Integer Encodings
Integers

• Topics
  – C integer data types
  – Unsigned encoding
  – Signed encodings
  – Conversions
## Integer data types in C99

<table>
<thead>
<tr>
<th>C data type</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>[signed] char</td>
<td>−127</td>
<td>127</td>
<td>1 byte</td>
</tr>
<tr>
<td>unsigned char</td>
<td>0</td>
<td>255</td>
<td></td>
</tr>
<tr>
<td>short</td>
<td>−32,767</td>
<td>32,767</td>
<td>2 bytes</td>
</tr>
<tr>
<td>unsigned short</td>
<td>0</td>
<td>65,535</td>
<td></td>
</tr>
<tr>
<td>int</td>
<td>−32,767</td>
<td>32,767</td>
<td>2 bytes</td>
</tr>
<tr>
<td>unsigned</td>
<td>0</td>
<td>65,535</td>
<td></td>
</tr>
<tr>
<td>long</td>
<td>−2,147,483,647</td>
<td>2,147,483,647</td>
<td>4 bytes</td>
</tr>
<tr>
<td>unsigned long</td>
<td>0</td>
<td>4,294,967,295</td>
<td></td>
</tr>
<tr>
<td>int32_t</td>
<td>−2,147,483,648</td>
<td>2,147,483,647</td>
<td>4 bytes</td>
</tr>
<tr>
<td>uint32_t</td>
<td>0</td>
<td>4,294,967,295</td>
<td></td>
</tr>
<tr>
<td>int64_t</td>
<td>−9,223,372,036,854,775,808</td>
<td>9,223,372,036,854,775,807</td>
<td>8 bytes</td>
</tr>
<tr>
<td>uint64_t</td>
<td>0</td>
<td>18,446,744,073,709,551,615</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 2.11** Guaranteed ranges for C integral data types. The C standards require that the data types have at least these ranges of values.
## Integer data types on stu

All sizes in bytes; sizes in red are larger than mandated by C99

<table>
<thead>
<tr>
<th>Type</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>char</td>
<td>1</td>
</tr>
<tr>
<td>unsigned char</td>
<td>1</td>
</tr>
<tr>
<td>short</td>
<td>2</td>
</tr>
<tr>
<td>unsigned short</td>
<td>2</td>
</tr>
<tr>
<td>int</td>
<td>4</td>
</tr>
<tr>
<td>unsigned int</td>
<td>4</td>
</tr>
<tr>
<td>long</td>
<td>8</td>
</tr>
<tr>
<td>unsigned long</td>
<td>8</td>
</tr>
<tr>
<td>long long</td>
<td>8</td>
</tr>
<tr>
<td>unsigned long long</td>
<td>8</td>
</tr>
<tr>
<td>int8_t</td>
<td>1</td>
</tr>
<tr>
<td>uint8_t</td>
<td>1</td>
</tr>
<tr>
<td>bool</td>
<td>1</td>
</tr>
<tr>
<td>int16_t</td>
<td>2</td>
</tr>
<tr>
<td>uint16_t</td>
<td>2</td>
</tr>
<tr>
<td>int32_t</td>
<td>4</td>
</tr>
<tr>
<td>uint32_t</td>
<td>4</td>
</tr>
<tr>
<td>int64_t</td>
<td>8</td>
</tr>
<tr>
<td>uint64_t</td>
<td>8</td>
</tr>
<tr>
<td>size_t</td>
<td>8</td>
</tr>
</tbody>
</table>
Unsigned integer encoding

• Bit $i$ represents the value $2^i$
  - Bits typically written from most to least significant (i.e., $2^3 \cdot 2^2 \cdot 2^1 \cdot 2^0$)
  - This is the same encoding we saw last time!
  - No representation of negative numbers

1 = $1 = 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = [0001]$
5 = $4 + 1 = 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = [0101]$
11 = $8 + 2 + 1 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = [1011]$
15 = $8 + 4 + 2 + 1 = 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = [1111]$
Unsigned integer encoding

- Textbook’s notation
  - Each bar represents a bit
  - Add together bars to represent the contributions of each bit value to the overall value

Figure 2.12
Unsigned number examples for $w = 4$.
When bit $i$ in the binary representation has value 1, it contributes $2^i$ to the value.
Signed integer encodings

