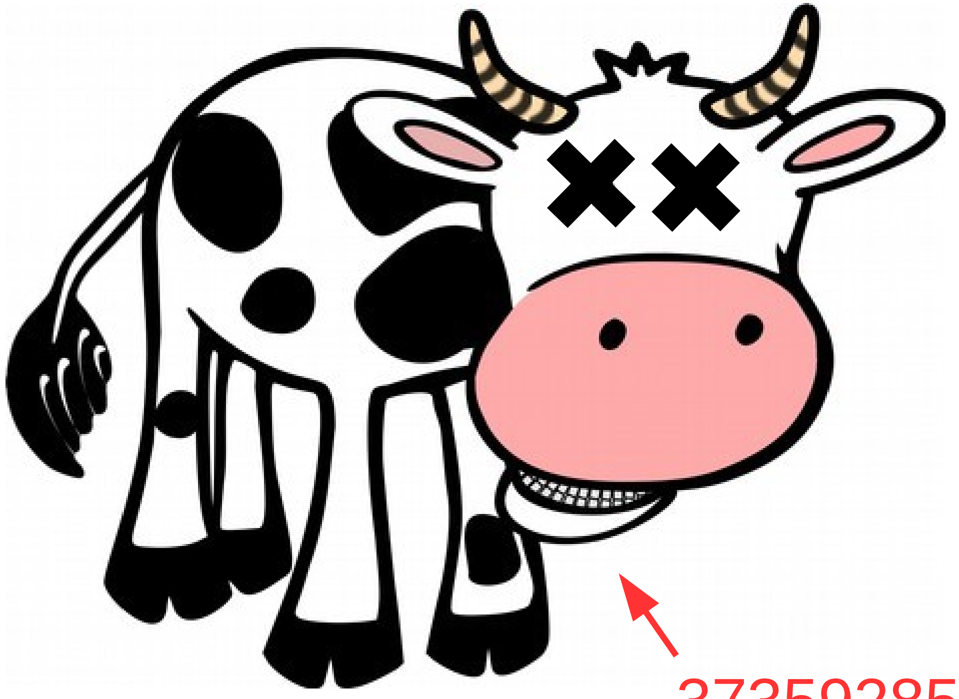
CS 261
Fall 2021

Mike Lam, Professor



Binary Information

Binary information

- Topics
  - Base conversions (bin/dec/hex)
  - Data sizes
  - Byte ordering
  - Character and program encodings
  - Bitwise operations

Core theme

What does this mean?

IOO

Core theme

Information = Bits + Context

Why binary?

- Computers store information in binary encodings
  - 1 bit is the simplest form of information (on / off)
  - Minimizes storage and transmission errors
- To store more complicated information, use more bits
  - However, we need **context** to understand them
  - Data encodings provide context
  - For the next two weeks, we will study encodings
  - First, let’s become comfortable working with binary

Base conversions

- Binary encoding is base-2: bit *i* represents the value 2^i
  - Bits typically written from most to least significant (i.e., $2^3\ 2^2\ 2^1\ 2^0$)

$1 = 1$	$= 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$	= [0001]	1-**1**=0
$5 = 4 + 1$	$= 0 \cdot 2^3 + 1 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$	= [0101]	5-**4**=1 1-**1**=0
$11 = 8 + 2 + 1$	$= 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$	= [1011]	11-**8**=3 3-**2**=1 1-**1**=0
$15 = 8 + 4 + 2 + 1$	$= 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$	= [1111]	15-**8**=7 7-**4**=3 3-**2**=1 1-**1**=0

Binary to decimal:

Add up all the powers of two (memorize powers of two to make this go faster!)

Decimal to binary:

Find highest power of two and subtract to find the remainder
Repeat above until the remainder is zero
Every power of two become 1; all other bits are 0

Remainder system

- Quick method for decimal → binary conversions
  - Repeatedly divide decimal number by two until zero, keeping track of remainders (either 0 or 1)
  - Read in reverse to get binary equivalent

```
11
5   r   1
2   r   1       =>     1011      (8 + 2 + 1)
1   r   0
0   r   1
```

Question

- What is the decimal number 25 when represented in binary?

