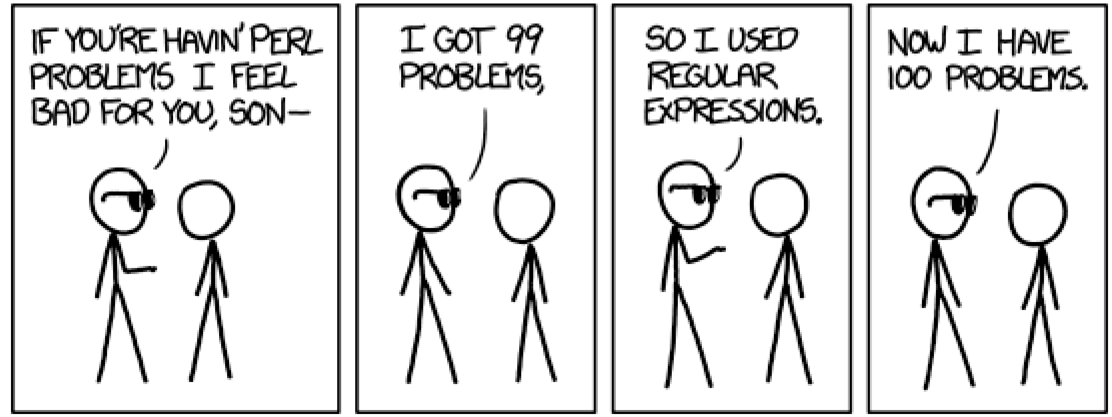
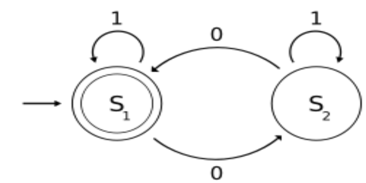
CS 432
Fall 2017