• Sign magnitude
  – Most natural/intuitive but hardest to implement

• Ones’ complement
  – Cleaner arithmetic but less intuitive

• Two’s complement
  – Cleanest arithmetic but most complicated
  – Most modern signed integer types use this!
Sign magnitude

- Interpret most-significant bit as a sign bit
- Interpret remaining bits as unsigned number \( x \) (the magnitude)
  - If negative, absolute value is \( x \)
- To negate: **flip the sign bit**
- Disadvantages:
  - Two zeros: \(-0\) and \(+0\) [1000 and 0000]
  - Less useful for arithmetic because the sign bit has no relationship with the magnitude--cannot use unsigned arithmetic logic!

\[
\begin{array}{ccc}
0 & 011 & = 3 \\
1 & 011 & = -3 \\
0 & 111 & = 7 \\
? & 010 &
\end{array}
\]
Question

What is the negation of 10110 in sign magnitude?

A) 10110
B) 10111
C) 01001
D) 01011
E) 01010
F) 00110
Question

• Which of the following are negative numbers if interpreted as a sign magnitude integer?
  - A) 10110
  - B) 10111
  - C) 01001
  - D) 01011
  - E) 01010
  - F) 00110
Ones’ complement

- Interpret most-significant bit as a sign bit
- Interpret ALL bits as unsigned integer \( x \)
  - If negative, absolute value is \([1111\ldots1] - x\)
- To negate: **flip all the bits** (binary NOT)
- Disadvantages:
  - Still have two representations of zero (1111 and 0000)
  - Also, less useful for arithmetic than two’s complement
    - Must “end-around carry” to preserve results

\[
\begin{align*}
0 \ 011 & = 3 \\
1 \ 100 & = -3 \\
0 \ 111 & = 7 \\
1 \ \ \ 111 \ (7) & +1 \ (end-around \ carry) \\
\hline
1 \ 100 & \ ( -3 ) \\
0 \ 100 & \ \\
\end{align*}
\]
• What is the negation of 10110 in ones’ complement?
  - A) 10110
  - B) 10111
  - C) 01001
  - D) 01011
  - E) 01010
  - F) 00110
Which of the following are negative numbers if interpreted as a ones’ complement integer?

- A) 10110
- B) 10111
- C) 01001
- D) 01011
- E) 01010
- F) 00110
Two’s complement

- Interpret most-significant bit as a sign bit
- Interpret ALL bits as unsigned integer $x$
  - If negative, absolute value is $2^N - x$ where $N$ is the number of bits
- To negate: **subtract value from $2^N$** where $N$ is the number of bits
- One zero; positive numbers wrap to negative ones halfway through

<table>
<thead>
<tr>
<th>2’s Comp.</th>
<th>Unsigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1111</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>-7</td>
<td>1001</td>
</tr>
<tr>
<td>-8</td>
<td>1000</td>
</tr>
<tr>
<td>7</td>
<td>0111</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0001</td>
</tr>
<tr>
<td>0</td>
<td>0000</td>
</tr>
</tbody>
</table>
Two’s complement advantage: uses unsigned arithmetic logic

- (ignore carries out of the sign bit for now)

- Ex: $5 - 3 = 5 + (-3) = 0101 + 1101 = 0010 \ (2)$
- Ex: $1 - 3 = 1 + (-3) = 0001 + 1101 = 1110 \ (-2)$
- Ex: $-2 - 3 = (-2) + (-3) = 1110 + 1101 = 1011 \ (-5)$

\[
\begin{array}{l}
0011 = 3 \\
1100 \\
1101 = -3 \\
0111 = 7 \\
\end{array}
\]

\[
\begin{array}{l}
0111 \ (7) \\
1101 \ (-3) \\
0100 \ (4) \\
\end{array}
\]
Two’s complement

- Alternate interpretation: value of most significant bit is negated
  - i.e., start at most negative number and build back up towards zero

Figure 2.12
Unsigned number examples for $w = 4$.
When bit $i$ in the binary representation has value 1, it contributes $2^i$ to the value.