25					
12	r	1			
6	r	0	=>	11001	(16 + 8 + 1)
3	r	0			
1	r	1			
0	r	1			

Base conversions

- Hexadecimal encoding is base-16 (usually prefixed with “0x”)
  - Converting between hex and binary is easy
    - Each digit represents 4 bits; just substitute digit-by-digit or in groups of four!
  - You should memorize (at least some of) these equivalences

Dec	Bin	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7

Dec	Bin	Hex
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Base conversions

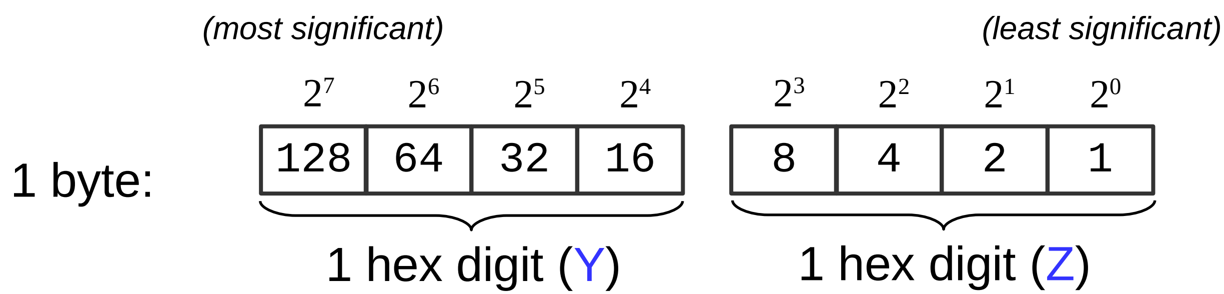
- Examples:
  - 0x4CA <=> 0100 1100 1010
  - 0x5F0 <=> 0101 1111 0000

Dec	Bin	Hex
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7

Dec	Bin	Hex
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Fundamental data sizes

- 1 byte = 2 hex digits (= *2 nibbles!*) = **8 bits**



Value of byte `0xYZ` is 16·Y + Z

- Machine word = size of an address
  - (i.e., the size of a pointer in C)
  - Early computers used 16-bit addresses
    - Could address 2^{16} bytes = 64 KB
  - Now 32-bit (4 bytes) or 64-bit (8 bytes)
    - Can address 4GB or 16 EB

Prefix	Bin	Dec
Kilo	2^{10}	$\sim 10^3$
Mega	2^{20}	$\sim 10^6$
Giga	2^{30}	$\sim 10^9$
Tera	2^{40}	$\sim 10^{12}$
Peta	2^{50}	$\sim 10^{15}$
Exa	2^{60}	$\sim 10^{18}$

Byte ordering

- Big endian: store **higher** place values at lower addresses
  - Most-significant byte (MSB) to least-significant byte (LSB)
  - Similar to standard way to write hex (implied with "0x" prefix)
- Little endian: store **lower** place values at lower addresses
  - Least-significant byte (LSB) to most-significant byte (MSB)
  - Default byte ordering on most Intel-based machines

```
                                     low            high
                                     addr           addr

0x11223344 in big endian:            11   22   33   44
0x11223344 in little endian:         44   33   22   11
```

Byte ordering examples

- Big endian: most significant byte first (MSB to LSB)
- Little endian: least significant byte first (LSB to MSB)

```
                                   low       high
0x11223344 in big endian:    11 22 33 44
0x11223344 in little endian: 44 33 22 11

Decimal: 1
16-bit big endian:     00000000 00000001  (hex: 00 01)
16-bit little endian:  00000001 00000000  (hex: 01 00)

Decimal: 19 (16+2+1)
16-bit big endian:     00000000 00010011  (hex: 00 13)
16-bit little endian:  00010011 00000000  (hex: 13 00)

Decimal: 256
16-bit big endian:     00000001 00000000  (hex: 01 00)
16-bit little endian:  00000000 00000001  (hex: 00 01)
```

Question

- What is the byte in the highest address when hexadecimal number 0x8345 is stored in little-endian ordering?
  - A) 0x83
  - B) 0x45
  - C) 0x34
  - D) 0x85
  - E) There is not enough information to tell.