Mike Lam, Professor

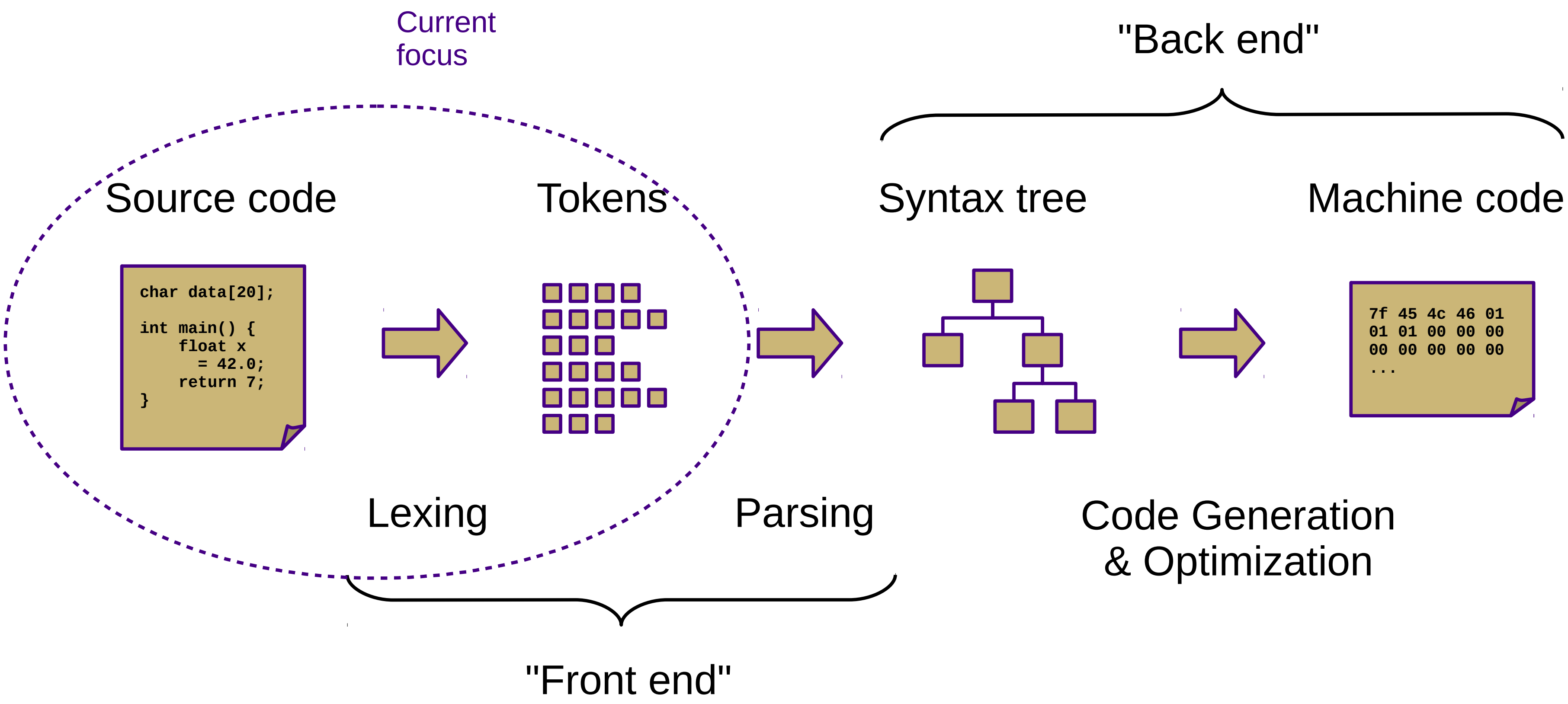


`a|(bc)*`



Regular Expressions
and
Finite Automata

Compilation



Lexical Analysis

- Lexemes or tokens: the smallest building blocks of a language's syntax
- Lexing or scanning: the process of separating a character stream into tokens

```
total = sum(vals) / n

total      identifier
=          equals_op
sum        identifier
(          left_paren
vals       identifier
)          right_paren
/          divide_op
n          identifier
```

```
char *str = "hi";

char       keyword
*          star_op
str        identifier
=          equals_op
"hi"       str_literal
;          semicolon
```

Discussion question

- What is a language?

Language

- A language is "a (potentially infinite) set of strings over a finite alphabet"

Discussion question

- How do we describe languages?

xyy
xy
xyyzzz
xyz
xyzz
xyyzz
xyyz
xyzzz
(etc.)

xy
xyy
xyz
xyyz
xyzz
xyyzz
xyzzz
xyyzzz
(etc.)

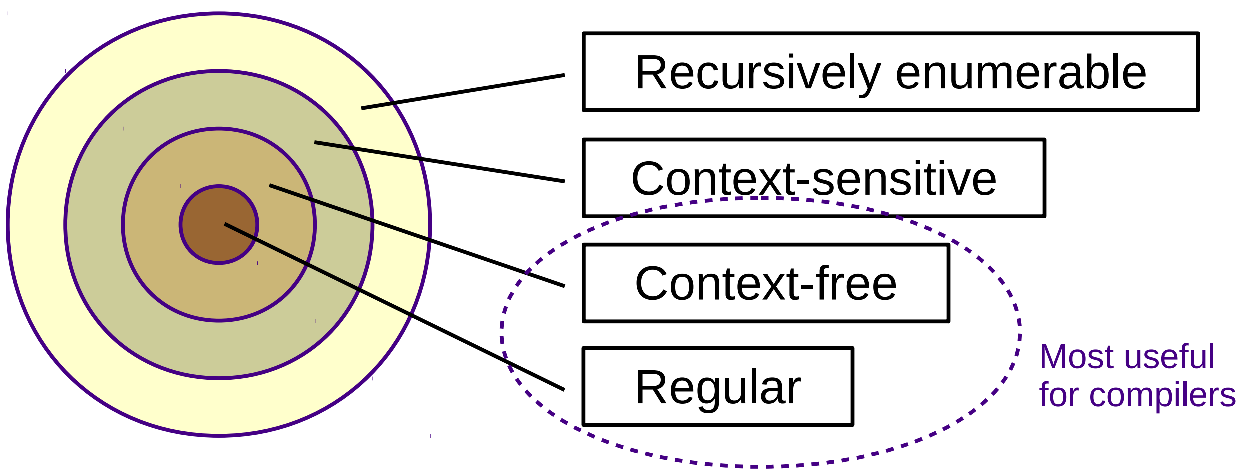
xy	**xyy**
xyz	**xyyz**
xyzz	**xyyzz**
xyzzz	**xyyzzz**
(etc.)	

