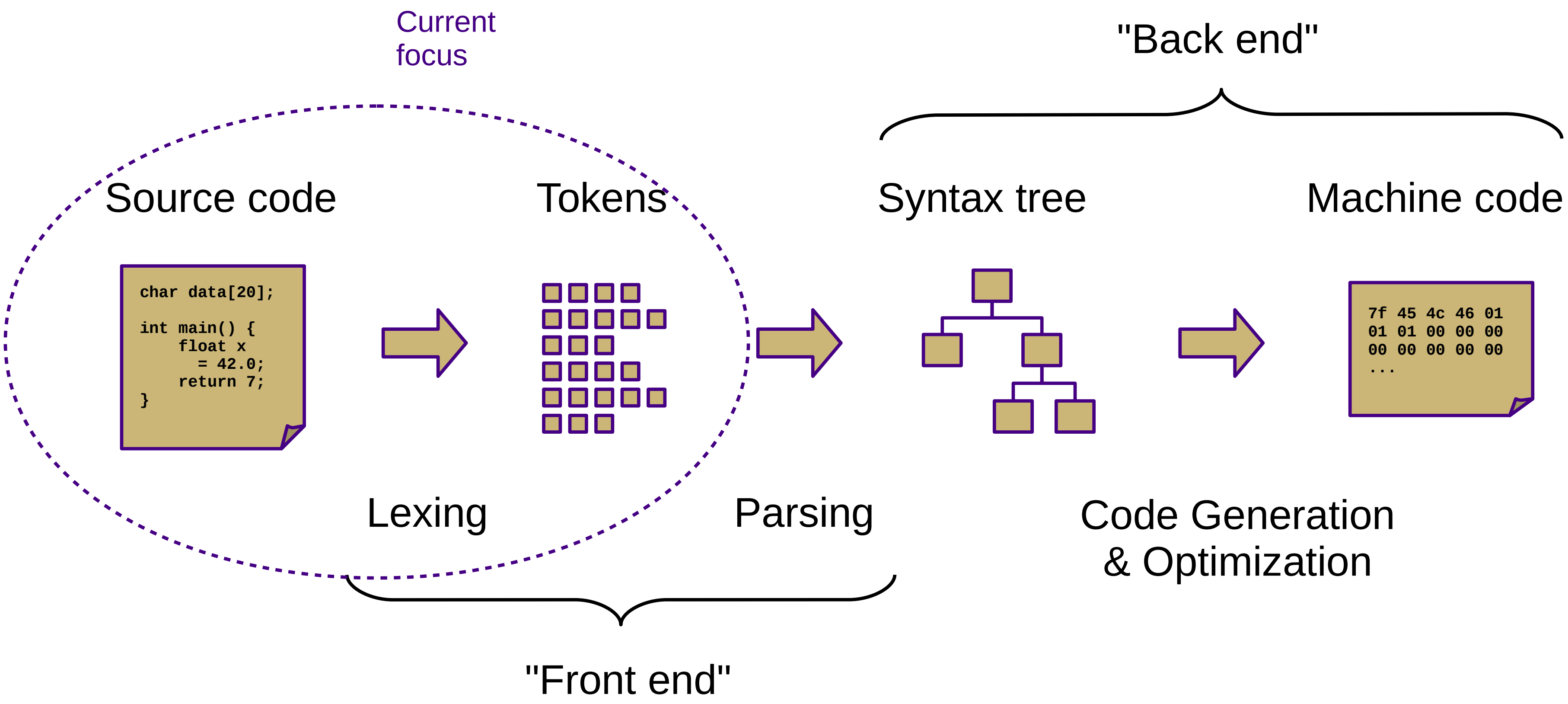



`a|(bc)*`




# Regular Expressions
and
# Finite Automata

# Compilation



# Lexical Analysis

- Lexemes or tokens: the smallest building blocks of a language's syntax
- Lexing or scanning: the process of separating a character stream into tokens

```
total = sum(vals) / n

total       identifier
=           equals_op
sum         identifier
(           left_paren
vals        identifier
)           right_paren
/           divide_op
n           identifier
```

```
char *str = "hi";

char        keyword
*           star_op
str         identifier
=           equals_op
"hi"        str_literal
;           semicolon
```

# Discussion question

- What is a language?