Character encodings

- ASCII ("American Standard Code for Information Interchange")
  - 1-byte code developed in 1960s
  - Limited support for non-English characters
- Unicode
  - Multi-byte code developed in 1990s
  - "All the characters for all the writing systems of the world"
  - Over 136,000 characters in latest standard
  - Fixed-width (UTF-16 and UTF-32) and variable-width (UTF-8)

UTF-8

Number of bytes	Bits for code point	First code point	Last code point	Byte 1	Byte 2	Byte 3	Byte 4
1	7	U+0000	U+007F	0xxxxxxx			
2	11	U+0080	U+07FF	110xxxxx	10xxxxxx		
3	16	U+0800	U+FFFF	1110xxxx	10xxxxxx	10xxxxxx	
4	21	U+10000	U+10FFFF	11110xxx	10xxxxxx	10xxxxxx	10xxxxxx

Program encodings

- Machine code
  - Binary encoding of **opcodes** and operands
  - Specific to a particular CPU architecture (e.g., x86_64)

```
int add (int num1, int num2)
{
    return num1 + num2;
}
```

```
0000000000400606 <add>:
  400606:       55                      push   %rbp
  400607:       48 89 e5                mov    %rsp,%rbp
  40060a:       89 7d fc                mov    %edi,-0x4(%rbp)
  40060d:       89 75 f8                mov    %esi,-0x8(%rbp)
  400610:       8b 55 fc                mov    -0x4(%rbp),%edx
  400613:       8b 45 f8                mov    -0x8(%rbp),%eax
  400616:       01 d0                   add    %edx,%eax
  400618:       5d                      pop    %rbp
  400619:       c3                      retq
```

Bitwise operations

- Basic bitwise operations

  `&` (and) `|` (or) `^` (xor)

- Not boolean algebra!

  `&&` (and) `||` (or) `!` (not)

  `0` (false) non-zero (true)

- Important properties:

```
x & 0 = 0
x & 1 = x
x | 0 = x
x | 1 = 1
x ^ 0 = x
x ^ 1 = ~x
x ^ x = 0
```

- Commutative:

```
x & y = y & x
x | y = y | x
x ^ y = y ^ x
```

- Associative:

```
(x & y) & z = x & (y & z)
(x | y) | z = x | (y | z)
(x ^ y) ^ z = x ^ (y ^ z)
```

- Distributive:

```
x & (y | z) = (x & y) | (x & z)
x | (y & z) = (x | y) & (x | z)
```

&	0	1
0	0	0
1	0	1

AND

|	0	1
0	0	1
1	1	1

OR

^	0	1
0	0	1
1	1	0

XOR

Bitwise operations

- Bitwise complement (~) - "flip the bits"
  - `~0000 = 1111  (~0 = 1)     ~1010 = 0101  (~0xA = 0x5)`

- Left shift (<<) and right shift (>>)
  - Equivalent to multiplying (<<) or dividing (>>) by two
  - Left shift: `0110 << 1 = 1100`   `1 << 3` = 8
  - Logical right shift (fill zeroes): `1100 >> 2 = 0011`
  - Arithmetic right shift (fill most sig. bit): `1100 >> 2 = 1111`
    `0100 >> 2 = 0001`

On stu:

```
 int: 0f000000 >> 8 = 000f0000   (arithmetic, for signed integers)
 int: ff000000 >> 8 = ffff0000
uint: 0f000000 >> 8 = 000f0000   (logical, for unsigned integers)
uint: ff000000 >> 8 = 00ff0000
```

Masking

- Bitwise operations can extract parts of a binary value
  - This is referred to as masking; specify a bit pattern mask to indicate which bits you want
    - Helpful fact: 0xF is all 1's in binary!
  - Use a bitwise AND (&) with the mask to extract the bits
  - Use a bitwise complement (~) to invert a mask
  - Example: To extract the lower-order 16 bits of a larger value v, use "`v & 0xFFFF`"

```
0xDEADBEEF &      0xFFFF = 0x0000BEEF = 0xBEEF
0xDEADBEEF &  0x0000FFFF = 0x0000BEEF = 0xBEEF
0xDEADBEEF &  0xFFFF0000 = 0xDEAD0000
0xDEADBEEF &     ~0xFFFF = 0xDEAD0000
0xDEADBEEF & ~0x0000FFFF = 0xDEAD0000
```