Language description

- Ways to describe languages
  - Ad-hoc prose
    - “A single ‘x’ followed by one or two ‘y’s followed by any number of ‘z’s”
  - Formal regular expressions (current focus)
    - x(y|yy)z*
  - Formal grammars (in two weeks)
    - A → x B C
    - B → y | y y
    - C → z C | ε

Languages

Chomsky Hierarchy of Languages



- Alphabet:
  - Σ = { set of all characters }
- Language:
  - L = { set of sequences of characters from Σ }

Regular expressions

- Regular expressions describe regular languages
  - Can also be thought of as generalized search patterns

- Three basic recursive operations:
  - Alternation: **a|b**
  - Concatenation: **ab**
  - ("Kleene") Closure: **a***

- Extended constructs:
  - Character sets: **[0-9]** == **0|1|2|3|4|5|6|7|8|9**
  - Grouping: **(a|b)c** == **ac|bc**
  - Positive closure: **a+** == **aa***

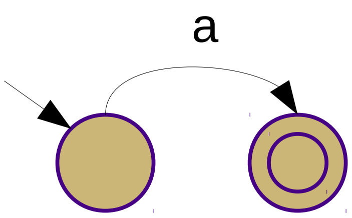
Discussion question

- How would you implement regular expressions?
  - Given a regular expression and a string, how would you tell whether the string belongs to the language described by the regular expression?

Lexical Analysis

- Implemented using state machines (finite state automata)
  - Set of states with a single start state
  - Transitions between states on inputs (w/ implicit dead states)
  - Some states are final or accepting
- Deterministic vs. non-deterministic
  - Non-deterministic: multiple possible states for given sentence
  - One edge from each state per character (deterministic)
  - Multiple edges from each state per character (non-deterministic)
  - Empty or ε-transitions (non-deterministic)

Regex: a



Deterministic finite automata

- Formal definition

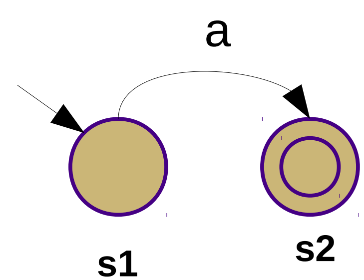
  S: set of states

  Σ: alphabet (set of characters)

  δ: transition function: (S, Σ) → S

  $s_{0:}$ start state

  S_A: accepting/final states



S = { s1, s2 }
Σ = { a }
δ = { (s1,a → s2), (s2,a → Ø) }
$s_{0:}$ = s1

S_A = { s2 }

- Acceptance algorithm

  $s := s_0$

  for each input c:

  $\quad s := \delta(s,c)$

  return $s \in S_A$

Alternative δ representation:

	a
s1	s2
s2	Ø

Non-deterministic finite automata

- Formal definition
  - DFA w/ multiple paths and ε-transitions
  - δ: (S, (Σ ∪ {ε})) -> [S]
  - ε-closure: all states reachable from s via ε-transitions
    - Formally: $\{s\} \cup \{ t \in S \mid (s,\varepsilon \rightarrow t) \in \delta \}$ (extended to sets by union)