# Language

- A language is "a (potentially infinite) set of strings over a finite alphabet"

# Discussion question

- How do we describe languages?

xyy
xy
xyyzzz
xyz
xyzz
xyyzz
xyyz
xyzzz
(etc.)

xy
xyy
xyz
xyyz
xyzz
xyyzz
xyzzz
xyyzzz
(etc.)

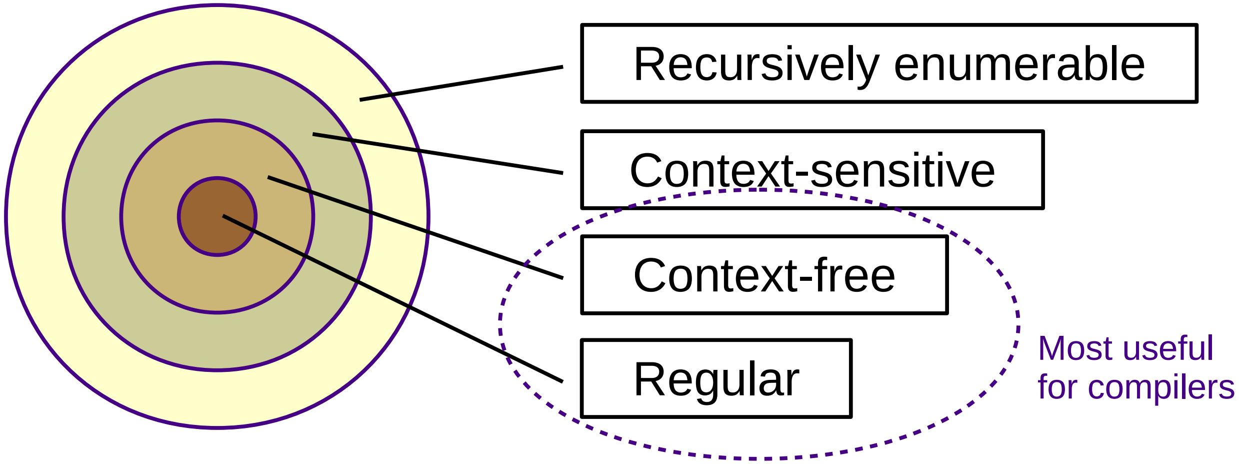
| | |
|---|---|
| xy | xyy |
| xyz | xyyz |
| xyzz | xyyzz |
| xyzzz | xyyzzz |
| (etc.) | |

# Language description

- Ways to describe languages
  - Ad-hoc prose
    - “A single ‘x’ followed by one or two ‘y’s followed by any number of ‘z’s”
  - Formal regular expressions (current focus)
    - x(y|yy)z*
  - Formal grammars (in two weeks)
    - $A \rightarrow x\ B\ C$
    - $B \rightarrow y \mid y\ y$
    - $C \rightarrow z\ C \mid \varepsilon$

# Languages

**Chomsky Hierarchy of Languages**




- Alphabet:
  - Σ = { set of all characters }
- Language:
  - L = { set of sequences of characters from Σ }

# Regular expressions

- Regular expressions describe regular languages
  - Can also be thought of as generalized search patterns

- Three basic recursive operations:
  - Alternation: **a|b** — Lowest precedence
  - Concatenation: **ab**
  - ("Kleene") Closure: **a*** — Highest precedence

**Additionally:** **ε** is a regex that matches the empty string

- Extended constructs:
  - Character sets/classes: **[0-9] ≡ [0...9] ≡ 0|1|2|3|4|5|6|7|8|9**
  - Positive closure: $a^2$ ≡ **aa**    $a^3$ ≡ **aaa**    **a+ ≡ aa***
  - Grouping: **(a|b)c ≡ ac|bc**