Figure 2.16
Comparing unsigned and two’s-complement representations for $w = 4$. The weight of the most significant bit is $-8$ for two’s complement and $+8$ for unsigned, yielding a net difference of 16.
Two’s complement trick

• Alternate way to negate in two’s complement
  – **Flip the bits** (binary NOT) then **add one**

Ex: 5 = 0101 → (binary NOT) → 1010 → (add one) → 1011 = -5 (-8 + 2 + 1)

*Aside*: Why does this work? The sum of a number $x$ and $\sim x$ is all ones (or $2^N-1$ where $N$ is the number of bits), so $\sim x$ can be expressed as $2^N-1 - x$. Because negating $x$ in two’s complement is equivalent to subtracting $x$ from $2^N$, if we add one to $\sim x$ the results are equal:

$\sim x + 1 = (2^N-1 - x) + 1 = 2^N - x$
Question

- What is the negation of 10110 in two’s complement?
  - A) 10110
  - B) 10111
  - C) 01001
  - D) 01011
  - E) 01010
  - F) 00110
Which of the following are negative numbers if interpreted as a two’s complement integer?

- A) 10110
- B) 10111
- C) 01001
- D) 01011
- E) 01010
- F) 00110
Ones’ vs. Two’s

- **Ones’ complement**
  - Interpret all bits as unsigned integer $x$
    - Value is $[11111\ldots1] - x$
    - I.e., the complement with respect to ones

- **Two’s complement**
  - Interpret all bits as unsigned integer $x$
    - Value is $2^N - x$ where $N$ is the number of bits
    - I.e., the complement with respect to a power of two
• **Ones’ complement** and **two’s complement** are both an **operation** and an **encoding**
  - E.g., “perform two’s complement” vs “the number is stored in two’s complement”
• The operation represents the action necessary to **negate** a number in that encoding.
  - E.g., performing two’s complement (ones’ complement and add one) negates a number in two’s complement encoding
• If you have a value in a particular encoding:
  - If the sign bit is not set, it’s a positive number
  - If it is set, perform the operation to recover the positive value

**We will avoid using the operation terminology in this course!**
Integer encodings

• Information = Bits + Context
  – What does “1011” mean? It depends!

  Unsigned: 11
  Sign magnitude: -3
  Ones' complement: -4
  Two's complement: -5
**Comparison**

- We’ll see one more signed integer encoding next week: “offset binary” / “biased” / “excess”
  - For now, here’s a comparison (for 1-byte integers):

<table>
<thead>
<tr>
<th>Binary</th>
<th>Unsigned</th>
<th>Sign Mag</th>
<th>Ones’ C</th>
<th>Two’s C</th>
<th>Offset-127</th>
</tr>
</thead>
<tbody>
<tr>
<td>1111 1111</td>
<td>255</td>
<td>-127</td>
<td>-0</td>
<td>-1</td>
<td>128</td>
</tr>
<tr>
<td>1111 1110</td>
<td>254</td>
<td>-126</td>
<td>-1</td>
<td>-2</td>
<td>127</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>1000 0001</td>
<td>129</td>
<td>-1</td>
<td>-126</td>
<td>-127</td>
<td>2</td>
</tr>
<tr>
<td>1000 0000</td>
<td>128</td>
<td>-0</td>
<td>-127</td>
<td>-128</td>
<td>1</td>
</tr>
<tr>
<td>0111 1111</td>
<td>127</td>
<td>127</td>
<td>127</td>
<td>127</td>
<td>0</td>
</tr>
<tr>
<td>0111 1110</td>
<td>126</td>
<td>126</td>
<td>126</td>
<td>126</td>
<td>-1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>0000 0001</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-126</td>
</tr>
<tr>
<td>0000 0000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-127</td>
</tr>
</tbody>
</table>
Question

Which of the following are guaranteed to be “safe” (i.e., the value will always be preserved)?

- A) Smaller unsigned → larger unsigned
- B) Smaller two’s comp. → larger two’s comp.
- C) Larger → smaller (unsigned or two’s comp.)
- D) Unsigned → two’s comp.
- E) Two’s comp. → unsigned
Conversions

- Smaller unsigned → larger unsigned
  - Safe; zero-extend to preserve value

- Smaller two’s comp. → larger two’s comp.
  - Safe; sign-extend to preserve value

- Larger → smaller (unsigned or two’s comp.)
  - Overflow if new type isn’t large enough to fit (truncate)

- Unsigned → two’s comp.
  - Overflow if first bit is non-zero (otherwise, no change)

- Two’s comp. → unsigned
  - Overflow if value is negative (otherwise, no change)