- Acceptance algorithm

T := ε-*closure*(s_0)

for each input *c*:

  N := {}

  for each *s* ***in*** *T*:

    N := $N \cup \varepsilon$-*closure*($\delta(s,c)$)

  T := N

return $|T \cap S_A| > 0$

Summary

DFAs

- S: set of states
- Σ: alphabet (set of characters)
- δ: transition function: (S, Σ) → S
- $s_{0:}$ start state
- S_A: accepting/final states

accept():

$s := s_0$

for each input c:

$s := \delta(s,c)$

return $s \in S_A$

NFAs

- δ may contain ε-transitions
- δ may contain transitions to multiple different states on the same symbol

accept():

$T := \varepsilon\text{-}closure(s_0)$

for each input c:

$N := \{\}$

for each s ***in*** T:

$N := N \cup \varepsilon\text{-}closure(\delta(s,c))$

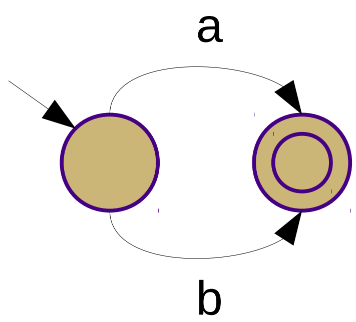
$T := N$

return $|T \cap S_A| > 0$

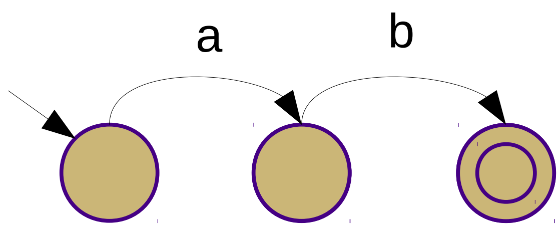
Lexical Analysis

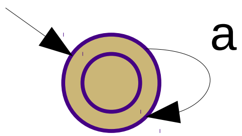
- Examples:

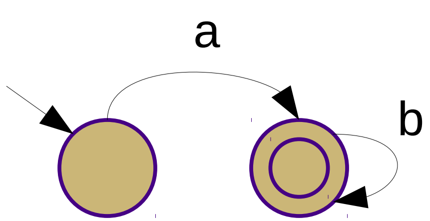
a|b



ab



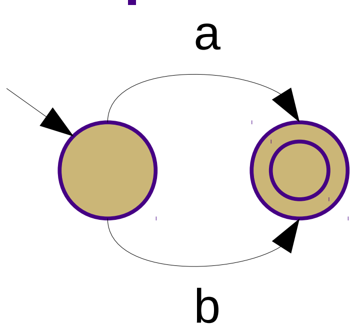




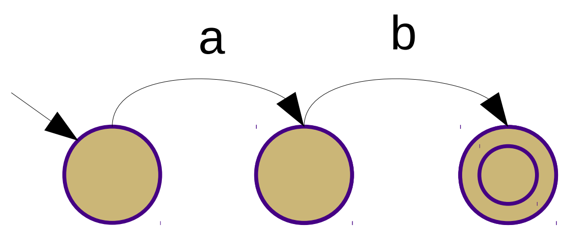
Lexical Analysis

- Examples:

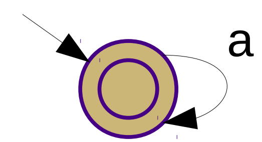
a|b



ab

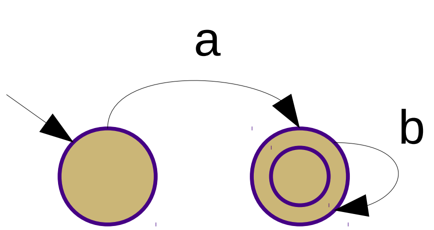


a*



aa*|b

ab*

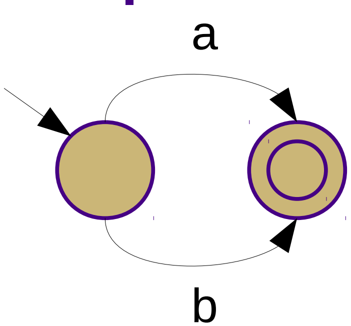


a(bc|c*)