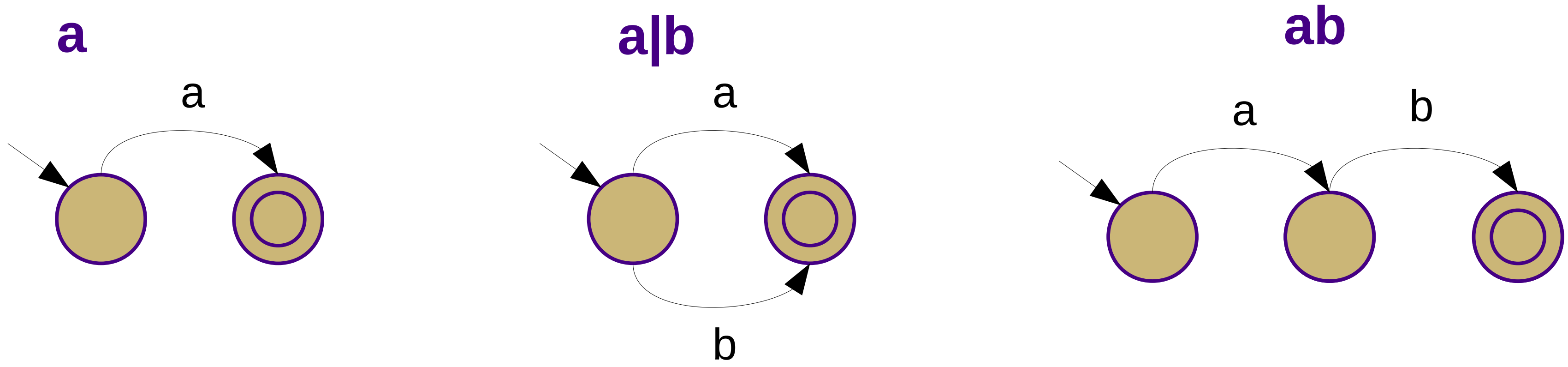
These are not covered extensively in your textbook!

# Discussion question

- How would you implement regular expressions?
  - Given a regular expression and a string, how would you tell whether the string belongs to the language described by the regular expression?

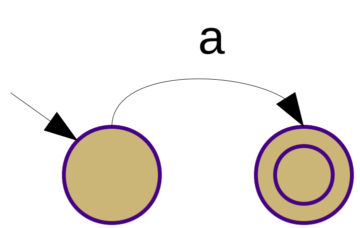
# Lexical Analysis

- Implemented using state machines (finite state automata)
  - Set of states with a single start state
  - Transitions between states on inputs (w/ implicit dead states)
  - Some states are final or accepting

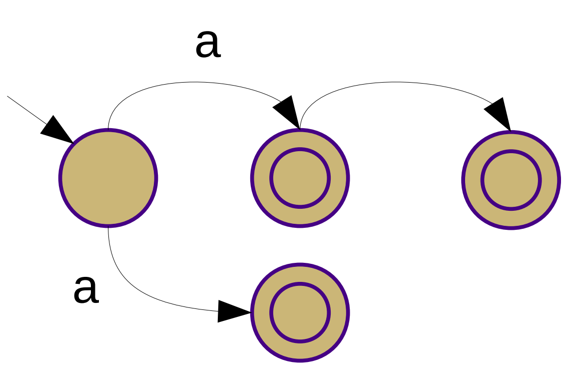


# Lexical Analysis

- Deterministic vs. non-deterministic
  - Non-deterministic: multiple possible states for given sentence
  - One edge from each state per character (deterministic)
  - Multiple edges from each state per character  (non-deterministic)
  - Empty or ε-transitions (non-deterministic)




**Deterministic (DFA)**




**Non-deterministic (NFA)**

# Deterministic finite automata

- Formal definition

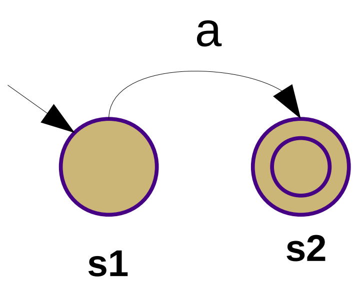
  S: set of states

  Σ: alphabet (set of characters)

  δ: transition function: (S, Σ) → S

  $s_0$: start state

  $S_A$: accepting/final states




S = { s1, s2 }
Σ = { a }
δ = { (s1,a → s2), (s2,a → Ø) }
$s_0$ = s1

$S_A$ = { s2 }

- Acceptance algorithm

  $s := s_0$

  ***for each input*** $c$:

  $\quad s := \delta(s,c)$

  ***return*** $s \in S_A$

Alternative δ representation:

| | a |
|---|---|
| s1 | s2 |
| s2 | Ø |

# Non-deterministic finite automata

- Formal Definition
  - S, Σ, $s_0$, and $S_A$ same as DFA
  - δ: (S, (Σ ∪ {ε})) → [S]
  - ε-closure: all states reachable from s via ε-transitions
    - Formally: ε-closure(s) = {s} ∪ { t ∈ S | (s,ε→t) ∈ δ }
    - Extended to sets by union over all states in set
- Acceptance algorithm

$T := \varepsilon\text{-}closure(s_0)$

***for each input*** *c*:

  $N := \{\}$

  ***for each*** *s* ***in*** *T*:

    $N := N \cup \varepsilon\text{-}closure(\delta(s,c))$

  $T := N$

***return*** $|T \cap S_A| > 0$

# Summary

## DFAs

- S: set of states
- Σ: alphabet (set of characters)
- δ: transition function: (S, Σ) → S
- $s_0$: start state
- $S_A$: accepting/final states

**accept()**:

$s := s_0$

***for each input*** $c$:

$s := \delta(s,c)$

***return*** $s \in S_A$

## NFAs

- δ may return a set of states
- δ may contain ε-transitions
- δ may contain transitions to multiple states on a symbol

**accept()**:

$T := \varepsilon\text{-}closure(s_0)$

***for each input*** $c$:

$N := \{\}$

***for each*** $s$ ***in*** $T$:

$N := N \cup \varepsilon\text{-}closure(\delta(s,c))$

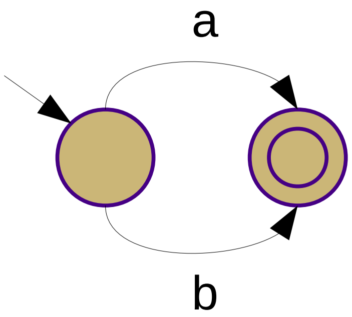
$T := N$

***return*** $|T \cap S_A| > 0$

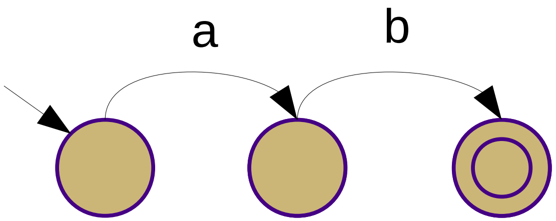
# Lexical Analysis

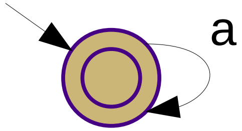
- Examples:

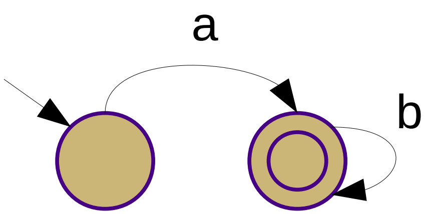
**a|b**




**ab**



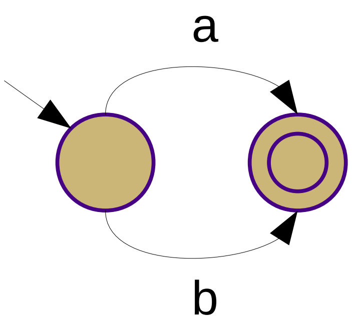



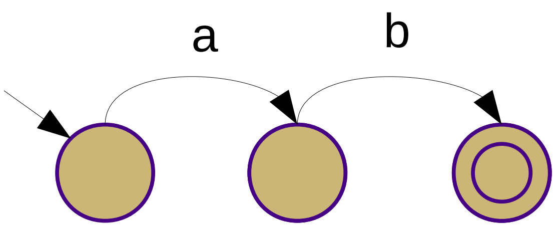
# Lexical Analysis

- Examples:

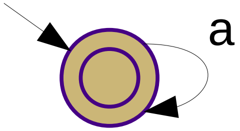
**a|b**




**ab**

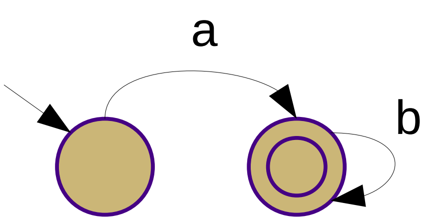



**a***




**aa*|b**

**ab***

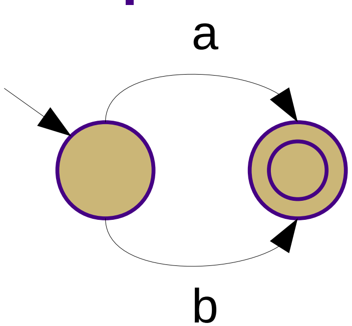



**a(bc|c*)**

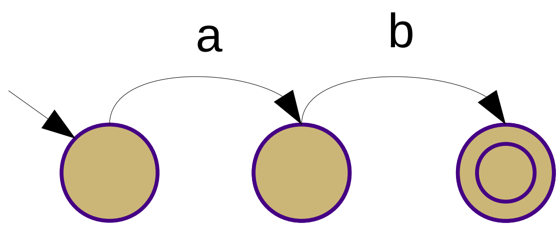
# Lexical Analysis

- Examples:

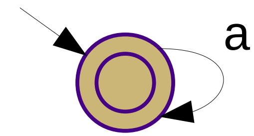
**a|b**



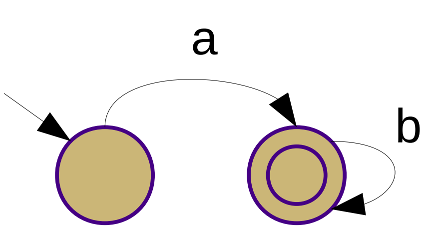

**ab**



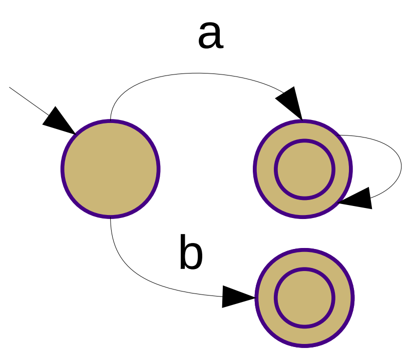

**a***



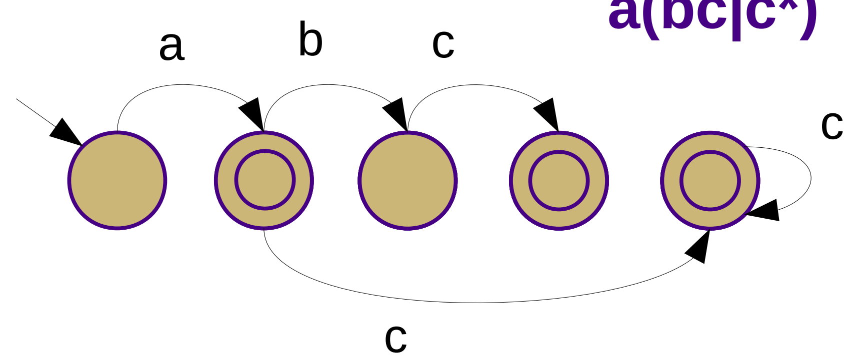

**ab***




**aa*|b**




**a(bc|c*)**



# Equivalence

- A regular expression and a finite automaton are equivalent if they recognize the same language
  - Same applies between different REs and between different FAs
- Regular expressions, NFAs, and DFAs all describe the same set of languages
  - "Regular languages" from Chomsky hierarchy
- Next week, we will learn how to convert between them

# Application

- PA2: Use Java regular expressions to tokenize Decaf files
  - Process the input one line at a time
  - Generally: one regex per token type
  - Each regex begins with “^” (only match from beginning)
  - Prioritize regexes and try each of them in turn
  - When you find a match, extract the matching text
  - Repeat until no match is found or input is consumed
  - Less efficient than an auto-generated lexer
  - However, it is simpler to understand
  - (Our approach to PA3 will be similar)

# Activity

- Construct state machines for the following regular expressions:

**x*yz***     **1(1|0)***     **1(10)***     **(a|b|c)(ab|bc)**

**(dd*.d*)|(d*.dd*)**     ← ε-transitions may make this one slightly easier