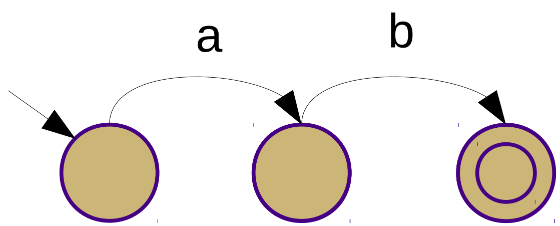
Lexical Analysis

- Examples:

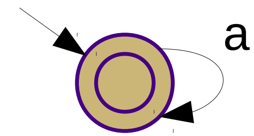
a|b



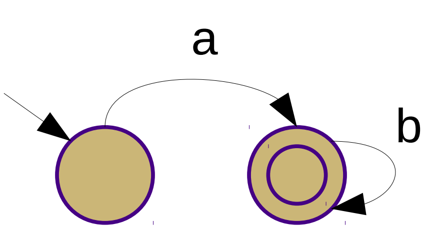
ab



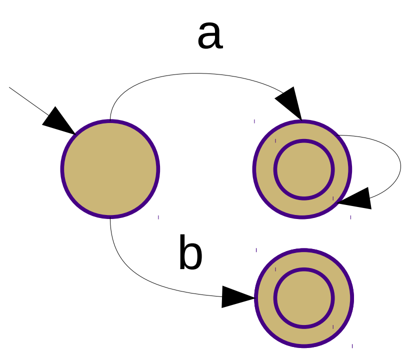
a*



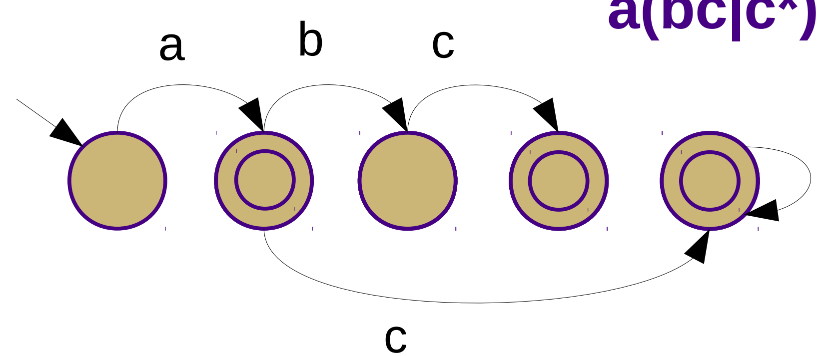
ab*



aa*|b



a(bc|c*)



Equivalence

- A regular expression and a finite automaton are equivalent if they recognize the same language
  - Same applies between different REs and between different FAs
- Regular expressions, NFAs, and DFAs all describe the same set of languages
  - "Regular languages" from Chomsky hierarchy
- Next week, we will learn how to convert between them

Application

- PA2: Use Java regular expressions to tokenize Decaf files
  - Process the input one line at a time
  - Generally: one regex per token type
  - Each regex begins with “^” (only match from beginning)
  - Prioritize regexes and try each of them in turn
  - When you find a match, extract the matching text
  - Repeat until no match is found or input is consumed
  - Less efficient than an auto-generated lexer
  - However, it is simpler to understand
  - (Our approach to PA3 will be similar)

Activity

- Construct state machines for the following regular expressions:

x*yz* **1(1|0)*** **1(10)*** **(a|b|c)(ab|bc)**

(dd*.d*)|(d*.dd*) ← ε-transitions may make this one slightly easier